Lab: AP Review Sheets — AP Physics

Chapter 8: Capacitance — by Hadi Ammar

Background/summary:

Understanding capacitors and capacitance is foundational. This review covers how capacitors store and release electrical energy and how this process is measured. Hint: capacitance measures a capacitor's ability to store charge per unit voltage. This unit helps us understand its applications in circuits.

Capacitance Formulae

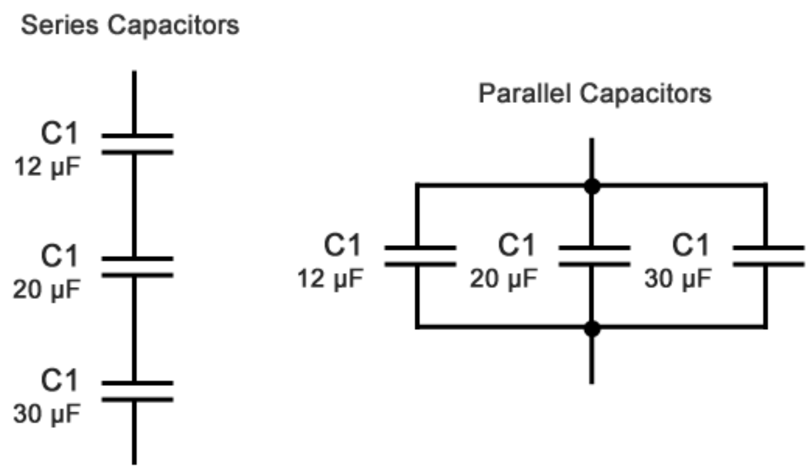
$\Delta V = Q/C$, can be written as $C = Q/\Delta V$

$C = k\dfrac{\varepsilon_0 A}{d}$, A= area of one plate, d = the distance between each plate

$C_p = \sum C_i$, when in parallel, the total Capacitance is the sum of each Capacitor’s capacitance

$1/C_s = \sum 1/C_i$, when in series, the total Capacitance = 1/ the sum of each Capacitor’s capacitance

Symbol for a capacitor in Circuit



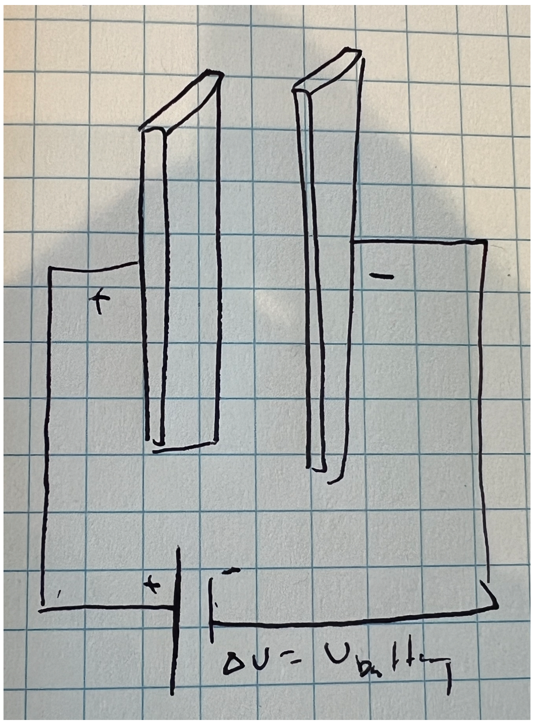
Wait, but what even is a capacitor?

A capacitor, in simple terms, is quite literally two metal plates placed close to each other with a bit of insulation between them

When connected to a battery, one plate becomes positively charged and the other negatively charged, creating an electrical potential difference (ΔV))between them. This setup allows the capacitor to store electrical energy, which can be released later for various applications in circuits.

Did you say Insulator? What? Why?

Unfortunately, yes. Usually, the "insulator" between the two plates is just air, in which case it will look like this.



