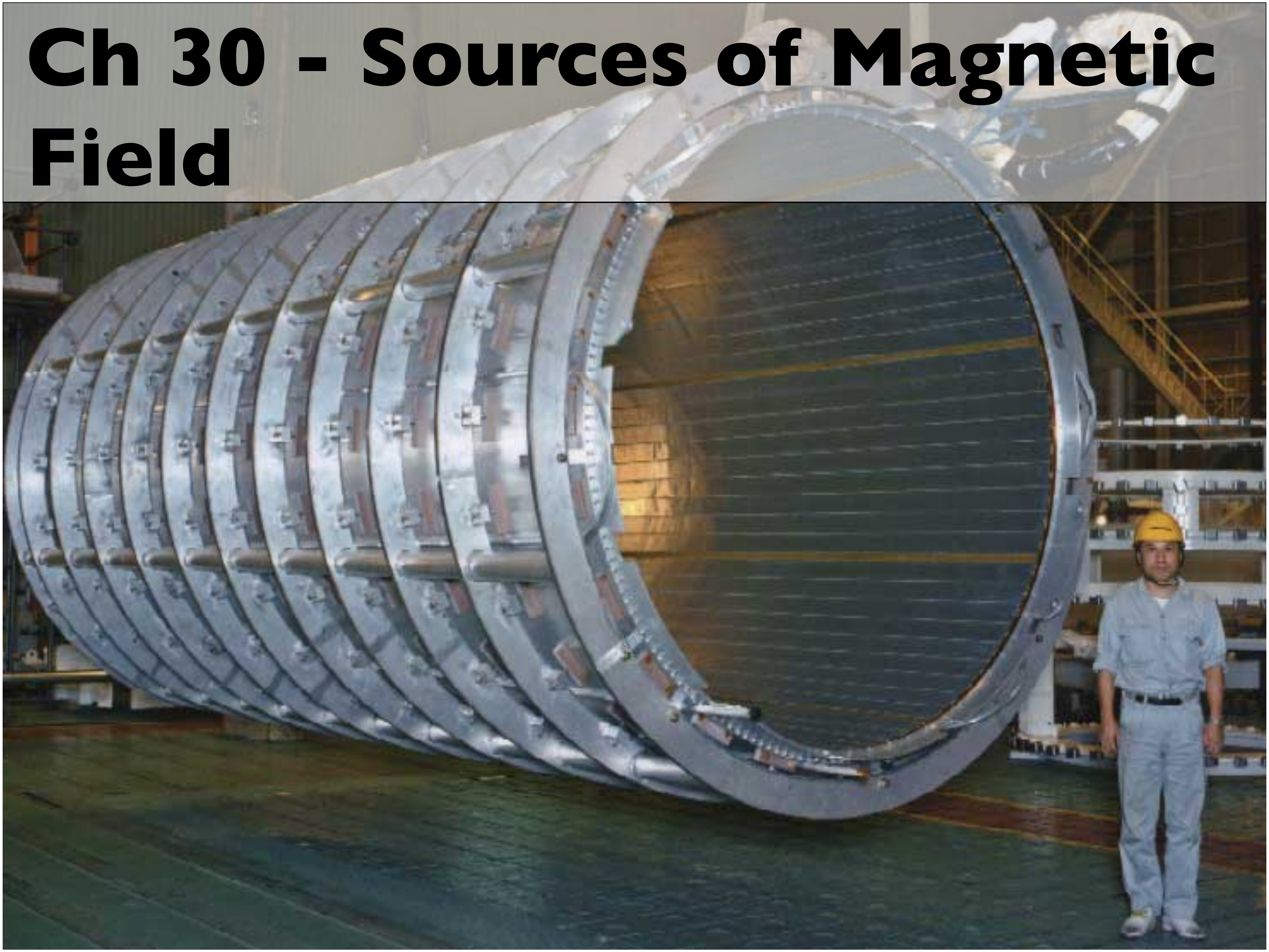
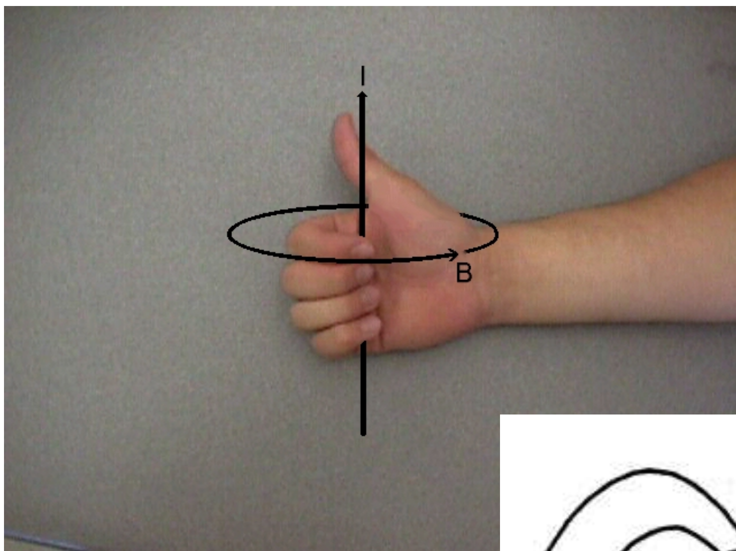
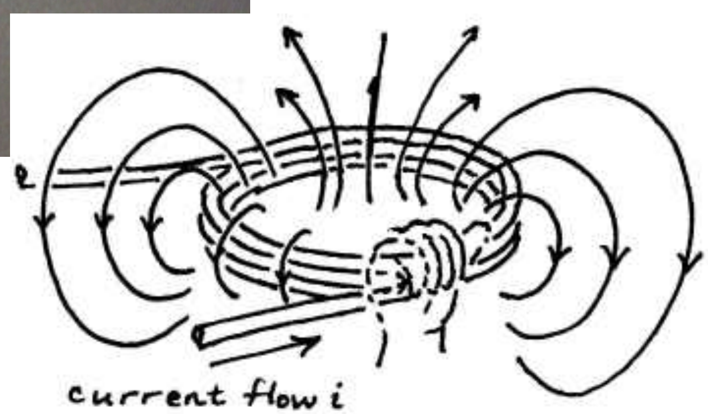
Ch 30 - Sources of Magnetic Field



