Lab: AP Review Sheets

Chapter 13: Universal Gravitation

By Chase Engstrom

Background: The universal gravitation unit explores Newton's Law of Gravitation, Kepler's Laws, orbital energy, angular momentum, and the relationships between gravitational force, gravitational energy, mass, and distance.

Key Points

1. There exists a force between any two masses that is proportional to the product of the masses and inversely proportional to the distance between the center of the two masses.

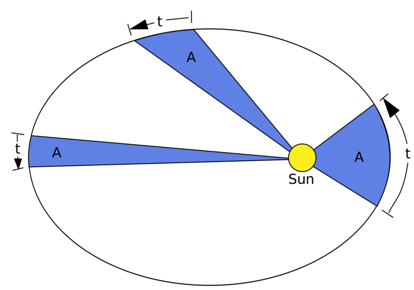
2. Planets move in elliptical orbits with the Sun at one focal point (Kepler's First Law).

3. Planetary motion adheres to conservation of angular momentum: $\vec{r_1} \times m\vec{v_1} = \vec{r_2} \times m\vec{v_2}$

4. The total amount of mechanical energy in a circular orbit system equals half the potential energy in the system: $E_{tot} = \frac{1}{2}U$

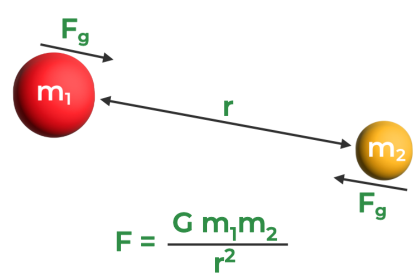
5. The square of a planet's period T^2 is directly proportional to the cube of its radius r^3, so $T^2 = \frac{4\pi^2}{GM_s}r^3$.

Kepler's Second Law

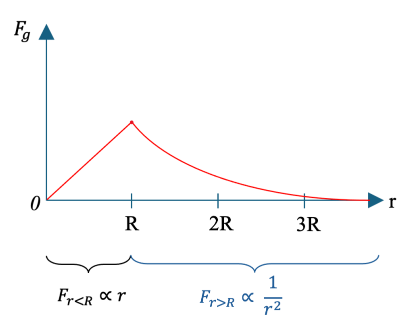


A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.

Newton's Law of Gravitation



Gravitational Force vs Distance from Earth



Equations

$$|\vec{F_g}| = \frac{Gm_1m_2}{r^2}$$

$$U_g = -\frac{Gm_1m_2}{r}$$

$$\Sigma KE = \frac{1}{2}mv^2$$

$$|\vec{F_g}| = \frac{mv^2}{r}$$

$$\vec{L} = \vec{r} \times m\vec{v}$$

$G = 6.67 \times 10^{-11} N * m^2/kg^2$

$r_{earth} = 6.37 \times 10^6\ m$

$m_{earth} = 5.98 \times 10^{24}\ kg$

Problem Set

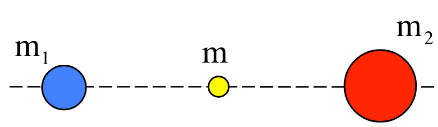
1. [Easy] A 300 kg object and a 700 kg object are 5.00 meters apart. What is the net gravitational force on a 75.0 kg mass located halfway in between the two?

2. [Medium] A 200 kg satellite orbits the earth at $2.50 \times 10^6\ km$. How much potential energy is wrapped up in the earth/satellite system? What is the gravitational force exerted by the earth on the satellite?

3. [Hard] A 150 kg satellite orbits the earth in a circular orbit above the equator at $2.50 \times 10^5\ m$. What is the period of the orbit? What is the speed of the satellite?

Solutions

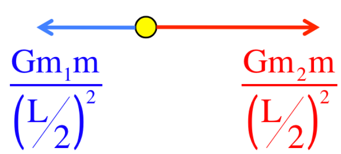
1. We will use Newton's universal gravitational force function, stated $|\vec{F}_g| = \frac{Gm_1m_2}{r^2}(-\vec{r})$



As there are two masses, m_1 and m_2, there will be two forces. Summing them up:

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_{net} = -\left(\frac{Gm_1m}{\left(\frac{L}{2}\right)^2}\right) + \left(\frac{Gm_2m}{\left(\frac{L}{2}\right)^2}\right)$$



$$\vec{F}_{net} = \left(\frac{Gm}{\left(\frac{L}{2}\right)^2}\right)(-m_1 + m_2)$$

$$\vec{F}_{net} = \left(\frac{(6.67 \times 10^{-11} N * m^2/kg^2)(75.0kg)}{(5.00\ m)^2}\right)(-(300\ kg) + (700\ kg)) = \boxed{8.00 \times 10^{-8}\ N}$$

