Ch 31 – Faraday's Law

Electricity & Magnetism

Up to this point, we've seen electric fields produced by electric charges...

Sign on pg 25

$$E = \int \frac{k \bullet dq}{r^2} \rightarrow \oint E \bullet dA = \frac{q_{in}}{\varepsilon_o}$$

... and magnetic fields produced by moving charges...

$$\oint B \bullet d\ell = \mu_o I$$

If currents produce magnetic fields, can magnetic fields produce currents?

Faraday's Experiment

Michael Faraday designed an experiment to demonstrate that magnetic fields produce a flow of charge (a current).

Demo

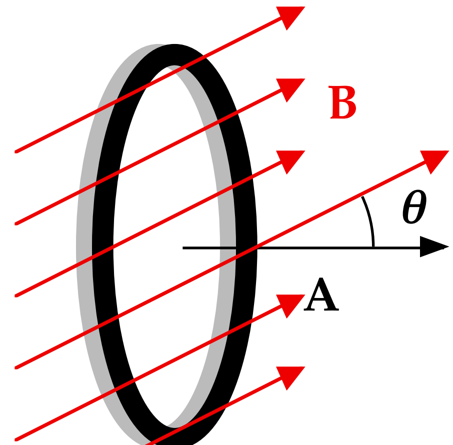
A changing magnetic field in the vicinity of a loop of wire produces a flow of current.

Faraday's Law of Induction

The emf $\boxed{?}$ induced in a circuit is directly proportional to the time rate of change of magnetic flux through the circuit.

$$\mathcal{E} = \frac{-d\Phi_B}{dt}$$

$$\text{where } \Phi_B = \int \vec{\mathbf{B}} \bullet d\vec{\mathbf{A}}$$



If we want to induce a flow of current, we need to change the magnetic flux, which we can do by changing *B*, *A*, or *q*.

$$\mathcal{E} = N\frac{-d}{dt}\left(BA\cos\theta\right)$$

Example 1

A coil is wrapped with 200 turns of wire on the perimeter of a square frame of sides 18cm. The total resistance of the coil is 2.0 ohms. A uniform magnetic field is turned on perpendicular to the plane of the coil.

a. If the field changes linearly from 0 to 0.50 Wb/m^2 in a time of 0.80s, find the magnitude of the induced emf in the coil while the field is changing.

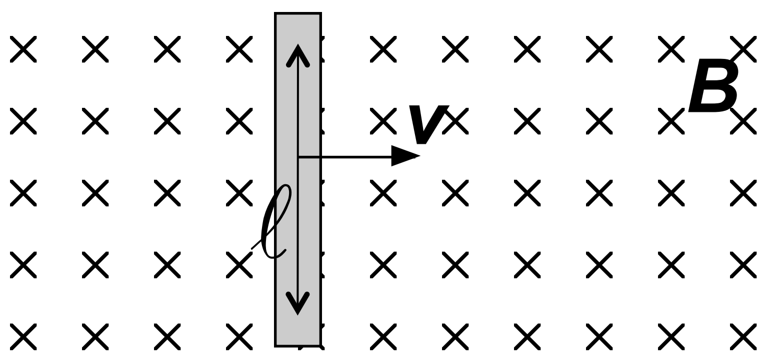
a. A=0.0324 m^2 , emf=4.1V

b. Find the magnitude of the current induced in the coil while the field is changing.

b. 2.05A

Motional EMF

When we move a conductor through a magnetic field, motional emf occurs. How does moving a conductor through a field cause charges to migrate in the conductor?



$$F = qv \times B$$

$$F_B = F_E$$

$$qv \times B = qE$$

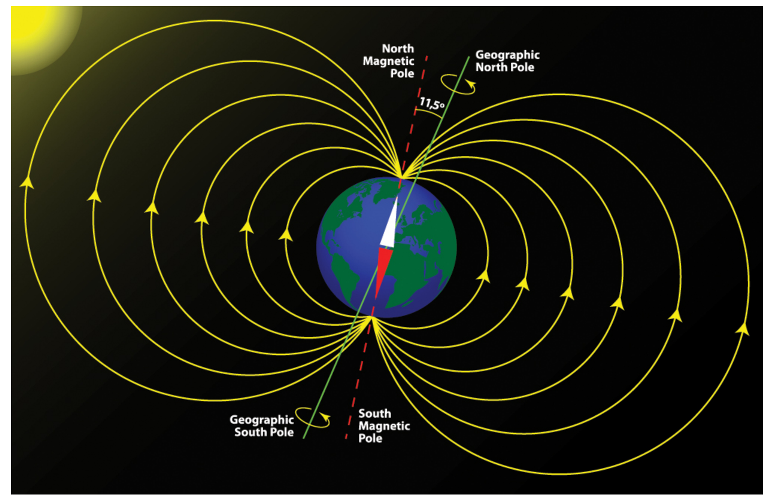
$$E = vB$$

$$V = Ed = E\ell = B\ell v$$

Charges accumulate at the ends of the conductor until equilibrium is reached, at which point F(B) = F(E)

Example 2

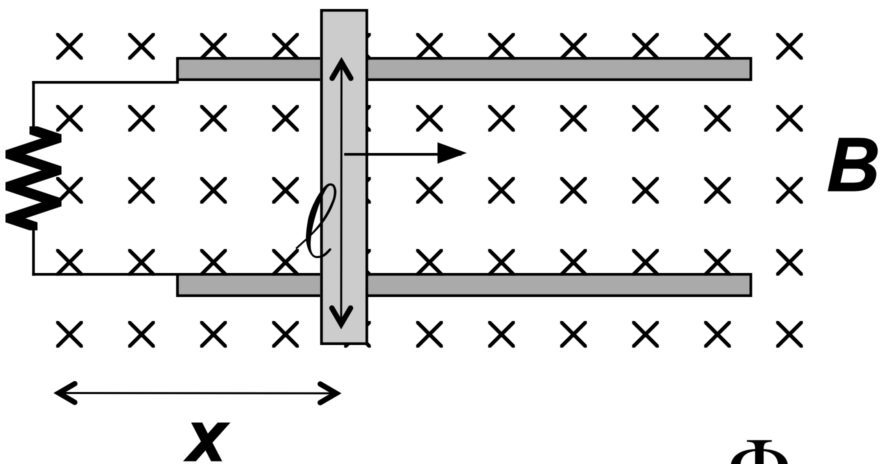
An airplane flies from LA to Seattle, and due to its motion through the Earth's magnetic field, undergoes motional emf. Which wingtip is positively charged: the *left* or the *right*?



Answer: Earth's magnetic field points slightly downward as one flies north—cross product produces leftward EMF, putting positive charges on the left.

Changing flux

We can look at motional emf in another way: as the bar moves through the field below, what's happening to the flux through the area enclosed by the circuit?