Currents produce Magnetism?

1820, Hans Christian Oersted: moving charges produce a magnetic field. The direction of the field is determined using a RHR.

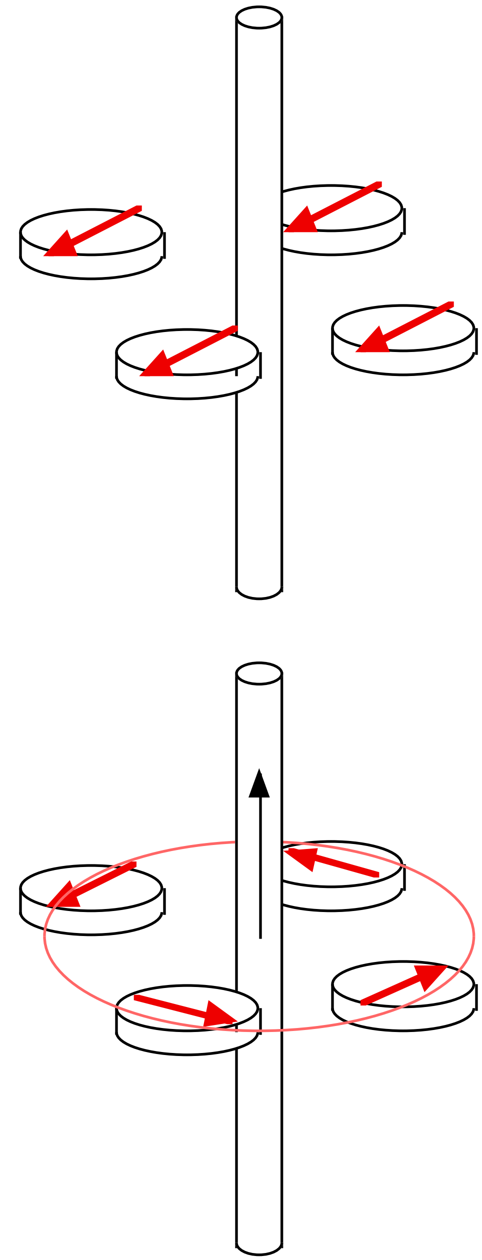




Oersted (1820)

