

Chapter 7 Review: Energy and Energy Transfer

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Background / Summary:

One fundamental law of the universe and the way it works is that energy cannot be created or destroyed, only transformed and transferred through different inputs and outputs. This is known as the conservation of energy.

Key Ideas and Points for the Subject:

- Energy cannot be created or destroyed, only transformed and transferred.
- Potential energy (U) is energy that is stored in a mass and has potential to be released in the form of kinetic energy.
- Kinetic energy (KE) is energy that an object has while in motion.
- The “k” constant of a spring tells you how much force is required to elongate the spring per meter length.
- Work is a force applied over a distance, such as friction, and can affect the amount of energy in a system.

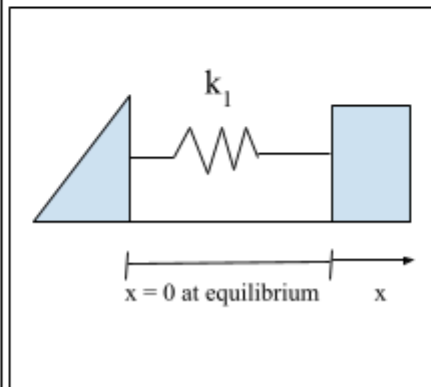
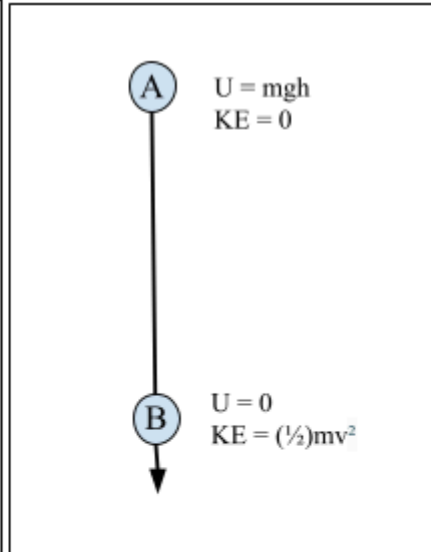


Diagram Explanation:

When the ball is held at point A, there is no movement and therefore no kinetic energy, and only potential. As the ball drops, the potential energy is converted into kinetic energy as it gains speed, and finally as it is about to hit the floor, there will only be kinetic energy.

Key Equations and Relationships:

$$\Sigma U_1 + \Sigma KE_1 + W_{\text{ext}} = \Sigma U_2 + \Sigma KE_2$$

$$U_{\text{gravity}} = mgh$$

$$U_{\text{spring}} = (\frac{1}{2})kx^2$$

$$KE = (\frac{1}{2})mv^2$$

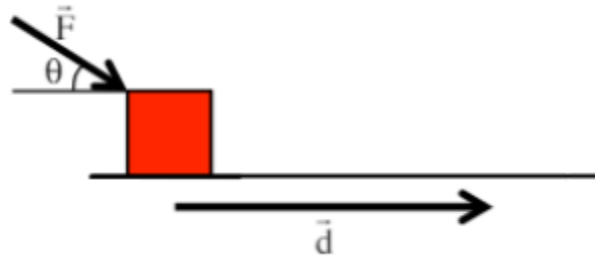
$$KE_{\text{rotational}} = (\frac{1}{2})I\omega^2$$

$$\text{Work}_{\text{ext}} = \text{Force} \cdot \text{distance} \cdot \cos(\theta)$$

$$W_{\text{net}} = \Delta KE$$

$$F_{\text{friction}} = \mu_k F_{\text{normal}}$$

Problem Set:



1. [Easy] A block of mass $m = 1.80$ kg sits on a flat, frictionless table. A constant force $F = 16.0$ N acts at an angle of $\theta = 15^\circ$ as shown in the diagram above. If the force is applied over a distance $d = 3.40$ meters, how much work does force F do on the object, how much work does the normal force do on the object, and how much work is done by the gravitational force?
2. [Medium] An ideal spring is attached to a hook on the ceiling. A 3.00 kg mass attached to the spring will elongate the spring a total distance of $.045$ meters. If the 3.00 kg mass is swapped for a 1.50 kg mass, by how much will the spring elongate? Additionally, how much work does the spring do when it is elongated by the new mass?
3. [Hard] If a 4.00 kg mass is found to have a velocity of $(7.00 \text{ (i)} - 2.00 \text{ (j)})$ m/s, what is its kinetic energy? Additionally, how much work must be done to change the body's velocity to $(8.50 \text{ (i)} + 3.50 \text{ (j)})$ m/s?