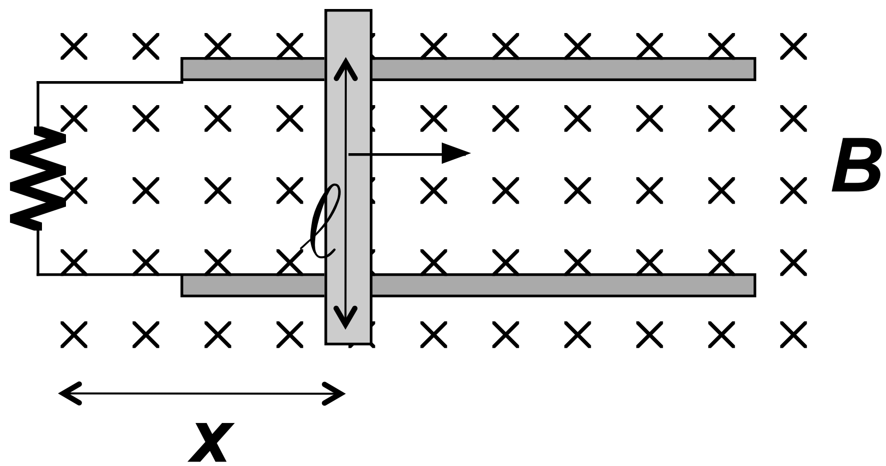


$$\Phi_B = B\ell x$$

$$\varepsilon = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}B\ell x = -B\ell v$$

Changing flux—part 2

As the current-carrying bar moves through the magnetic field, though... there's a magnetic force that acts on it.



$$P = \frac{W}{t} = \frac{Fx}{t} = F_{app}v$$

$$P = (I\ell B)v$$

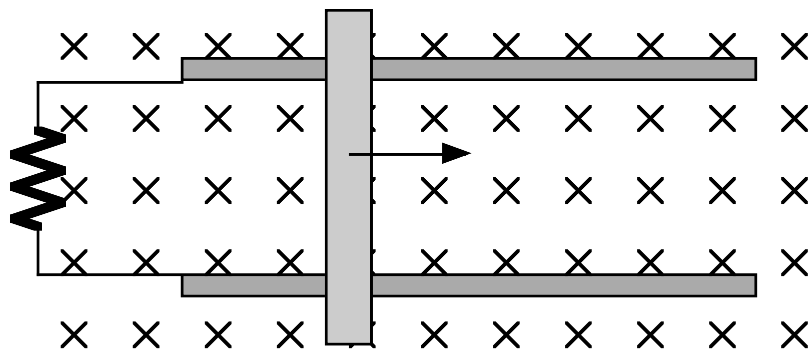
$$P = \left(\left(\frac{B\ell v}{R}\right)\ell B\right)v$$

$$P = \frac{B^2\ell^2v^2}{R}\left(= \frac{V^2}{R}\right)$$

If the bar is moving with a constant velocity *v*, we can calculate the Power delivered by the force acting on the bar.

Example 3

A bar of mass m and length *l* is given an initial velocity v_0 and released so that it slides on two parallel frictionless rails in the presence of a magnetic field *B* as shown, with load resistance *R* attached. Find *v* as a function of time.



Example 3—Answer

$$F = -I\ell B \text{ and } F = ma = m\frac{dv}{dt}, \text{ so } -I\ell B = m\frac{dv}{dt}$$

$$I = \frac{B\ell v}{R}$$

$$-\frac{B\ell v}{R}(\ell B) = m\frac{dv}{dt} \quad \text{Rearrange to get :}$$

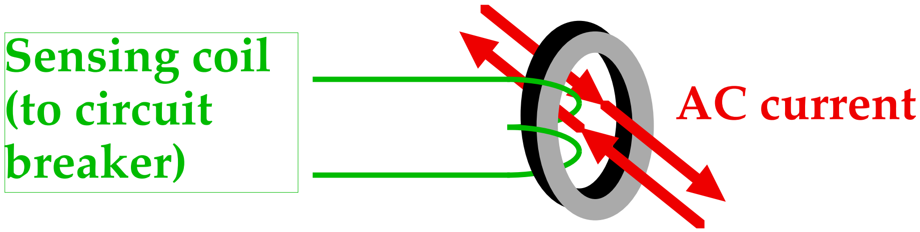
$$\frac{dv}{v} = -\frac{B^2\ell^2}{mR}dt, \text{ then integrate both sides to get}$$

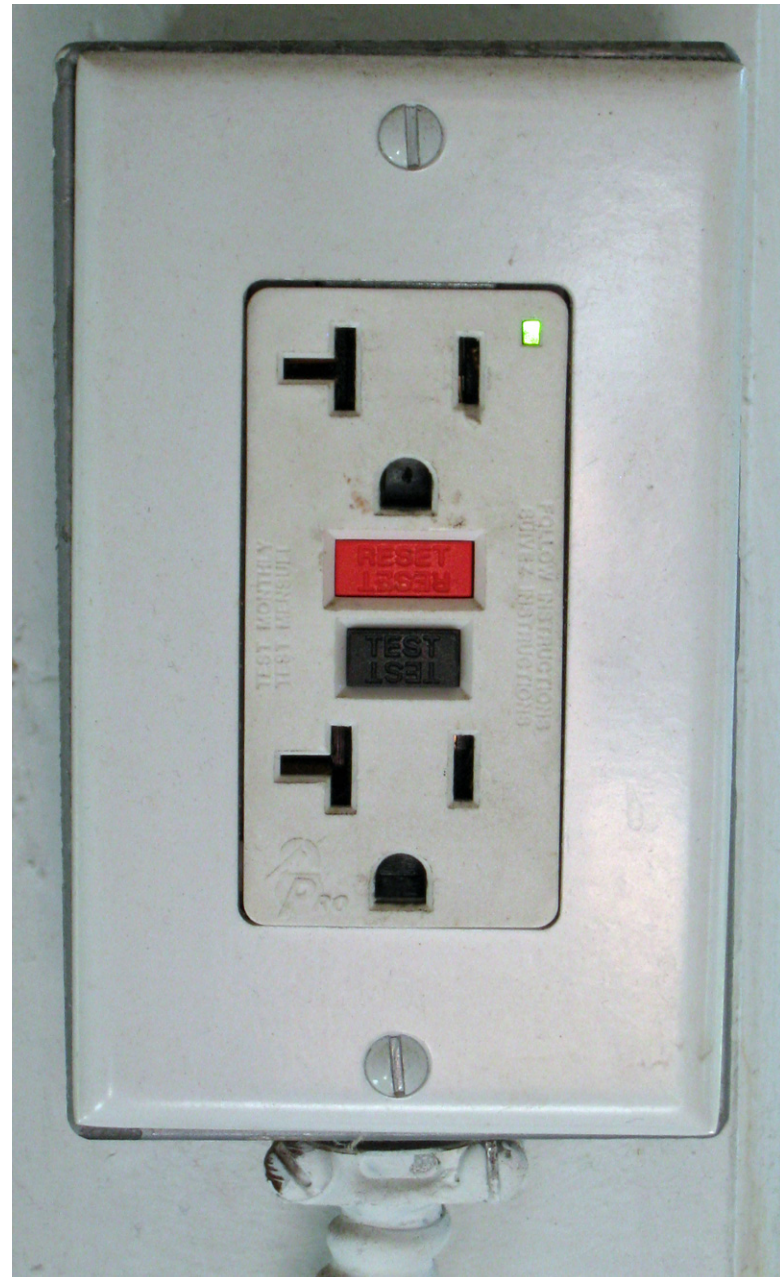
$$\int_{v_o}^{v}\frac{dv}{v} = \int_{0}^{t} -\frac{B^2\ell^2}{mR}dt$$