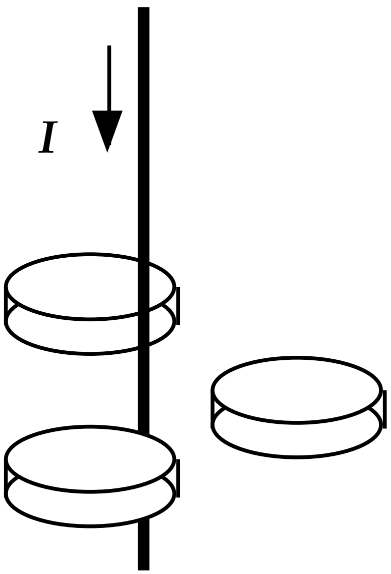
If the wire is grasped with the right hand, with the thumb in the direction of current flow, the fingers curl around the wire in the direction of the magnetic field.

The magnitude of B is the same everywhere on a circular path perpendicular to the wire and centered on it. Experiments reveal that *B* is proportional to *I*, and inversely proportional to the distance from the wire.



Example 3

Predict the orientation of the compass needles.



The Biot-Savart Law

$$d\vec{\mathbf{B}} = k_m \frac{I \; d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

determines direction of field

magnetic field at a point P perpendicular to segment of wire *ds* & perpendicular to unit vector **r** from *ds* to P.

steady current in direction *ds*

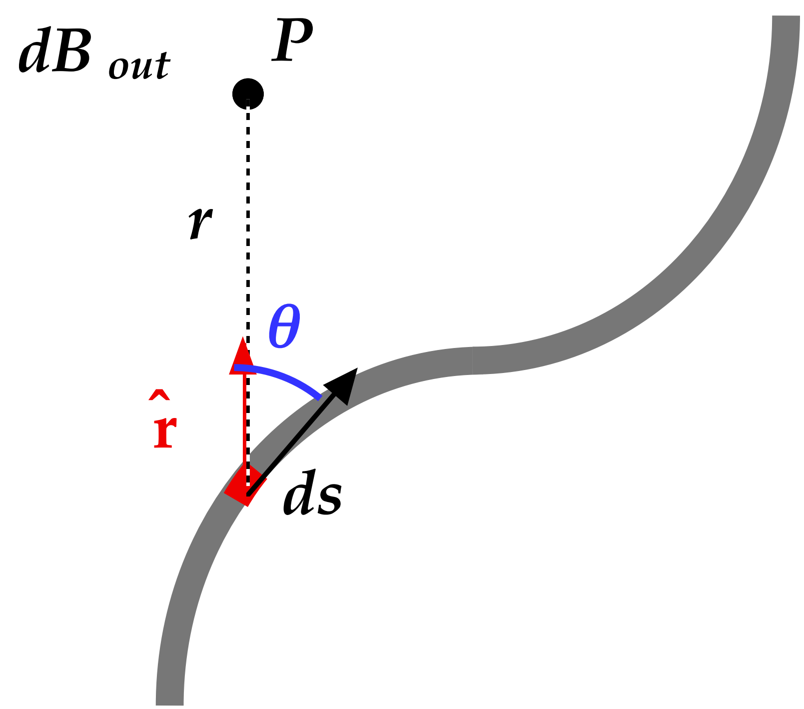
element of wire

$k_m = 10^{-7}$ T•m/A

$k_m = \mu_o / 4\pi$, where

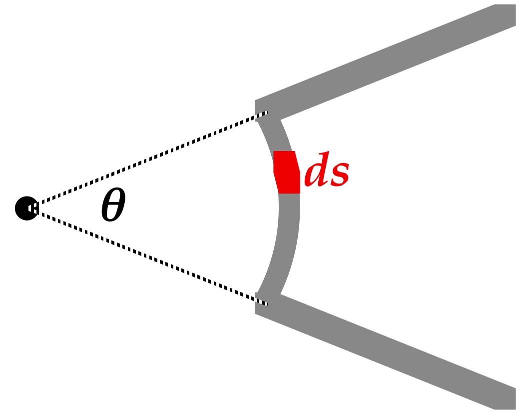
μ_o = *permeability* of free space

$\mu_o = 4\pi \times 10^{-7}$



Example 1

Determine the magnitude and direction of the magnetic field at the point *O* in the diagram. (Current *I* flows from top to bottom, radius of curvature = *r*.)