2. To find potential energy, use $U_g = -\frac{Gm_1m_2}{r}$

$$U_g = -\frac{GmM}{(R+h)}$$

$$U_g = -\frac{(6.67\times10^{-11}N*m^2/kg^2)(200\ kg)(5.98\times10^{24}kg)}{(6.37\times10^6\ m)+(2.50\times10^6\ m)}$$

$$\boxed{U_g = -8.99\times10^9\ J}$$

Can use $|\vec{F}_g| = \frac{Gm_1m_2}{r^2}$ for force exerted on satellite by Earth:

$$|\vec{F}_g| = \frac{GmM}{(R+h)^2}$$

$$|\vec{F}_g| = \frac{\left(6.67\times10^{-11}N*\frac{m^2}{kg^2}\right)(200\ kg)(5.98\times10^{24}kg)}{[(6.37\times10^6\ m)+(2.50\times10^6\ m)]^2}$$

$$\boxed{|\vec{F}_g| = 1014\ N}$$

3. Knowns:
   m = 150 kg,
   orbit above earth = 250 km
   period = ?
   U = ?
   $v_{satellite} = ?$
   To find the period T, we can use $T = \frac{2\pi(R+h)}{v}$
   We only need to calculate velocity v to find the period:
   $\Sigma F_{centripetal}$:

$$F_c = \frac{GmM}{(R_e+h)^2}$$

$$\frac{GmM}{(R_e+h)^2} = m\left(\frac{v^2}{R_e+h}\right)$$

$$v = \left(\frac{GM}{R_e+h}\right)^{1/2}$$

Plugging back into $T = \frac{2\pi(R+h)}{v}$

$$T = \frac{2\pi(R_e+h)}{\left(\frac{GM}{R_e+h}\right)^{1/2}} = 2\pi(R_e+h)\left(\frac{R_e+h}{GM}\right)^{\frac{1}{2}} = 2\pi(R_e+h)^{3/2}\left(\frac{1}{GM}\right)^{\frac{1}{2}}$$

$$T = 2\pi\big((6.37\times10^6\ m)+(2.50\times10^5\ m)\big)^{3/2}\left(\frac{1}{(6.67\times10^{-11}N*m^2/kg^2)(5.98\times10^{24}\ kg)}\right)^{\frac{1}{2}}$$

$$T = 5.36\times10^3 s = \boxed{1.49\ hr}$$

Calculating the value for velocity:

$$v = \frac{2\pi(R+h)}{T}$$

$$v = \frac{2\pi\big((6.37 \times 10^6\ m) + (2.50 \times 10^5\ m)\big)}{5.36 \times 10^3 s}$$

$$\boxed{v = 7.76 \times 10^3\ m/s}$$

Minimum energy required for orbit:

Satellite's takeoff speed will be the speed of the earth:

$$v_{earth} = \frac{2\pi R_e}{T_{earth}} = \frac{2\pi(6.37 \times 10^6 m)}{(24\ hr)(3600\frac{s}{hr})} = 4.63 \times 10^3\ m/s$$

Now it's just conservation of energy:

$$\Sigma KE_1 + \Sigma U_1 + \Sigma W_{\min energy} = \Sigma KE_2 + \Sigma U_2$$

$$\frac{1}{2}mv_e{}^2 + \left(-\frac{GmM_e}{R_{earth}}\right) + \Sigma W_{\min energy} = \frac{1}{2}m(v_{orbit})^2 + \left(-\frac{GmM_e}{(R_{earth}+h)}\right)$$

$$\Sigma W_{\min energy} = -\frac{1}{2}mv_e{}^2 - \left(-\frac{GmM_e}{R_{earth}}\right) + \frac{1}{2}m(v_{orbit})^2 + \left(-\frac{GmM_e}{(R_{earth}+h)}\right)$$

$$\Sigma W_{\min energy} = \frac{1}{2}m((v_{orbit})^2 - v_e{}^2)) + GmM_e\left(\frac{1}{R_e} - \frac{1}{(R_e+h)}\right)$$

$$\Sigma W_{\min energy} = \frac{1}{2}m((v_{orbit})^2 - v_e{}^2)) + GmM_e\left(\frac{1}{R_e} - \frac{1}{(R_e+h)}\right)$$

Writing it all out (without units because that takes up too much space):

$$\Sigma W_{\min energy} = \frac{1}{2}(150)[(7.76 \times 10^3)^2 - (4.63 \times 10^3)^2] + (6.67 \times 10^{-11})(150)(5.98 \times 10^{24})\left(\frac{1}{6.37 \times 10^6} - \frac{1}{(6.37 \times 10^6) + (2.50 \times 10^5)}\right)$$

$$\boxed{\Sigma W_{\min energy} = 4.85 \times 10^9\ J}$$