$$\ln\left(\frac{v}{v_o}\right) = -\frac{B^2\ell^2}{mR}t, \text{ or (with } \tau = 1\Big/\frac{B^2\ell^2}{mR}\text{): } \ln\left(\frac{v}{v_o}\right) = -t/\tau$$

$$\frac{v}{v_o} = e^{-t/\tau}, \text{ or } \quad v = v_o e^{-t/\tau}$$

Real Life Application





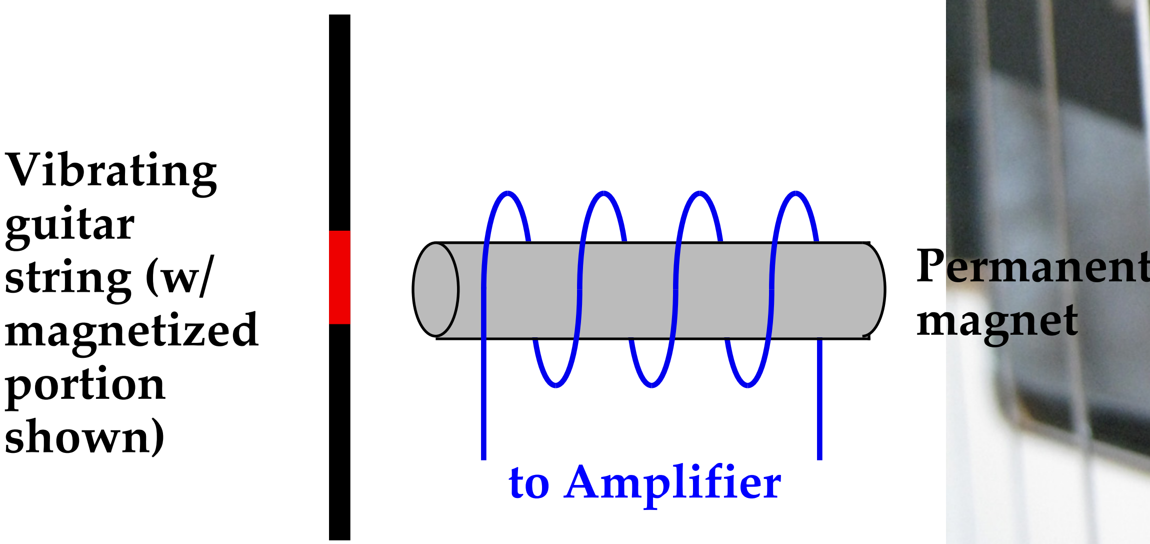
Ground Fault Interrupter

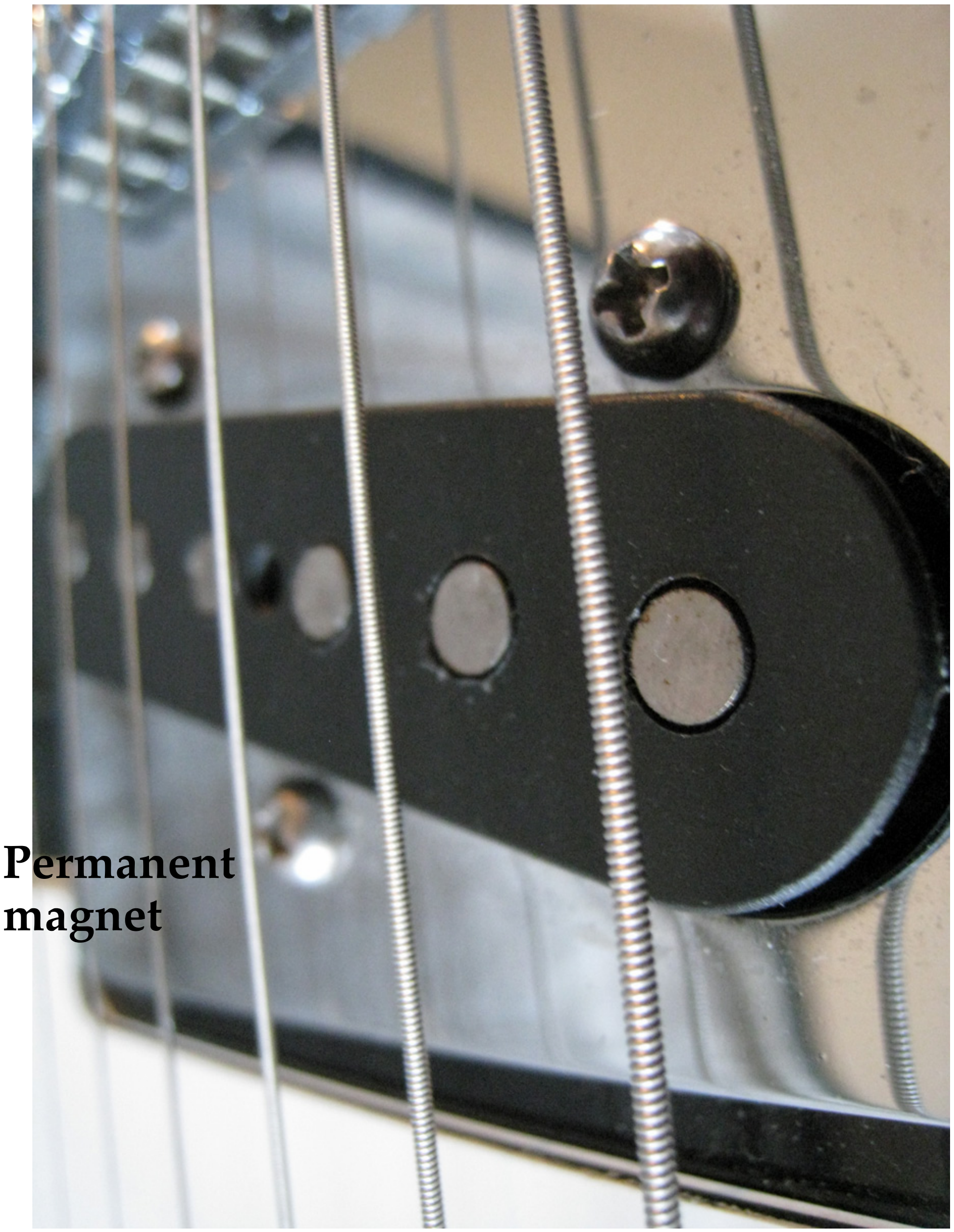
1. No net current enclose by sending coil, so no magnetic field.
2. If there's a *ground fault* (current going out but not back), magnetic field is created, increasing flux, and inducing an emf that activates the circuit breaker.