For straight segments, $\mathrm{ds} \times \mathrm{r} = 0$.

For curved segment, $\mathrm{ds} \times \mathrm{r} = \mathrm{ds}$, because s and r are $\perp$.

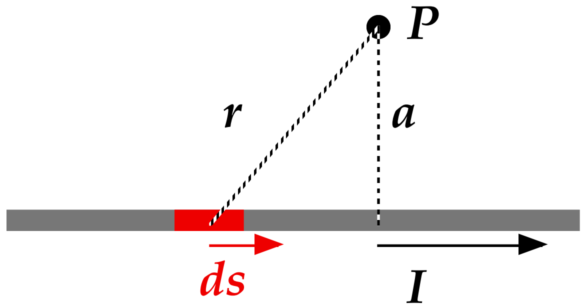
$$B = \frac{\mu_o I}{4\pi} \int \frac{ds}{r^2}$$

$$B = \frac{\mu_o I}{4\pi r^2} \int r d\theta$$

$$B = \frac{\mu_o I \theta}{4\pi r}$$

Example 2

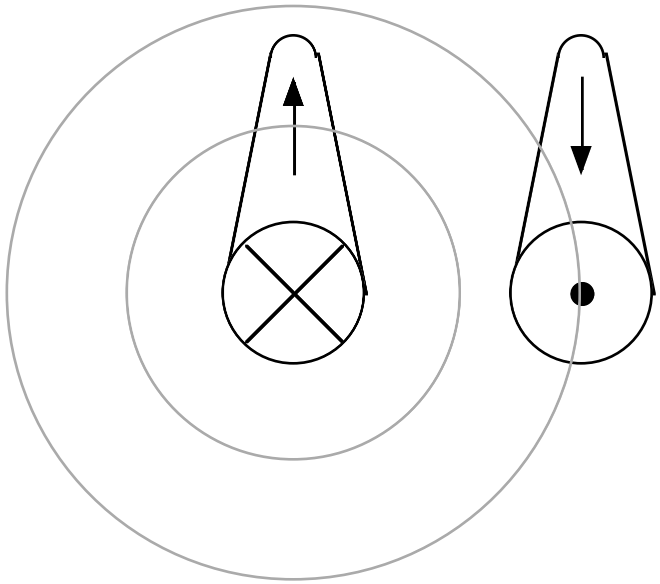
Determine the direction and total magnitude of the magnetic field at the point *P* shown here, near a long, thin, wire.



The magnetic field for an infinitely long straight wire is

$$B = \frac{\mu_o I}{2\pi a}$$

Magnetic Force between wires



$$F = I\ell \times B = I\ell B \sin\theta$$

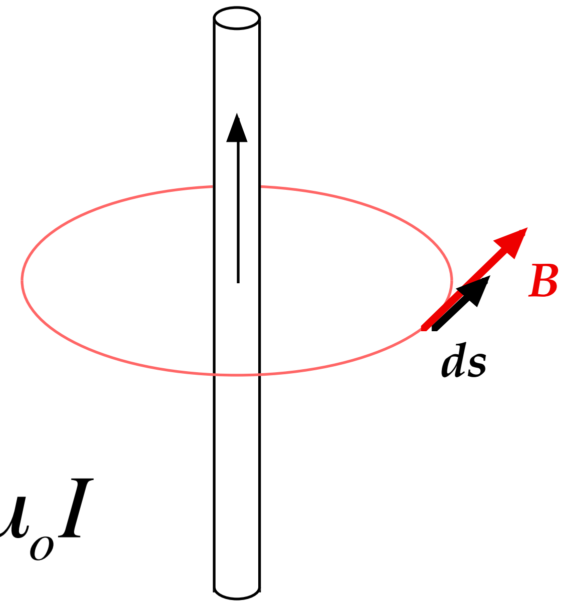
$$B \text{ (due to wire)} = \frac{\mu_o I}{2\pi a}$$