Based off of all of this, How can we increase capacitance?

1) Use a dielectric

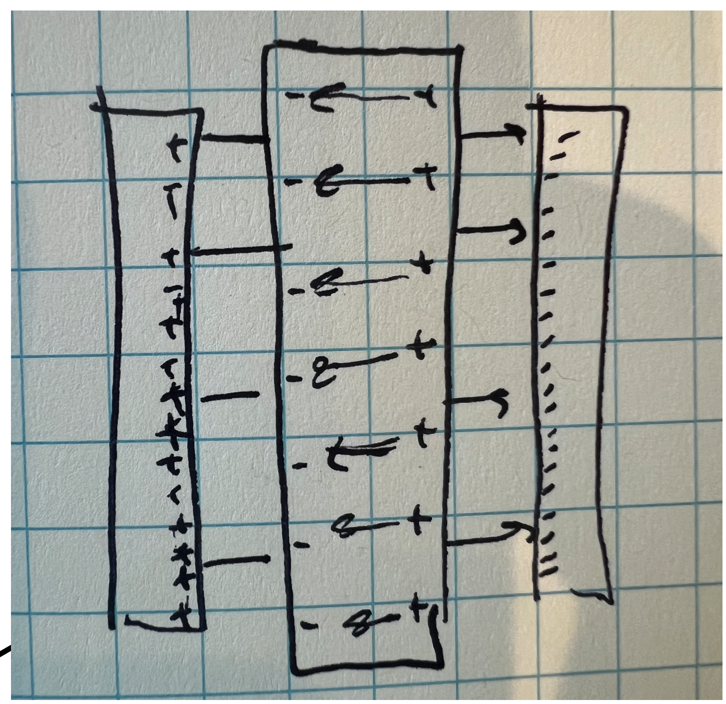
2) Decrease the d (bring the plates closer together)

3) Increase the A (Area of the plates)

$$C = k\frac{\varepsilon_0 A}{d}$$

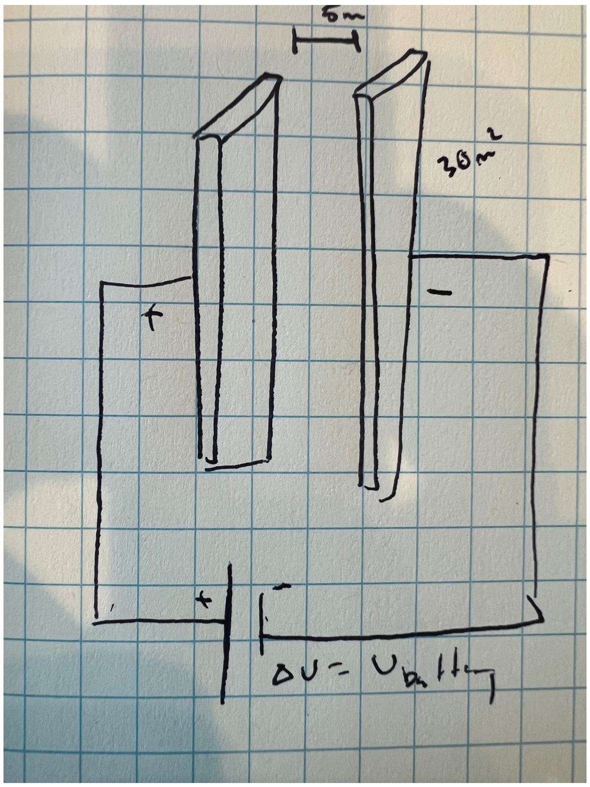
Sometimes though the AP folks use something called dielectrics. Due to the electric field between the capacitors, these dielectrics feel a Van der Waal effect which induces a reverse electric field that lowers the net electric field across the plates. Why is this important? Well, if you remember our equation $C = Q/\Delta V$, when the ΔV goes down as a result of the decreasing E-field, the capacitance actually increases!