Real Life Application

Electric Guitar Pickup

Vibrating string produces a change in magnetic flux in the coil, which is transmitted as an emf to the amplifier.



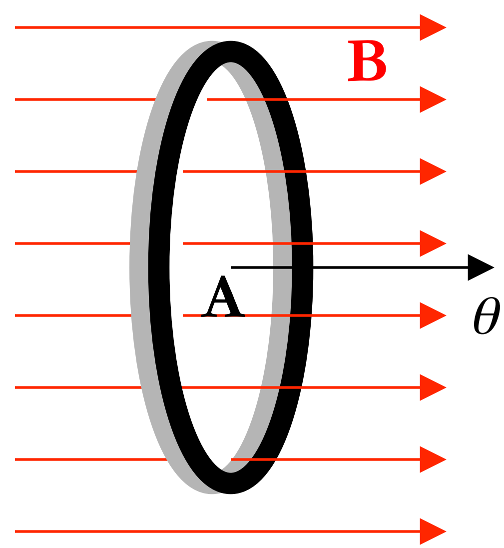


Lenz's Law

Faraday's Law allows us to calculate the magnitude of an induced emf (and thus current), but to determine *direction* of current flow, we use Lenz's Law.

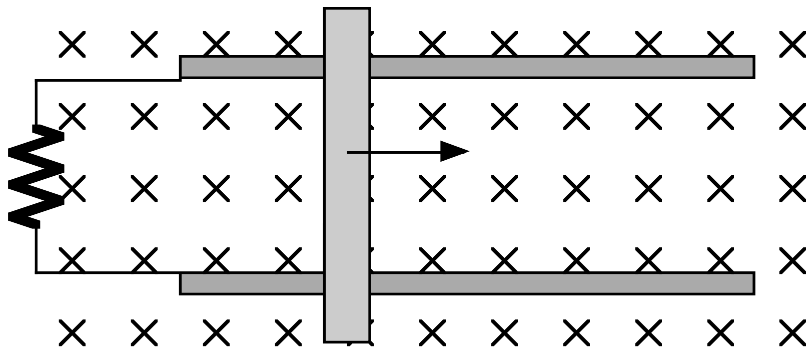
$$\mathcal{E} = \frac{-d\Phi_B}{dt}$$

Lenz's Law: the polarity of the induced emf is such that it tends to produce a current that will create a magnetic flux to oppose the change in magnetic flux through the loop.



Example 4

What direction is the current flowing in this loop?

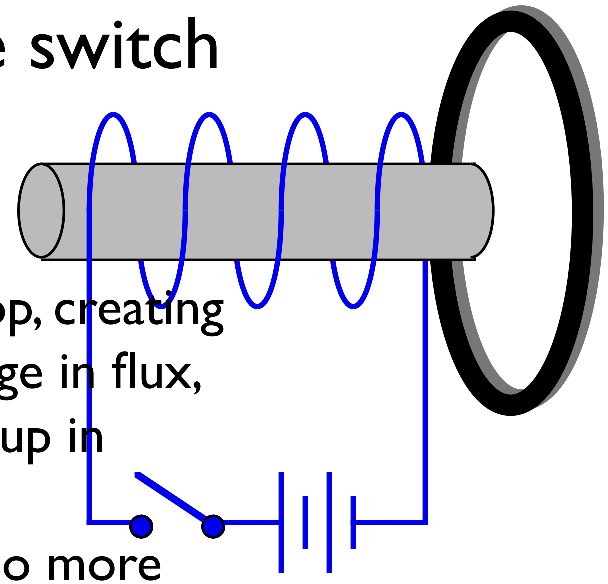


Procedure:

1. Determine direction of magnetic field
2. Determine whether flux in the area is increasing or decreasing in the direction of magnetic field
3. Use second RHR, with fingers pointing in opposite direction, to determine direction of current flow.

Example 5

Examine the following diagram. Find the direction of the induced current in the loop at the instant the switch is closed, after the switch has been closed for several seconds, and when the switch has been opened again.

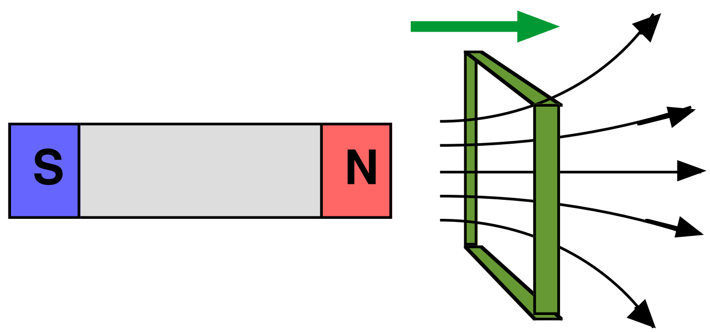


Procedure:

1. When switch is closed, current begins to flow in loop, creating increasing magnetic field to left. To oppose this change in flux, current in the loop is induced iin an "down in front, up in back" direction.
2. When switch has been closed for several seconds, no more current flows in the loop, because there is no more change in magnetic flux.
3. When the switch is opened again, the magnetic field to the left begins to decrease. Loop current opposes this decrease by having a direction that is in the same direction as the magnetic field, ie. "up in front, down in back."

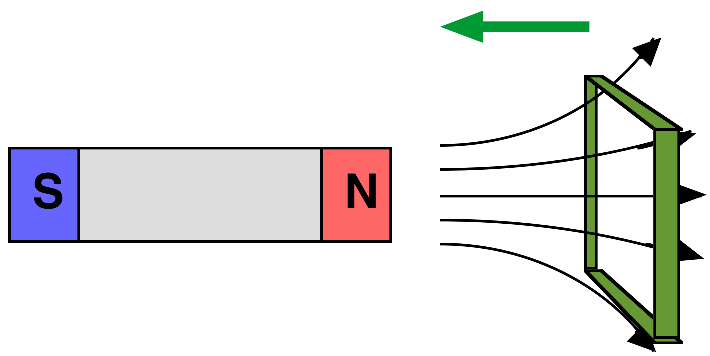
Example 6

Which direction will the induced current flow in a loop of wire as shown?



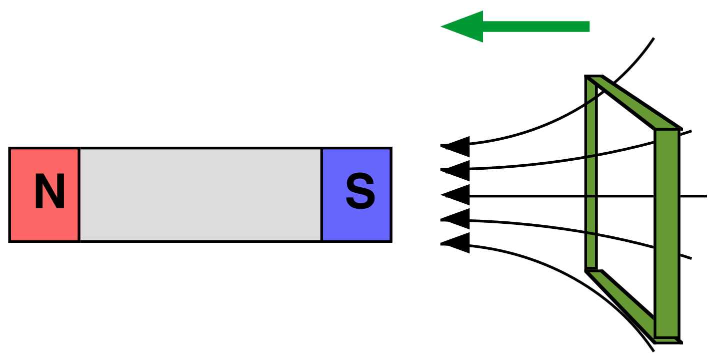
Example 7

Which direction will the induced current flow if a loop of wire as shown?