$$F = I_1 \ell \left(\frac{\mu_o I_2}{2\pi a} \right), \text{ or}$$

$$\frac{F}{\ell} = \frac{\mu_o I_1 I_2}{2\pi a}$$

Ampère's Law

If we evaluate the dot-product $\vec{\mathbf{B}} \bullet d\vec{\mathbf{s}}$ in a circle around this wire, and sum this product over the entire path of the circle, we get



$$\oint \vec{\mathbf{B}} \bullet d\vec{\mathbf{s}} = B\oint ds = \left(\frac{\mu_o I}{2\pi r}\right)(2\pi r) = \mu_o I$$

Line integral, taken around a closed path that surrounds the current.

This result is valid for *any* closed path that encloses a wire conducting a steady current, and is known as Ampère's Law:

$$\boxed{\oint \vec{\mathbf{B}} \bullet d\vec{\mathbf{s}} = \mu_o I}$$

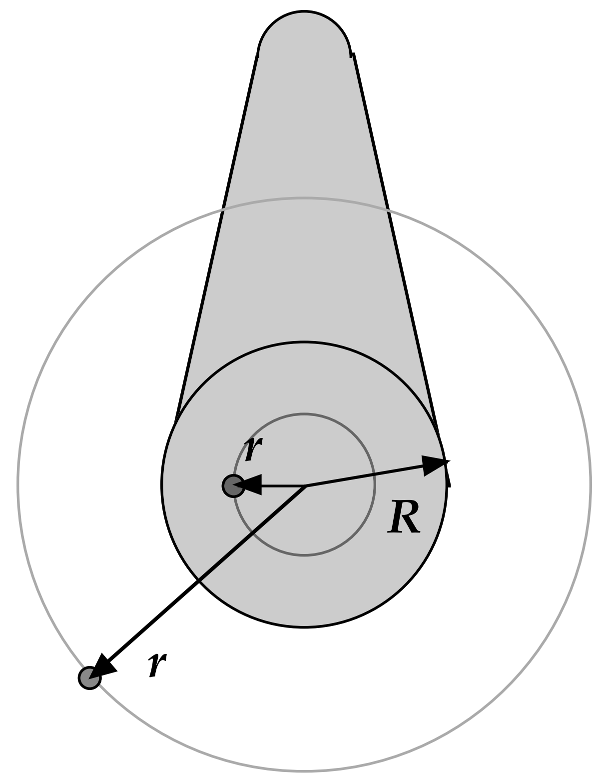
Gauss & Ampère

$$\oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_o}$$

$$\oint \vec{\mathbf{B}} \bullet d\vec{\mathbf{s}} = \mu_o I$$

Example 3

Use Ampère's Law to calculate the magnetic field at a distance *r* away from a current-carrying wire.



$$\oint B \bullet ds = \mu_o I$$

$$B\oint ds = \mu_o I$$

$$B(2\pi r) = \mu_o I$$

$$B = \frac{\mu_o I}{2\pi r} \text{ (for r > R)}$$

$$I_r < I_o$$

$$\frac{I_r}{I_o} = \frac{\pi r^2}{\pi R^2}, \; so \;\; I_r = \frac{r^2}{R^2} I_o$$

$$B = \frac{\mu_o \frac{r^2}{R^2} I_o}{2\pi r} = \frac{\mu_o r I_o}{2\pi R^2} \text{ (for r < R)}$$

