

Sources of Magnetic Fields

AP Physics, G Period
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Background

Moving charge/current produces a magnetic field. This sheet will review magnetic fields on wires, solenoids, and important laws/formulas.

Formulas

Magnetic field of a current-carrying wire: $B = \frac{\mu_o I}{2\pi r}$

$$* \mu_o = 4\pi \cdot 10^{-7}$$

Magnetic field of a solenoid: $B = \mu_o nI$

Biot-Savart: $dB = \frac{\mu_o}{4\pi} \int \frac{Id\mathbf{s} \times \mathbf{r}}{r^2}$

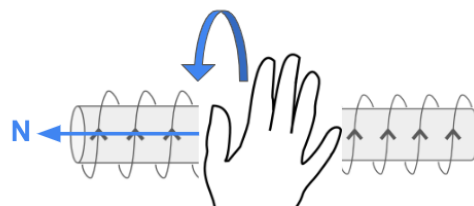
Ampere's Law: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_o I$

Magnetic flux: $\Phi_B = \oint \mathbf{B} \cdot d\mathbf{A} = 0$

Key concepts/terms

The **right thumb rule** determines the direction of the B-fld around a wire. Point your right thumb along the current, and your fingers will curl in the direction of the B-fld.

The other **right hand rule** is for solenoids. B-flds in solenoids run in one direction down the center of the coil. Lay your fingers on the coil following the current, and your thumb will point in the direction of the B-fld and toward the North pole.



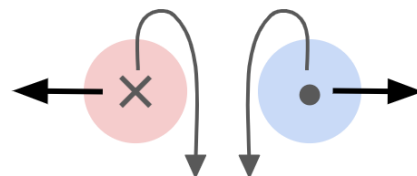
Biot-Savart determines the magnitude of the B-fld due to a segment of wire at a specific point, where ds is the small section of wire, and r is the vector in the direction toward the point. Use Biot-Savart when there is no symmetry.

Ampere's Law is like the magnetic equivalent of Gauss's Law, stating that the B-fld around a closed Amperian path is proportional to the current through the face. Use Ampere's Law to find the B-fld in symmetrical situations so the B-fld will either be constant or equal 0. This is wayyy easier than Biot-Savart, so use it whenever you can.

Magnetic flux through a closed path is 0 since field lines circle or cancel out (North/South pole).

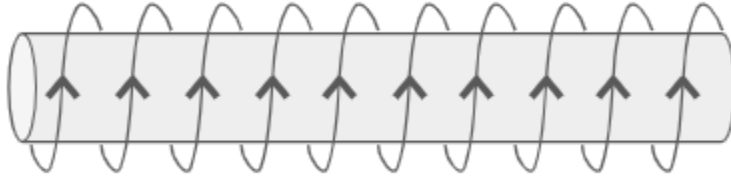
Forces between wires

When B-fld lines of two objects are parallel, they will repel each other. If they are anti-parallel, they will attract.

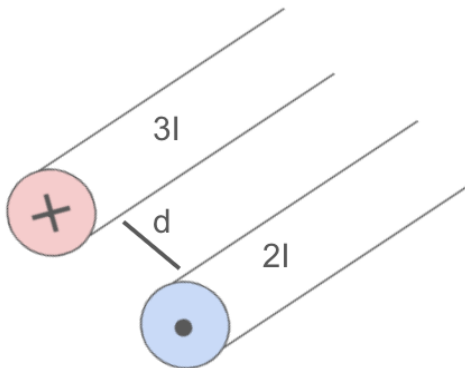


Problems

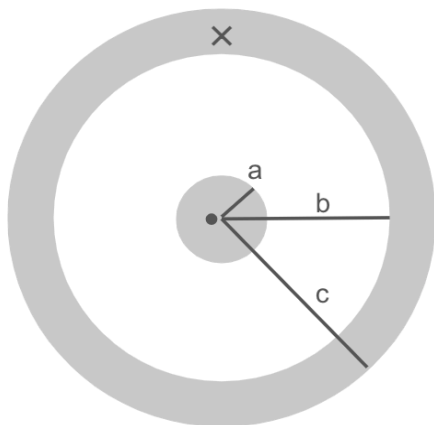
1. **Easy:** Calculate the B-fld through the 0.01 m long solenoid with 55 turns and 12 A of current. Indicate the direction of the B-fld.