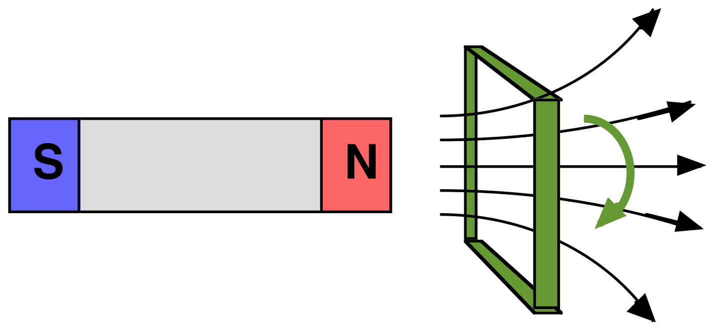
Example 8

Which direction will the induced current flow if a loop of wire as shown?



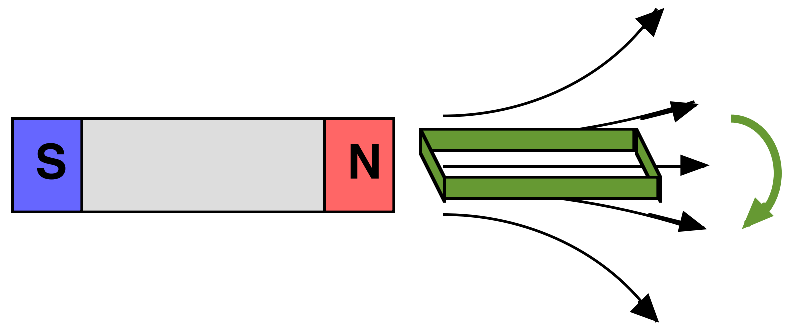
Example 9

Which direction will the induced current flow if a loop of wire as shown?



Example 10

Which direction will the induced current flow if a loop of wire as shown?

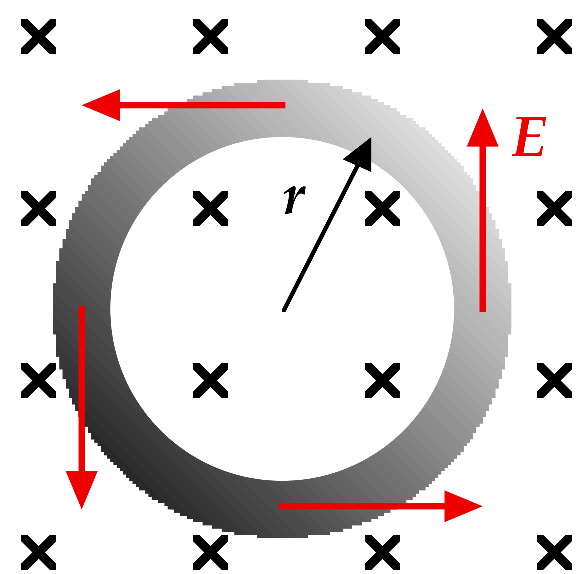


Induced EMF & Electric Field

We've determined that a changing magnetic flux induces an emf and a resulting current flow in a conducting loop, so... an electric field must be present! ($V = -\int E \bullet ds$)

For the conducting loop in a magnetic field shown here, if the magnetic field changes over time, Faradays law

($\mathcal{E} = \frac{-d\Phi_B}{dt}$) implies that the induced E field must be *tangent* to the loop.

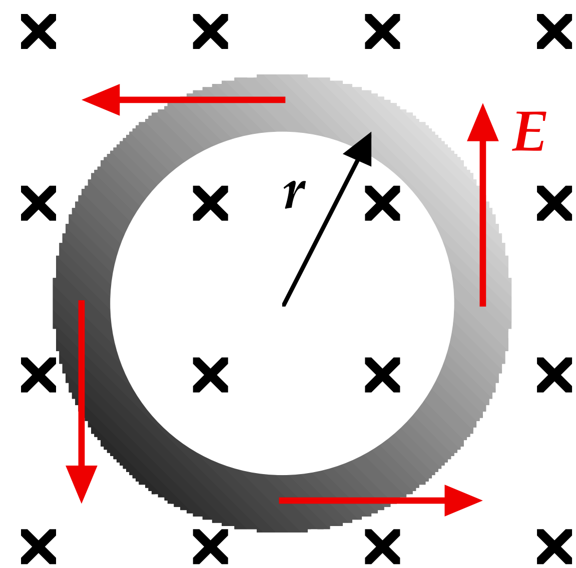


Faraday's Law, revised

Because we can determine the electric potential V (or ?) for any closed path integral $V = -\int E \bullet ds$, we can write

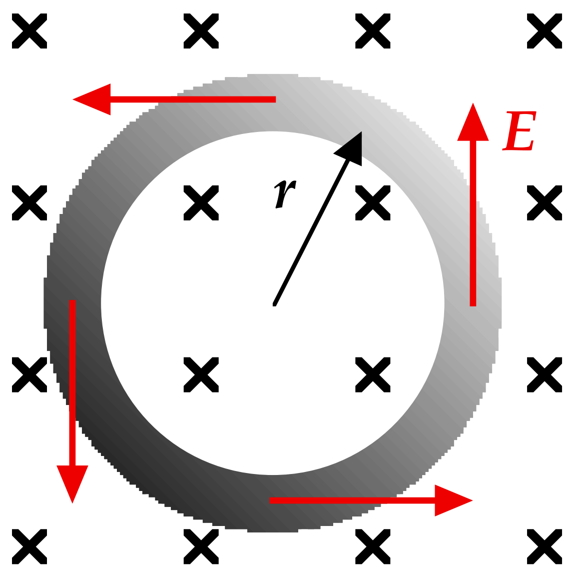
$$\oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

E is a nonconservative, time-varying field generated by the changing magnetic field B.



Example 11

Determine magnitude of E as a function of r and changing B.