Example 5

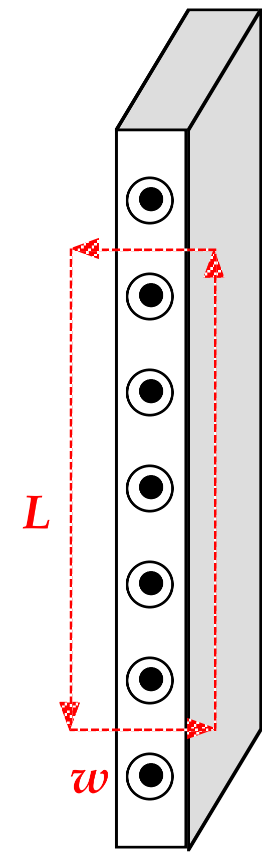
Find the magnetic field B for a thin, infinite sheet of current, carrying current of linear density ***J*** in the z direction.

$$\oint B \bullet ds = \mu_o I$$

$$B \oint ds = \mu_o (J\ell)$$

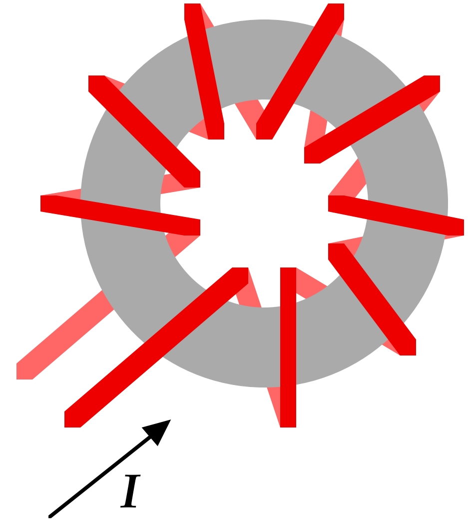
$$B(2\ell) = \mu_o (J\ell)$$

$$B = \frac{\mu_o J}{2}$$



Example 4

Use Ampère's Law to calculate to calculate the magnetic field inside a toroid of *N* loops, at a distance *r* from the center.



To determine magnetic field inside torus, draw Amperian line inside coil. The wire passes around that Amperian line *N* times.

$$\oint B \bullet ds = \mu_o I$$

$$B \oint ds = \mu_o I$$

$$B(2\pi r) = \mu_o NI$$

$$B = \frac{\mu_o NI}{2\pi r}$$

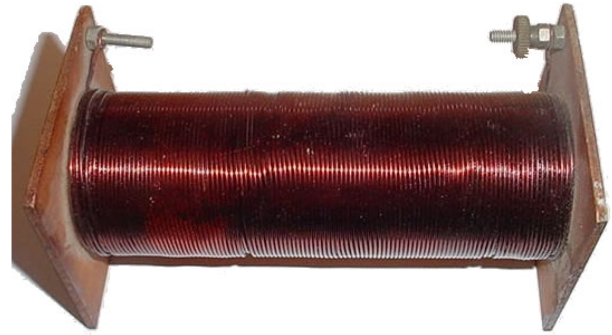
Assumptions: *r* of torus is large compared to cross-sectional radius, so B is ~ uniform within torus.

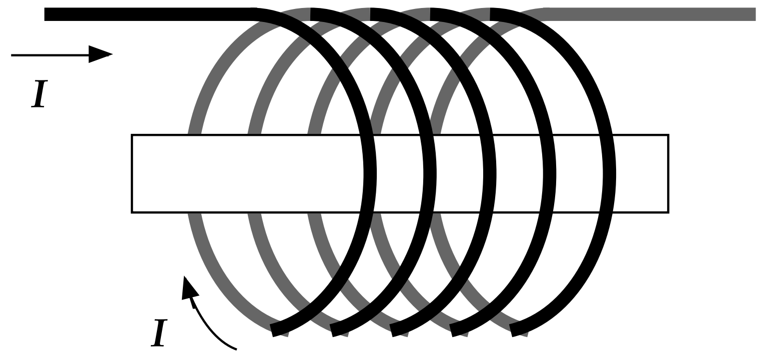
Also, for an ideal torus (closely spaced turns of wire), the external magnetic field is close to 0.

Electromagnets & Solenoids

A solenoid is a long wire wound in the form of a helix.

If the turns are closely space and the solenoid is finite in length, the field lines from a solenoid look very similar to those of a bar magnet.



