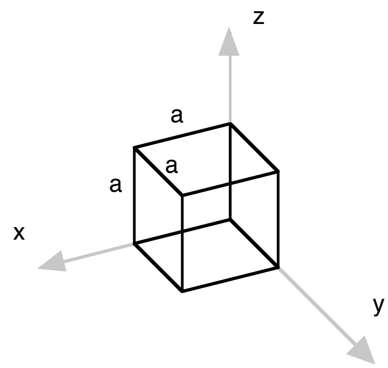
AP Physics
Gauss's Law; Electric Potential

Part I. Multiple Choice (5 points each)
Choose the one best answer to each of the following problems.

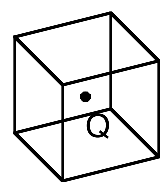
1 (AP). What is the radial component of the electric field associated with the potential $V = ar^{-2}$, where a is a constant:

a. $-2ar^{-3}$ b. $-2ar^{-1}$ c. ar^{-1} d. $2ar^{-1}$ e. $2ar^{-3}$

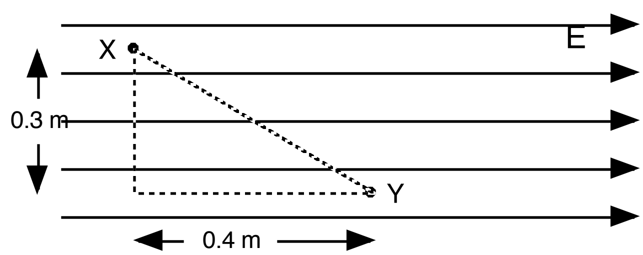


2 (AP).
A closed surface, in the shape of a cube of side a, is oriented as shown above in a region where there is a constant electric field of magnitude E parallel to the x-axis. The total electric flux through the cubical surface is:

a. $-Ea^2$ b. zero c. Ea^2 d. $2\ Ea^2$ e. $6\ Ea^2$



3 (AP).
The point charge Q shown above is at the center of a metal box that is isolated, ungrounded, and uncharged. Which of the following is true?
a. The net charge on the outside of surface of the box is Q.
b. The potential inside the box is zero.
c. The electric field inside the box is constant.
d. The electric field outside the box is zero everywhere.
e. The electric field outside the box is the same as if only the point charge (and not the box) were there.



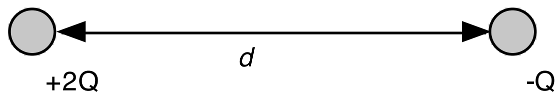
4. (AP).
A uniform electric field E of magnitude 6000 volts per meter exists in a region of space as shown above. What is the electric potential difference, $V_x - V_y$, between points X and Y?

a. -12,000 V b. 0 V c. 1,800 V d. 2,400 V e. 3,000 V

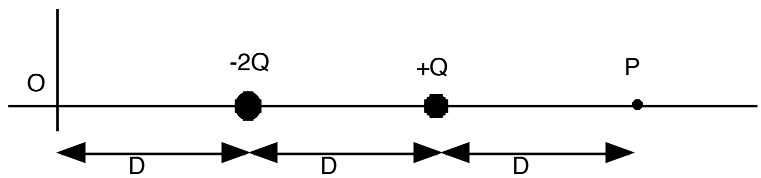
5 (AP). A positive charge of 3.0×10^{-8} Coulombs is placed in an upward-directed uniform electric field of 4.0×10^{4} Newtons per Coulomb. When the charge is moved 0.5 meters upward, the work done by the electric force on the charge is:

a. 6×10^{-4} J b. 12×10^{-4} J c. 2×10^{4} J d. 8×10^{4} J e. 12×10^{4} J

6 (AP). Which of the following statements about conductors under electrostatic conditions is true?
a. Positive work is required to move a positive charge over the surface of a conductor.
b. Charge that is placed on the surface of a conductor always spreads evenly over the surface.
c. The electric potential inside a conductor is always zero.
d. The electric field at the surface of a conductor is tangent to the surface.
e. The surface of a conductor is always an equipotential surface.



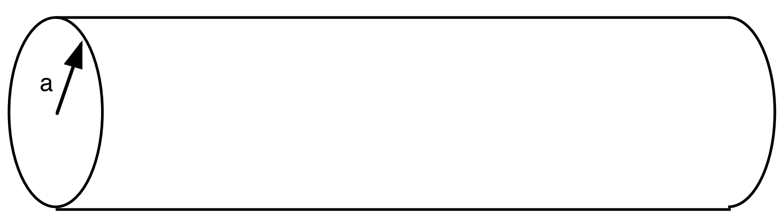
7 (AP).
Two identical conducting spheres are charged to +2Q and -Q, respectively, and are separated by a distance *d* (much greater than the radii of the spheres) as shown above. The magnitude of the force of attraction on the left sphere is F_1. After the two spheres are made to touch and then are reseparated by distance *d*, the magnitude of the force on the left sphere is F_2. Which of the following relationships is correct?
a. $2F_1 = F_2$ b. $F_1 = F_2$ c. $F_1 = 2F_2$ d. $F_1 = 4F_2$ e. $F_1 = 8F_2$