$$W = qV$$

$$\mathcal{E} = \frac{-d\Phi_B}{dt}$$

$$W = q\left(\frac{-d\Phi_B}{dt}\right)$$

$$W = Fd$$

$$W = (qE)(2\pi r)$$

$$(qE)(2\pi r) = q\left(\frac{-d\Phi_B}{dt}\right)$$

$$E = \frac{-d\Phi_B / dt}{2\pi r}$$

For a circle, $A = \pi r^2$

$$E = -\frac{1}{2\pi r}\frac{dB\pi r^2}{dt} = -\frac{r}{2}\frac{dB}{dt}$$

Demo

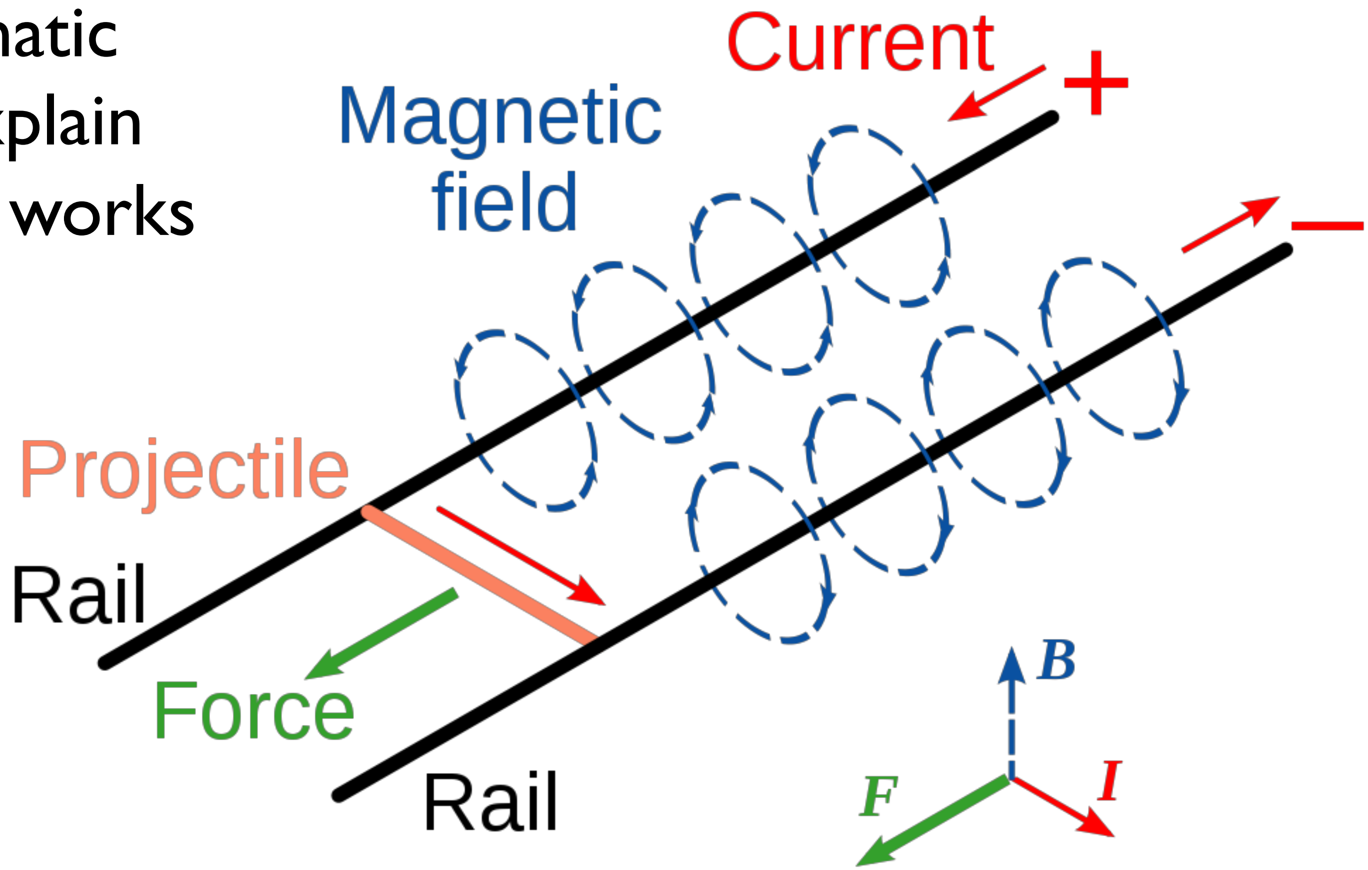
A changing magnetic field in the vicinity of a loop of wire produces a flow of current.

Demo

A changing current flow in a loop produces a changing magnetic field in the vicinity of that current, which induces a changing current in other loops in the vicinity of that magnetic field, which produces a changing magnetic field...

Example 12

Given the schematic diagram here, explain how a "rail gun" works to accelerate a projectile.



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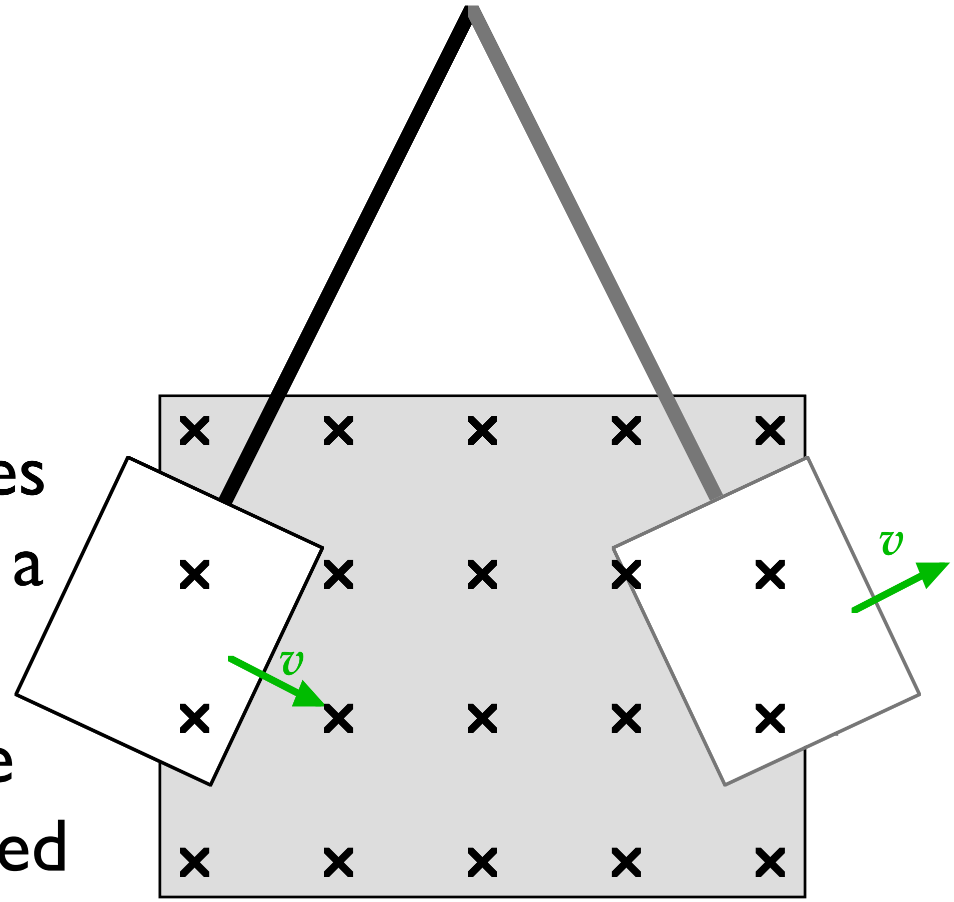
https://www.youtube.com/watch?v=eObepuHvYAw

Eddy Currents

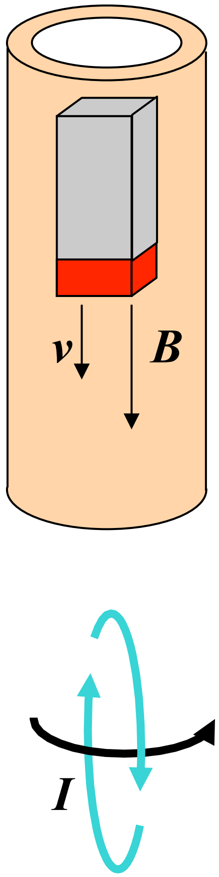
If changing magnetic flux produces emf and currents in a circuit, those same changes in magnetic flux can induce eddy currents (circulating free charges in a bulk metal moving through a magnetic field).