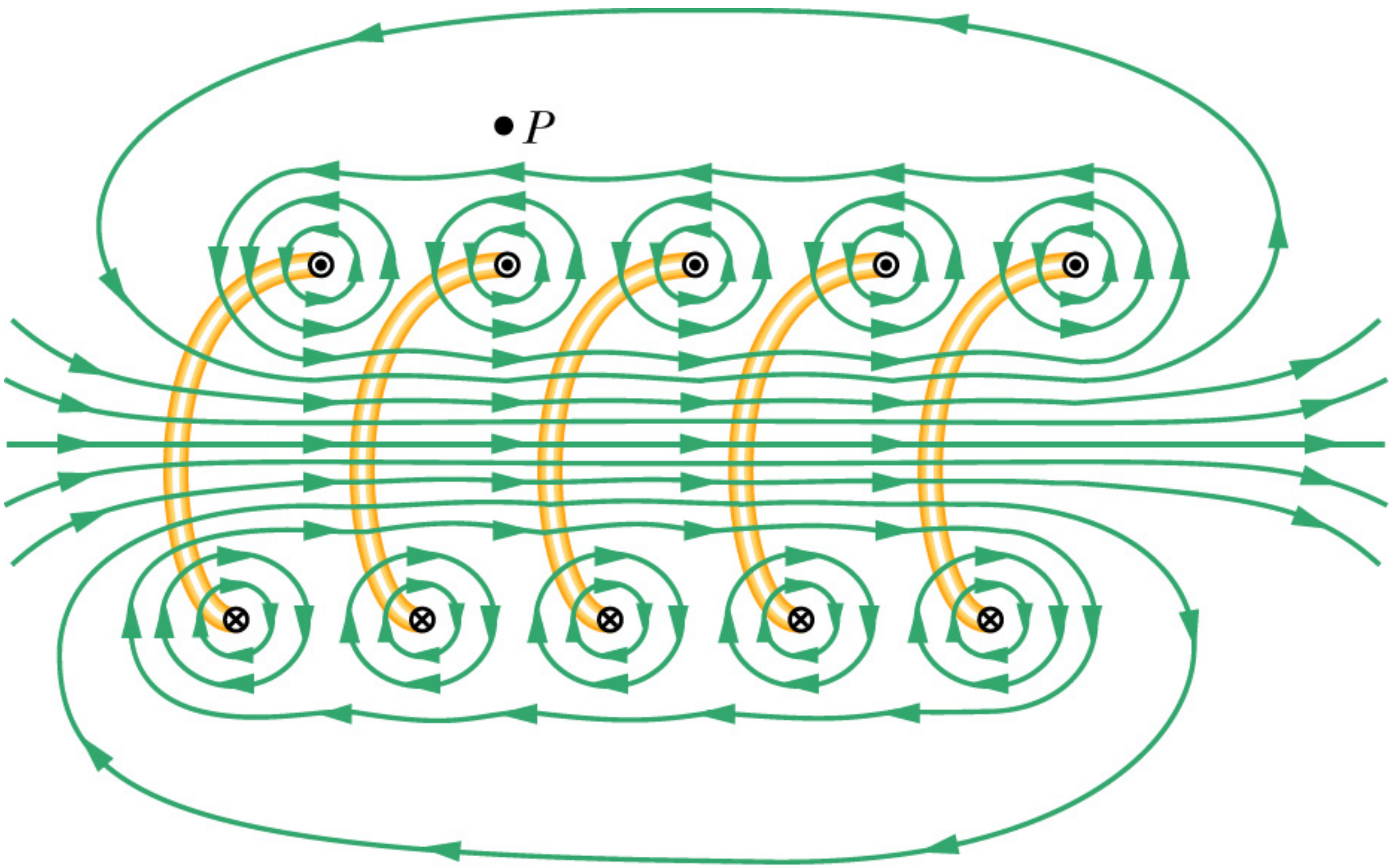


B Field for a Solenoid

Inside = strong B field

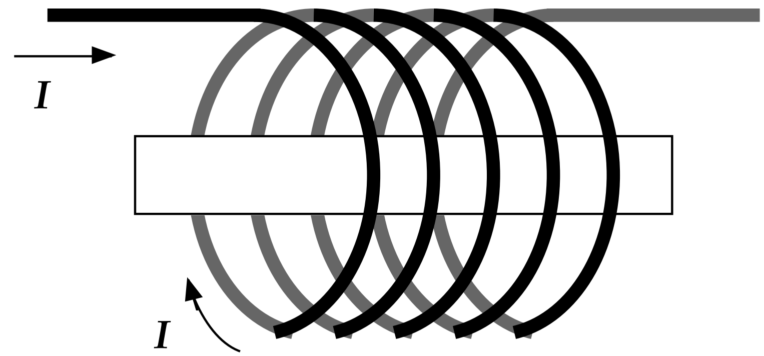
Outside = weak B field

Between coils = 0 field



Example 6

Use Ampère's Law to calculate the magnetic field of a solenoid

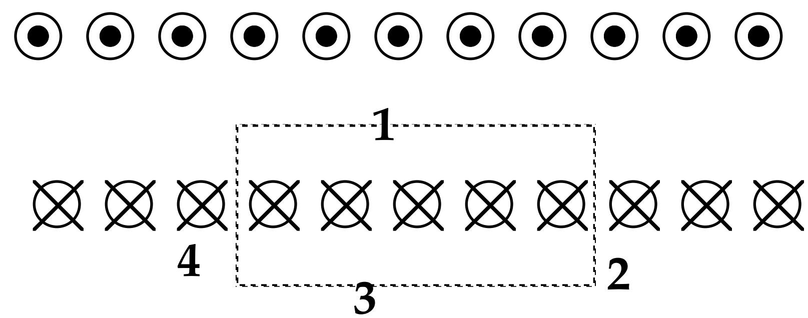


$$\oint B \bullet ds = \mu_o I$$

$$B \oint ds = \mu_o I$$

$$Bl = \mu_o NI$$