Here's what this looks like, thanks to Fletch for these sketches…



Lets Do Some Problems!

Level 1 (Easy): Find the capacitance of a resistor with an plate area of 30 m^2 and 5 m between each plate

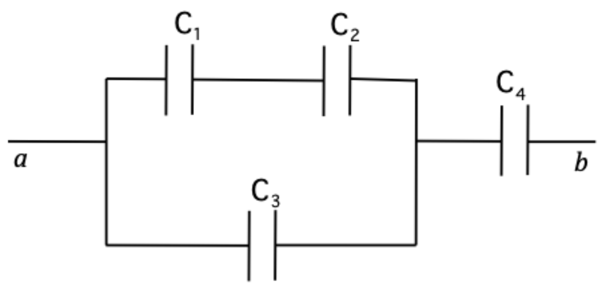


$$C = k\frac{\varepsilon_0 A}{d}$$

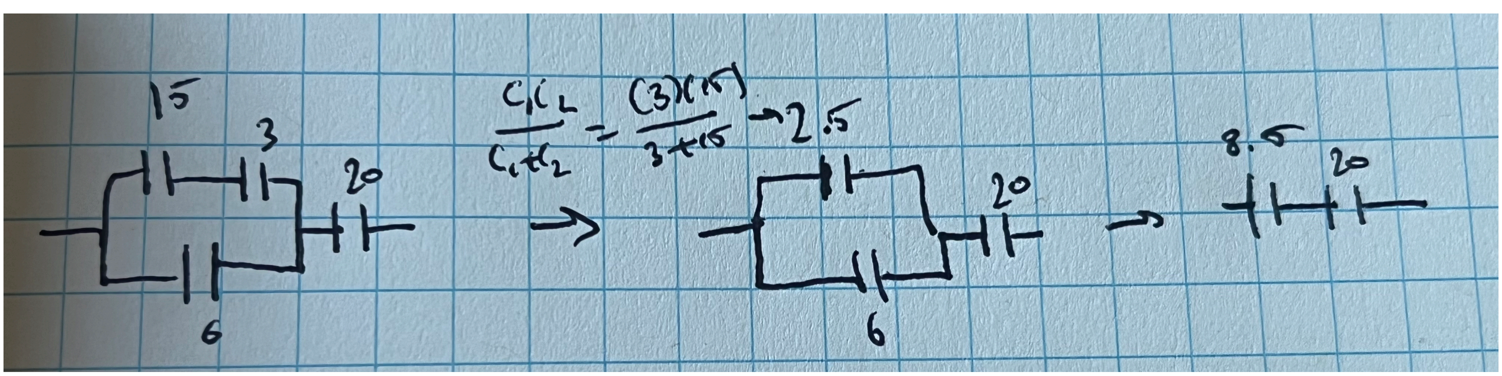
$$C = (9x10^9)\frac{(8.85x10^-12)(30)}{5}$$

$$c = 4.779e - 19$$

Level 2 (Medium): Find the equivalent capacitance for this circuit if C1 = 15.0 μF, C2 = 3.00 μF, C3 = 6.00 μF and C4 = 20.0 μF