Solutions:

1. Known values:

$$\text{Mass} = 1.8 \text{ kg}$$

$$\text{Force} = 16 \text{ N}$$

$$\text{Angle of force} = 15^\circ$$

$$\text{Distance} = 3.4 \text{ meters}$$

To find how much work that force F does over distance d, we must use the relationship:

$$W = F \cdot d \cdot \cos(\theta)$$

$$\rightarrow = 16 * 3.4 * \cos(15)$$

$$\rightarrow = 52.55 \text{ Joules}$$

Because the distance that the object is moving due to the force is only in the x-direction, neither the normal force nor the gravitational force will do work on the mass.

$$W = F \cdot d \cdot \cos(\theta)$$

$$\rightarrow = F * 0 * \cos(\theta)$$

$$\rightarrow = 0 \text{ Joules}$$

2. Known values:

$$\text{Mass}_1 = 3.00 \text{ kg}$$

$$\text{Elongation}_1 = 0.045 \text{ meters}$$

$$\text{Mass}_2 = 1.50 \text{ kg}$$

$$\text{Elongation}_2 = 0.05 \text{ meters}$$

Before we start solving either of the questions asked, we should first find the “k” constant of the spring. Since k is the force needed to elongate a spring per meter length:

$$k = F / d$$

$$\rightarrow = (m \cdot g) / 0.045$$

$$\rightarrow = (3 \cdot 9.8) / 0.045$$

$$\rightarrow = 653.33$$

Then, using the spring constant, we can solve for the elongation after exchanging the initial mass for a 1.5 kg mass:

$$F_{\text{spring}} = -k \cdot y$$

$$|y| = F_{\text{spring}} / k$$

$$\rightarrow = (m \cdot g) / 653.33$$

$$\rightarrow = (1.5 \cdot 9.8) / 653.33$$

$$\rightarrow = 0.023 \text{ meters}$$

To find how much work the spring does when it is elongated by the new 1.5 kg mass, we can use the relationship:

$$W = \int |F| \cdot |dr| \cdot \cos(\theta)$$

$$\rightarrow = \int_0^{0.023} |(-ky)| \cdot |dy| \cdot \cos(0)$$

$$\rightarrow = k(y^2 / 2) \Big|_0^{0.023}$$

$$\rightarrow = 0.5 \cdot 653.33 \cdot (0.023^2 - 0^2)$$

$$\rightarrow = 0.17 \text{ Joules}$$

3. Known values:

$$\text{Mass} = 4.00 \text{ kg}$$

$$\text{Velocity}_i = (7.00 \text{ (i)} - 2.00 \text{ (j)}) \text{ m/s}$$

$$\text{Velocity}_f = (8.50 \text{ (i)} + 3.50 \text{ (j)}) \text{ m/s}$$

We can start this problem by finding the vector of the velocity:

$$v = ((7)^2 + (-2)^2)^{1/2}$$

$$\rightarrow = 7.28 \text{ m/s}$$

Then, we can use:

$$\text{KE} = (1/2)mv^2$$

$$\rightarrow = 0.5 \cdot 4 \cdot 7.28^2$$

$$\rightarrow = 105.99 \text{ Joules}$$

To find the work done to change the velocity of the object to $(8.50 \text{ (i)} + 3.50 \text{ (j)}) \text{ m/s}$, we once again need to find the vector of the new velocity.

$$v = ((8.5)^2 + (3.5)^2)^{1/2}$$

$$\rightarrow = 9.19 \text{ m/s}$$

Then, we can once again use:

$$W_{\text{net}} = \Delta \text{KE}$$

$$\rightarrow = \text{KE}_2 - \text{KE}_1$$

$$\rightarrow = (0.5 \cdot 4 \cdot 9.19^2) - 105.99$$

$$\rightarrow = 62.92 \text{ Joules}$$