2. **Medium:** Calculate the force on the red wire and determine if the wires attract or repel.



3. **Hard:** The two conductors each carry current, $2I$. Calculate the B-fld when $b < r < c$, and indicate the direction of the field. Then, find the magnetic flux through the inner conductor.



Solutions

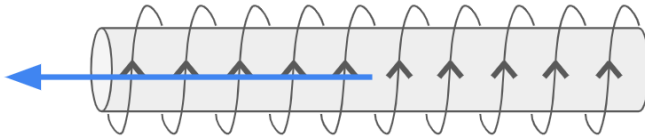
1. Easy

The B-fld of a solenoid is $B = \mu_o nI$ where n is the $\frac{\text{number of turns}}{\text{length}}$. Plugging the information into the formula, we get:

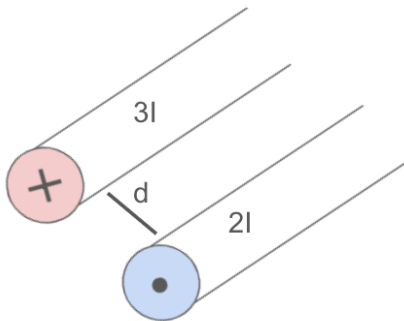
$$B = 4\pi \cdot 10^{-7} \left(\frac{55}{0.01}\right)(12)$$

$$B = 8.29 \cdot 10^{-2} T$$

Given the right hand rule for solenoids, the B-fld is to the left.



2. Medium



We can find the B-fld due to the blue wire with $B = \frac{\mu_o I}{2\pi r}$. Using the information from the

problem, $B = \frac{2\mu_o I}{2\pi d} = \frac{\mu_o I}{\pi d}$

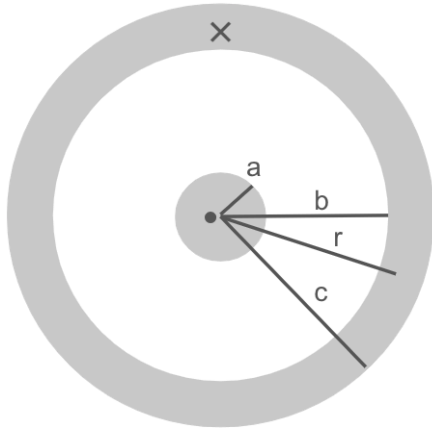
The force is $F = iL \times B$, and we can plug in the B-fld due to the blue wire that we just calculated.

$$F = 3IL \times \frac{\mu_o I}{\pi d}$$

$$F = \frac{3\mu_o I^2 L}{\pi d} N$$

Applying the right thumb rule, the B-fld on both the red and blue wires is downward. Because the B-fld lines are parallel, the two wires will repel.

3. Hard



We can use Ampere's Law: $\oint B \cdot ds = \mu_o I$

$$B \oint ds = \mu_o I$$

$$B(2\pi r) = \mu_o (I_{in} - I_{out})$$

$$I_{in} = 2I$$

$$I_{out} = 2 \frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)} I$$

Plugging in the currents:

$$B = \frac{\mu_o}{2\pi r} \left(2I - 2 \frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)} I \right)$$

$$B = \frac{\mu_o I}{\pi r} \left(1 - \frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)} \right)$$

The net current is outward because of $(I_{in} - I_{out})$, where I_{out} is only a fraction I_{in} (the center conductor)

Using the right thumb rule with the current outward, the B-fld is in the counterclockwise direction.

To calculate magnetic flux: $\Phi_B = \oint B \cdot dA$

$$\Phi_B = BA$$

$$\Phi_B = \frac{\mu_o I}{\pi r} \left(1 - \frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)} \right) (\pi a^2)$$