Chapter 24: Gauss’s Law  
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**Summary:** In this chapter, we continue learning about electric fields and what happens when their field lines penetrate the surface of an object. With knowledge of the definition of flux, we are also introduced to Gauss’s Law to help us analyze the relationship between the electric flux through an object and an enclosed charge.

**Topics**

- Electric Flux
- Gauss’s Law
- Application of Gauss’s Law to Various Charge Distribution
- Conductors in Electrostatic Equilibrium
- Formal Derivation of Gauss’s Law

**Equations**

- Electric Flux: \( \Phi_E = E \cos \theta \)
- Electric Flux through a Surface: \( \Phi_E = \int E \cdot dA \)
- Gauss’s Law: \( \Phi_E = \oint E \cdot dA = \frac{q_{\text{in}}}{\varepsilon_0} \)

**Conductors in Electrostatic Equilibrium**

The electric field anywhere inside a conductor is zero, and the charge accumulates on the outer surface of the conductor.

### Electric Flux

Electric flux is directly related to how many E field lines penetrate an object. The more perpendicular an object is to the electric field, the higher the electric flux (the maximum flux is \( EA \), when the angle between the electric field and the surface is 90 degrees).

Electric flux has the units \( \text{Nm}^2/\text{C} \).

### Gauss’s Law

Gauss’s Law Problems for an enclosed point charge:

1. Enclose a charged particle in a Gaussian surface (a sphere). If the charge is located outside the Gaussian surface, then the number of electric field lines entering the surface and exiting the surface is equal and the net electric flux is zero.
2. Since \( E \) is constant at every point (with a spherically or cylindrically symmetrical Gaussian surface), you can pull it out of the integral.
3. Substitute either \( 4\pi r^2 \) (spherical symmetry) or \( 2\pi rl \) (cylindrical symmetry) for \( A \).
4. For an enclosed point charge, use \( q \) and solve!

### Application of Gauss’s Law to Various Charge Distribution

**Spherically Symmetric Charge Distribution**

(a = radius of sphere)

- \( r > a \): Same as a point charge of charge \( Q \), \( E = k \frac{Q}{r^2} \)
- \( r < a \): \( E = k \frac{Q}{a^3} r \)

**Outside a Thin Spherical Shell:** \( E = k \frac{Q}{r^2} \)

**Cylindrically Symmetrical Charge Distribution:** \( E = 2k \frac{q}{r} \)

**Plane of Charge:** \( E = \frac{q}{2\varepsilon_0} \)
Practice Problems

1. What is the electric flux through a rectangular plate tilted at 25 degrees from the horizontal with a length of 43cm and a width of 21cm if the electric field (parallel to the ground) is 4.00kN/C?

2. Consider an infinitely long, positively charged wire with charge per unit length of \( \lambda \). What is the magnitude of the electric field at a distance \( r \) from the wire?

3. What is the magnitude of the electric field inside a conducting sphere of radius \( r \)? Derive an expression for the magnitude of the electric field at a distance \( a \) from the center of the sphere (\( a>r \)).

Solutions

1. To find out the electric flux through this rectangular plate, we can use \( \Phi_E = EA \cos \theta \).

We know the electric field to be 4.00kN/C, so let’s convert this into appropriate units...

\[(4.00\text{kN/C})(1000\text{N/1kN}) = 4.0\times10^{-3}\text{N/C}\]

Now we need to find out the area of the rectangular plate (while changing the centimeters to meters):

\[A = lw = (43\text{cm}/100)(21\text{cm}/100) = .090\text{m}^2\]

\(\theta = 25^\circ\)

Now that we know these variables, we can solve...

\[EA \cos \theta = (4.0\times10^{-3}\text{N/C})(.090\text{m}^2)\cos(25) = 3.26\times10^{-4}\text{Nm}^2/\text{C}\]
Since this is Gauss’s Law, we should start with the general equation for this problem.

\[ \Phi_E = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0} \]

We can use a cylinder to analyze the electric field emitted by this wire, so we can deal with the left side of Gauss’s Law.

Since E and dA don’t change, we can pull them both out of the integral...

\[ EA = \frac{q_{in}}{\epsilon_0} \]

Now that we know the following values, we can rearrange the equation for the electric field.

\[ A = 2\pi rl \text{ (since it’s a cylinder)} \]

\[ Q_{in} = \lambda l \]

\[ E(2\pi rl) = \frac{\lambda l}{\epsilon_0} \]

Solving for E and simplifying, we get:

\[ E = \frac{\lambda}{2\pi\epsilon_0 r} \]

\[ E = 2k_e \frac{\lambda}{r} \]
3. All of the charge accumulates on the surface of a conductor, so when you place a Gaussian surface on the inside of one, there is no electric field being generated.

Again, like in the last problem, we can start with the general expression for Gauss’s Law:

\[ \Phi_E = \oint E \cdot dA = \frac{q_{\text{in}}}{\varepsilon_0} \]

Since we have spherical symmetry, \( E \) remains constant. The area is also a constant \( 4\pi r^2 \), so we can pull these both out of the integral.

\[ EA = \frac{q_{\text{in}}}{\varepsilon_0} \]

\[ E(4\pi r^2) = \frac{q_{\text{in}}}{\varepsilon_0} \]

Now we can solve for \( E \)...

\[ E = \frac{q_{\text{in}}}{4\pi r^2 \varepsilon_0} \]

\[ E = k \frac{q}{r^2} \]