

Chapter 9 - Momentum

Summary - Momentum Is a measure of the quantity of motion that an object has; it takes into account both how fast the object is moving, and its mass. It characterizes the “quantity of motion” of an object.

Important Formulae	Uses
$p = mv$	Equation to calculate momentum
$F = dp/dt$	Equation relating force to change in momentum
$J = \int_{t_i}^{t_f} F dt = \Delta p$	Ways to calculate impulse/change in momentum
$p_1 + p_2 = p_1' + p_2'$	Law of conservation of momentum
$cm = \frac{\sum m_i x_i}{\sum m_i}$	Center of mass
$\sum p_{x1} + \sum F \Delta t = \sum p_{x2}$	Modified conservation of momentum

Key Points

linear momentum:

- Quantity of motion
- Caused by a force applied over time

impulse

- Amount of force applied over a given time
- A change in momentum

Conservation of Momentum

- Similarly to conservation of energy, this states that when two or more particles interact without outside forces, their total linear momentum stays the same
- $p_1 + p_2 = p_1' + p_2'$

Types of Collisions

- Inelastic collisions
 - Collision where some energy is lost to heat
- Perfectly inelastic collision
 - An inelastic collision where the objects stick together after collision
- Elastic collision
 - A collision where KE is conserved

FRQ #1

A 10.0 kg gun with an 80.0 cm long barrel fires a 130-gram bullet to the right with a velocity of +400 m/s.

1. Calculate the acceleration of the bullet while in the barrel of the gun.
 - a. The bullet starts out with an initial velocity of zero, but then after being fired, exits the gun with a velocity of +400 m/s (to the right). Using kinematics to find the acceleration:

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$a = \frac{v_f^2 - v_i^2}{2\Delta x}, a = 1.0 \times 10^5 \text{ m/s}^2$$

2. Calculate the time over which the bullet accelerated.

$$v_f = v_i + a\Delta t$$

$$\Delta t = \frac{v_f - v_i}{a}$$
$$\Delta t = \frac{400 \text{ m/s} - 0 \text{ m/s}}{1.0 \times 10^5 \text{ m/s}^2}$$
$$\Delta t = 0.0040 \text{ s}$$

3. Calculate the average Force, magnitude and direction, applied by the gun to the bullet.

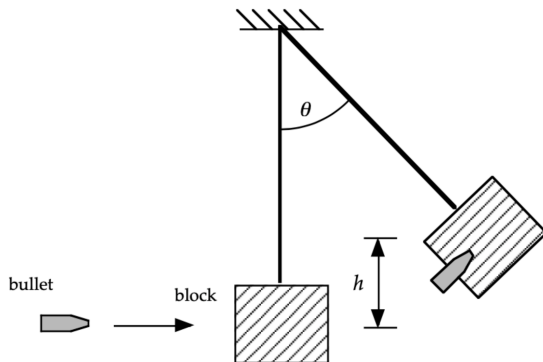
$$\overline{F_{net}} = ma \quad \overline{F_{net}} = (0.130 \text{ kg})(1.0 \times 10^5 \text{ m/s}^2)$$
$$F_{net} = 13000 \text{ N}$$

4. Calculate the impulse on the bullet from the gun, magnitude and direction.

$$J = F_{avg} \Delta t$$
$$J = (+13000N)(0.0040s) = 52N \cdot s$$

$$J = \Delta p$$
$$J = m(v_f - v_i)$$
$$J = (0.130kg)(400m/s - 0) = 52kg \cdot m/s$$

FRQ #2



A ballistic pendulum is used to measure the speed of a projectile: a 5.0 gram bullet is fired into a 1.0 kg block of wood. The bullet sticks into the wood, and the bullet-block swings up to a height of 5.0 cm.

1. Find the initial speed of the projectile just before it hits the block.
 - a. A pendulum swinging up is a conservation of energy problem, and I have enough information to be able to solve that. So I'll do that first, and then work my way backwards from there.

$$K_{initial} = U_{final}$$
$$\frac{1}{2}m_{bullet+bl}v'^2 = m_{bullet+bl}gh$$

- b. $v' = \sqrt{2gh}$
- c. This "initial velocity" for the energy part of the problem is the same as the "final velocity" from the collision part of the problem. So:

$$m_{bullet}v_{bullet} + 0 = (m_{bullet} + m_{bl})v'$$

$$v' = \sqrt{2gh}$$

$$m_{bullet}v_{bullet} = (m_{bullet} + m_{bl})\sqrt{2gh}$$

$$v_{bullet} = \frac{(m_{bullet} + m_{bl})\sqrt{2gh}}{m_{bullet}}$$

$$v_{bullet} = \frac{(0.005 + 1.00)\sqrt{2(9.8)(0.05)}}{0.005}$$

$$v_{bullet} = 199m/s$$

2. Find the energy lost in the collision between the block and the projectile.
- a. We now know the masses and velocities of the bullet and block both before and after the collision, so we should be able to use those with a conservation of energy equation to figure out what the ΔE_{int} was.

$$K_{i,bull} + K_{i,bl} = K_{f,bull+bl} + \Delta E_{int}$$

$$\frac{1}{2}mv_{i,bull}^2 + 0 = \frac{1}{2}(m_{bullet} + m_{bl})v_f^2 + \Delta E_{int}$$

$$\Delta E_{int} = \frac{1}{2}mv_{i,bull}^2 - \frac{1}{2}(m_{bullet} + m_{bl})v_f^2$$

$$\Delta E_{int} = \frac{1}{2}(0.005)(199)^2 - \frac{1}{2}(1.005)(0.99)^2$$

$$\Delta E_{int} = 98.5J$$

FRQ #3

Calculate X_{cm} and Y_{cm} for the system to the right

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{12(3) + 24(0) + 36(7)}{12 + 24 + 36}$$

$$X_{cm} = 4$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{12(0) + 24(2) + 36(5)}{12 + 24 + 36}$$

$$Y_{cm} = 3.17$$