8 (AP).
Two charges, -2Q and +Q, are located on the x-axis as shown above. Point P, at a distance of 3D from the origin), is one of two points on the positive x-axis at which the electric potential is zero. How far from the origin O is the other point?
a. 2/3 D b. D c. 3/2 D d. 5/3 D e. 2 D

9. A charge of 10 nC is distributed uniformly along the x axis from x = -2 m to x = +3 m. Which of the following integrals is correct for the electric potential (relative to zero at infinity) at the point x = +5 m on the x axis?
a. ∫90/x dx (evaluated from -2 to 3)
b. ∫90/(5-x) dx (evaluated from -2 to 3)
c. ∫18/x dx (evaluated from -2 to 3)
d. ∫ 18/(5-x) dx (evaluated from -2 to 3)
e. ∫ 18/(5+x) dx (evaluated from -2 to 3)

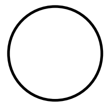
Part II. Free Response



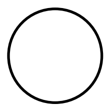
10 (AP) (20 points).
A very long nonconducting rod of radius *a* has positive charge distributed throughout its volume. The charge distribution is cylindrically symmetric, and the total charge per unit length of the rod is λ.
a. Use Gauss's law to derive an expression for the magnitude of the electric field *E* outside the rod.

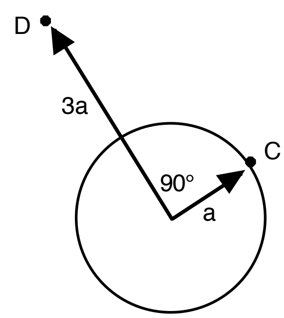
b. The diagrams below represent cross sections of the rod. On these diagrams, sketch the following.

i. Several equipotential lines in the region $r>a$.



ii. Several electric field lines in the region $r>a$.





c.

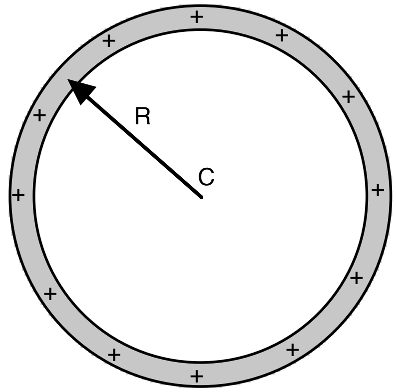
In the diagram above, point C is a distance a from the center of the rod (i.e., on the rod's surface), and point D is a distance $3a$ from the center on a radius that is 90° from point C. Determine the following.

i. The potential difference $V_C - V_D$ between points C and D

ii. The work required by an external agent to move a charge +Q from rest at point D to rest at point C.

Inside the rod ($r<a$), the charge density ρ is a function of radial distance r from the axis of the rod and is given by $\rho = \rho_o (r/a)^{1/2}$, where ρ_o is constant.

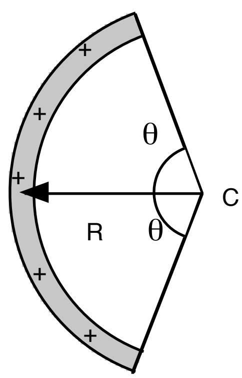
d. Determine the magnitude of the electric field E as a function of r for $r<a$. Express your answer in terms of ρ_o, a, and fundamental constants.



11 (AP) (15 points).
A thin nonconducting rod that carries a uniform charge per unit length of λ is bent into a circle of radius R as shown above. Express your answers in terms of λ, R, and the fundamental constants.

a. Determine the electric potential V at the center C of the circle.

b. Determine the magnitude E of the electric field at the center C of the circle above.

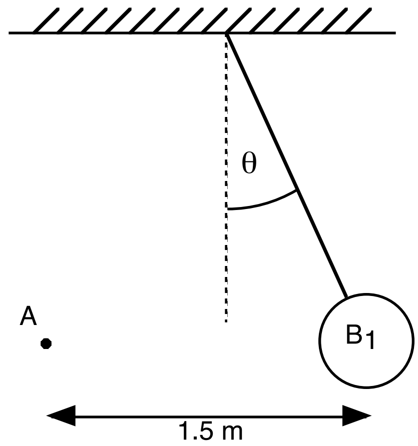


Another thin nonconducting rod that carries the same uniform charge per unit length λ is bent into an arc of a circle of radius R, which subtends an angle of 2θ as shown above. Express your answers in terms of θ and the quantities given above.

c. Determine the total charge on the rod.

d. Determine the electric potential V at the center of curvature C of the arc.

e. Determine the magnitude E of the electric field at the center of curvature C of the arc. Indicate the direction of the electric field on the diagram above.