Because eddy currents produce retarding forces, they can be used in as braking systems for mass transit systems. If eddy currents are not desired, then the bulk metal is often split into thin layers that are laminated together.



Demo

Examine these additional examples of eddy currents in action.



Maxwell's Equations

At this point in the year, we know all four of Maxwell's equations, which summarize everything we know about electricity and magnetism.

$$\oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \frac{Q}{\varepsilon_o}$$

Gauss's Law—electric charge distributions produce electric fields

$$\oint \vec{\mathbf{B}} \bullet d\vec{\mathbf{A}} = 0$$

Gauss's Law for Magnetism—net magnetic flux through any closed surface is 0 (no magnetic monopoles)

$$\oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

Faraday's Law of Induction—relates emf of a closed path to the rate of change of magnetic flux through surface.

$$\oint \vec{\mathbf{B}} \bullet d\vec{\mathbf{s}} = \mu_o I \quad (+\mu_o \varepsilon_o \frac{d\Phi_E}{dt})$$

Ampere's Law—relates currents & magnetic fields

Ch 32 – Inductance

Faraday's Law Implications

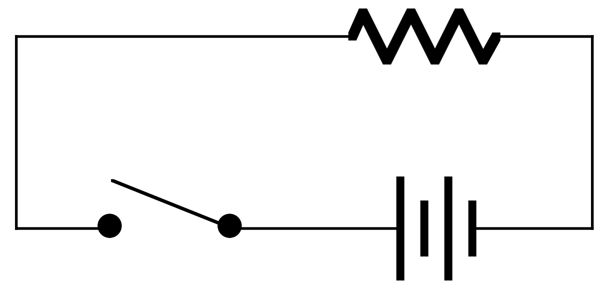
We've already learned that a changing magnetic flux induces emf and currents = *electromagnetic induction*.

The consequences of that induction include:

- Self-induction
- inductors
- mutual induction
- transformers
- RL circuits
- RLC circuits...

Self-Inductance - Qualitative

Examine this simple circuit. When we throw the switch, we don't just get an immediate current flow I=V/R from the battery...

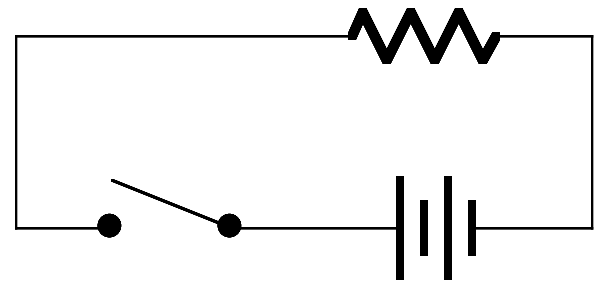


According to Faraday's Law:

- current begins to flow, increasing w/time
- As current increases in circuit, so does magnetic flux through the loop
- Increasing flux induces an emf that opposes the change in net magnetic flux through the loop (Lenz's Law)
- The opposing emf results in a gradual increase in current.

Self-Inductance - Quantitative

We can quantify the self-inductance as follows:



$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

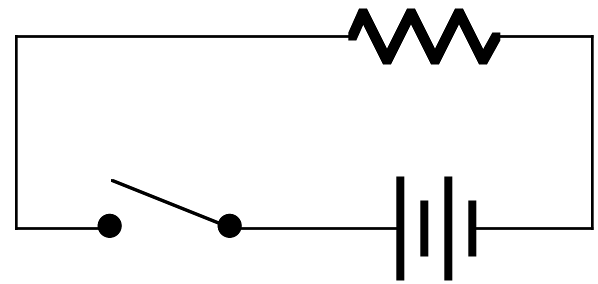
$\Phi_B \propto B \propto I$, so $\mathcal{E}$ is proportional to dI, and we can write

$$\mathcal{E} = -L\frac{dI}{dt}, \text{ or } \quad L = -\frac{\mathcal{E}_L}{dI/dt}$$

The unit of inductance is the [Volt-second]/[Ampere], also called a "Henry".

Self-Inductance cf. Resistance

Compare the inductance equation with Ohm's Law and we see some interesting similarities:



$$L = -\frac{\mathcal{E}_L}{dI/dt} \qquad R = \frac{\mathcal{E}}{I}$$

Inductance of a Coil

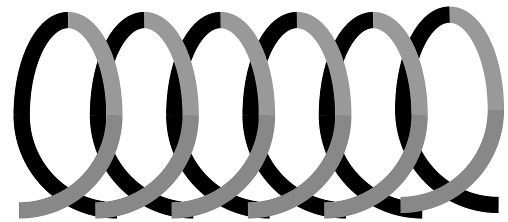
Increasing the number of loops in a circuit increases the magnetic flux proportionally, and thus increases the inductance of a circuit.

$$\mathcal{E} = -L\frac{dI}{dt}, \text{ and } \quad \mathcal{E} = -N\frac{d\Phi_B}{dt}$$

$$L_{coil} = N\frac{\Phi_B}{I}$$

Inductance of a Solenoid

In some circuits, this "resistance to a change in current flow" is actually desired. Such circuits include an "inductor," a coil that's often in the shape of a solenoid.



How do we calculate the inductance of a solenoid, with N turns of wire in a length ℓ ?

$$L = N\frac{\Phi_B}{I} = N\frac{BA}{I}$$

$$B_{solenoid} = \frac{\mu_o N I}{\ell}, so$$

$$L = N\frac{\left(\frac{\mu_o N I}{\ell}\right)A}{I} = \frac{\mu_o N^2 A}{\ell}$$

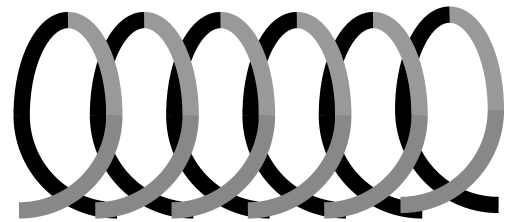
$$L = \frac{\mu_o N^2 A}{\ell}$$

$$N = n\ell$$

$$L = \mu_o n^2 \ell A = \boxed{\mu_o n^2 V}$$

Example 13

Calculate the inductance of a solenoid that has 300 turns of wire in a 25.0 cm length, w/ a cross-sectional area of 400 cm^2. Then calculate the self-induced emf in the solenoid if the current is decreasing at a rate of 50.0 A/s.