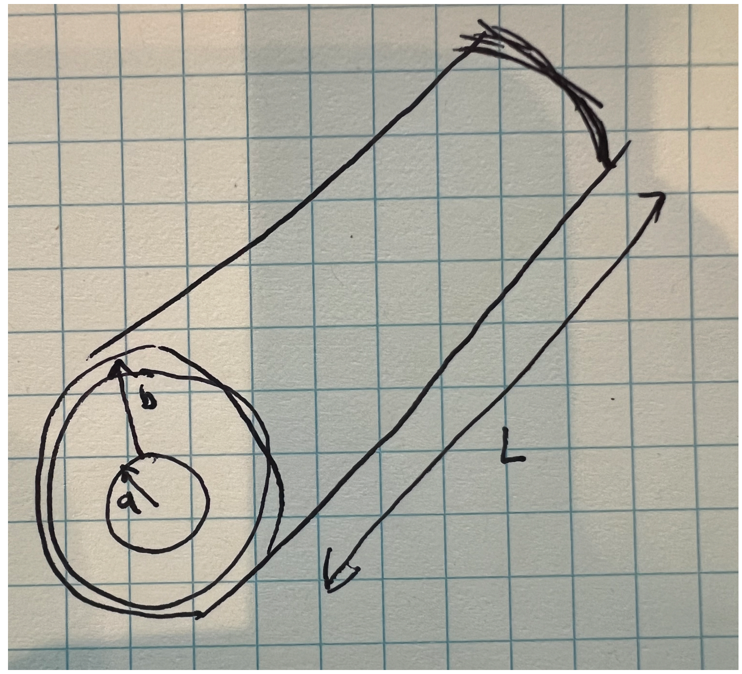
Step 1, simplify the circuit



Now that you are left with 2 capacitors in series employ the series equation

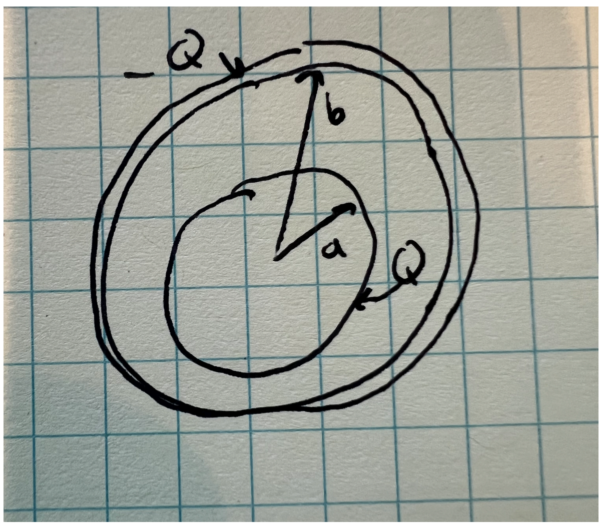
$$C_{equ} = \frac{C_{8.5}C_{20}}{C_{8.5} + C_{20}} = \frac{8.5(20)}{8.5 + 20} = 5.96 \times 10^{-6}\ \text{F}$$

Level 3 (Hard): Derive an expression for the capacitance of concentric spherical shells of inside radius a and



Solution:
Apparently, capacitors come in all shapes and sized, lets get started

Step 1: assume that the charges on both shells are Q



Step 2: find the electrical potential (ΔV) between the shells using the fact that we know that $E = \dfrac{Q}{4\pi\varepsilon_o r^2}$

$$V_{cap} = -\Delta V = +\int \vec{E} \cdot d\vec{r}$$

$$= \int_{r=a}^{b} \left(\frac{Q}{4\pi\varepsilon_o r^2} \hat{r} \right) \cdot (dr\hat{r}) = \frac{Q}{4\pi\varepsilon_o} \int_{r=a}^{b} \frac{1}{r^2} dr \cos 0^\circ$$

$$= \frac{Q}{4\pi\varepsilon_o} \left(-\frac{1}{r} \right) \Bigg|_{r=a}^{b} = \frac{Q}{4\pi\varepsilon_o} \left[\left(-\frac{1}{b} \right) - \left(-\frac{1}{a} \right) \right] = \frac{Q}{4\pi\varepsilon_o} \left(\frac{b-a}{ab} \right)$$

Step 3: find the Capacitance using the definition of capacitance.

$$C = \frac{\left(Q_{\text{on 1 shell}}\right)}{\left(V_{\text{across shells}}\right)}$$

$$= \frac{Q}{\frac{Q}{4\pi\varepsilon_o} \left(\frac{b-a}{ab} \right)}$$

$$\boxed{= \frac{4\pi\varepsilon_o (ab)}{(b-a)}}$$