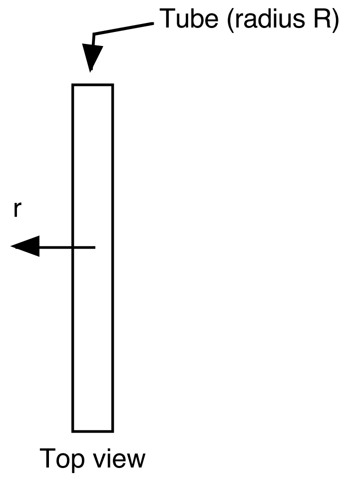


12 (AP) (20 points).
The small sphere A in the diagram above has a charge of 120μC. The large sphere B_1 is a thin shell of nonconducting material with a net charge that is uniformly distributed over its surface. Sphere B_1 has a mass of 0.025 kg, a radius of 0.05 m, and is suspended from an uncharged, nonconducting thread. Sphere B_1 is in equilibrium when the thread makes an angle $\theta = 20°$ with the vertical. The centers of the spheres are at the same vertical height and are a horizontal distance of 1.5 m apart, as shown.

a. Calculate the charge on sphere B_1.

b. Suppose that sphere B_1 is replaced by a second suspended sphere B_2 that has the same mass, radius, and charge, but that is conducting. Equilibrium is again established when sphere A is 1.5 m from sphere B_2 and their centers are at the same vertical height. State whether the equilibrium angle θ_2 will be less than, equal to, or greater than 20°. Justify your answer.

The sphere B_2 is now replaced by a very long, horizontal, nonconducting tube, as shown in the top view below. The tube is hollow with thin walls of radius R = 0.20 m and a uniform positive charge per unit length of $\lambda = +0.10\ \mu C/m$.



c. Use Gauss's law to show that the electric field at a perpendicular distance r from the tube is given by the expression $E = (1.8 \times 10^3 / r)$ N/C, where r > R and r is in meters.

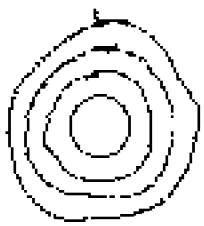
d. The small sphere A with charge 120 μC is now brought into the vicinity of the tube and is held at a distance of r = 1.5 m from the center of the tube. Calculate the repulsive force that the tube exerts on the sphere.

e. Calculate the work done against the electrostatic repulsion to move sphere A toward the tube form a distance r = 1.5 m to a distance r = 0.3 m from the tube.

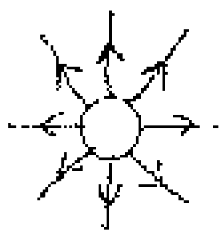
ANSWERS/SOLUTIONS:

1. e (Use $E=-dV/dr$)
2. b
3. a
4. d (Use $V=-Ed$)
5. a
6. e
7. e
8. d
9. d
10. a. Use a Gaussian cylinder concentric with the rod. $q_{in}=\lambda L$, and Area of cylinder = $2\pi rL$. Substitue into Gauss's Law to get $E=2k\lambda/r$.

b. i.



ii.



c. i. Use $\Delta V=-\int E\bullet ds$ and previous $E=k\lambda/r$ result to get $V_c - V_d=2k\lambda \ln 3$

ii. Use $\Delta V=U/q$ to get $2kQ\lambda \ln 3$

d. Hard problem! Use Gauss's law, and $q_{in}=\int\rho\, dV$. Integrate left side from 0 to q, integrate right side from 0 to R, with $dV=2\pi rL\bullet dr$, then sub in to Gauss's law to get $E=(2\rho\, r^{(3/2)})/(5\,\varepsilon_o\sqrt{a})$

11. a. Use $V=-\int E\bullet ds = \int k\bullet dq/r$, evaluated from 0 to 2π, to get $V=2k\lambda\pi$

b. E=0 at the center of the circle, because all of the vectors cancel.

c. $Q=\lambda L$, and $L=R2\theta$, so $Q=\lambda R2\theta$

d. Use $V=-\int E\bullet ds$ to get $k\lambda 2\theta$

e. y-components all cancel so field is only due to x-components. Use $E=\int k\, dq/R^2$ to get $E=2k\lambda\sin\theta / R$

12. a. Use $F_{net}=ma$ in both x and y directions to get q=1.86e-7 C

b. Chages are free to flow in B2, so negative chrages will be nearer, positive charges will be farther away. The effect of repulsion is less than the effect of attraction, so he angle θ will be *less* than 20°

c. Choose Gaussian cylinder to analyze, get E = (1798/r) N/C = 1.8e3/r N/C

d. Use F=Eq to get 0.144 N

e. Use $\Delta U=-\int F\bullet ds$ to get $\Delta U=-\int Eq\bullet ds$. Limits on integral are from 1.5 to 0.3. Subsitute in 1800/r expression (from above) for Electric field. Solve to get 0.348 J