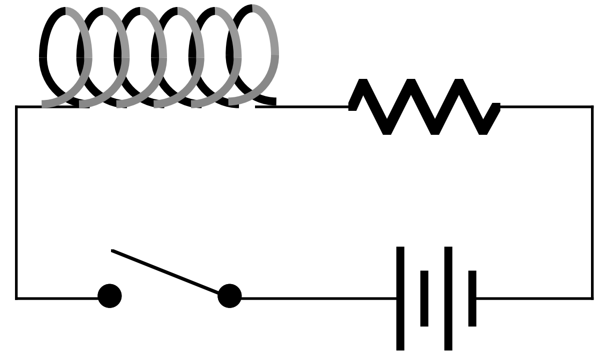
$$N = 300; \ell = 0.25m, A = 0.04m^2$$

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi e-7)(300)^2(0.04)}{(0.25)} = 1.8e-2H$$

$$\mathcal{E} = -L\frac{dI}{dt} = -(1.8e-2)(50) = 0.90V$$

RL Circuits

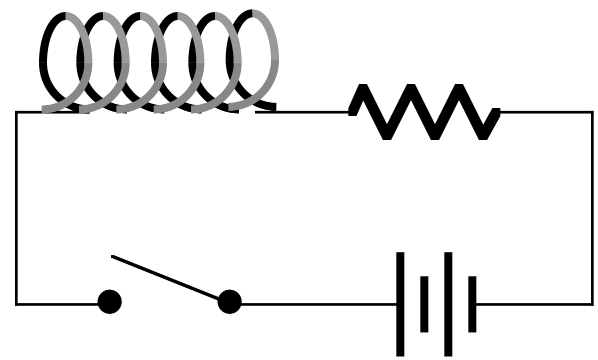
Circuits with a coil in them, as the one shown here, have a self-inductance due to the presence of the coil—the self inductance due to the geometry of the circuit, in these cases, is considered negligible.



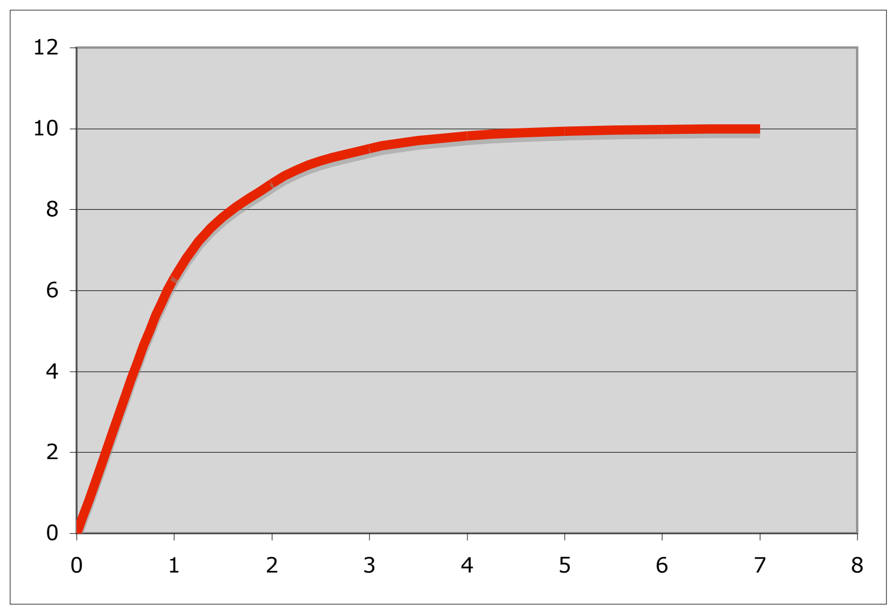
When the switch is thrown, current begins to flow in the circuit. As current increases with time, the inductor resists this increase with a self-induced *back emf* according to $\mathcal{E} = -L\dfrac{dI}{dt}$. Note that there is a *drop* in potential from left to right as current flows across the inductor.

RL Circuits, quantitatively

$$I = \frac{\varepsilon}{R}(1 - e^{-t/\tau}), \text{ where } \tau = \frac{L}{R}$$



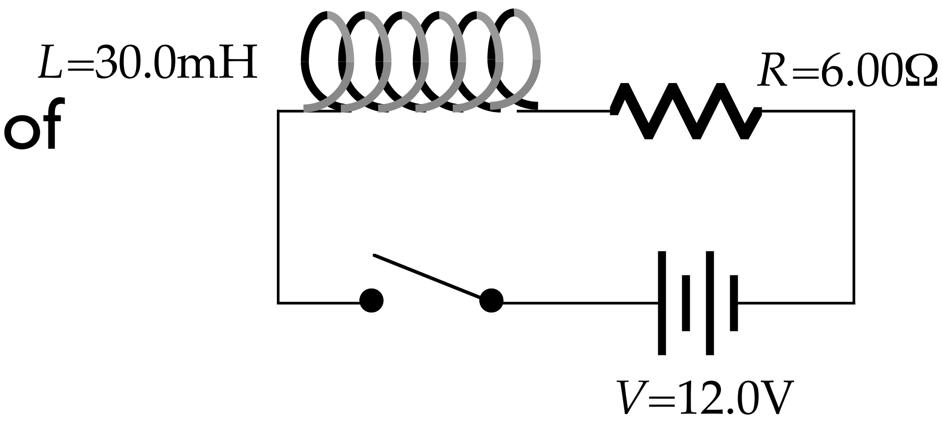
The time constant here represents the amount of time it takes for the current to reach 0.632 of final I (=V/R).



Example 14

The switch is closed at time t=0s. Find the time constant of the circuit, then find the current:

a. just after the switch is closed
b. at t=2.00 ms
c. at one time constant, and
d. a long time after the switch has been closed.



$$\tau = \frac{L}{R} = \frac{30e-3H}{6\Omega} = 5e-3s$$

$$I = \frac{\varepsilon}{R}\left(1-e^{-t/\tau}\right) = \frac{12}{6}\left(1-e^{-t/0.005}\right)$$

$$I = 0$$

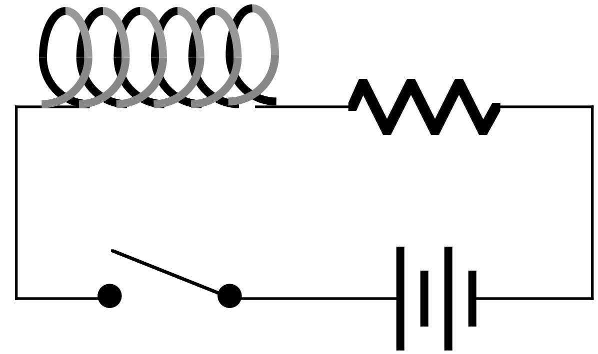
$$I = 0.66A$$

$$I = 1.26A$$

$$I = 2A$$

Energy in Inductors-cont'd

So $LI(dI/dt)$ represents the rate that energy is getting stored in the inductor. How can we determine the amount of energy stored in the inductor, U_B?



$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

$$\mathcal{E} = IR + L\frac{dI}{dt}$$

$$I\mathcal{E} = I^2R + LI\frac{dI}{dt}$$

$$\frac{dU_B}{dt} = LI\frac{dI}{dt}$$

$$\int_0^U dU_B = \int_0^I LI\,dI$$

$$U_B = \frac{1}{2}LI^2$$