$$B = \frac{\mu_o NI}{l} = \mu_o nI$$

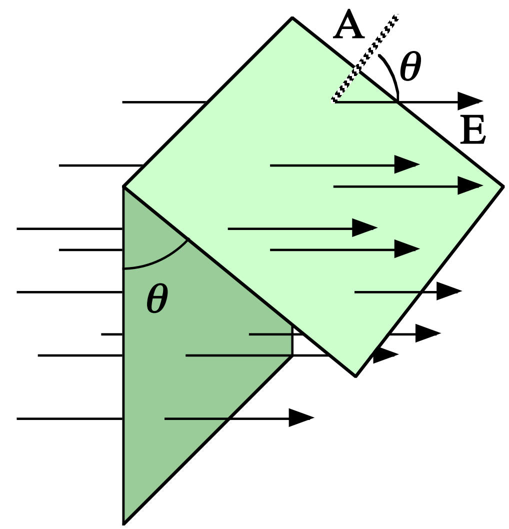


Electric Flux (Review)

Brief review of Electric Flux:

$$\Phi = \vec{\mathbf{E}} \bullet \vec{\mathbf{A}}$$

$$\Phi = EA\cos\theta$$



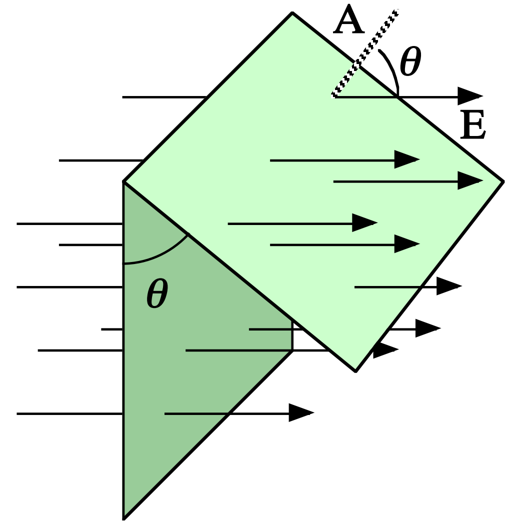
> "The net electric flux through any closed surface is equal to the net charge inside the surface divided by ε_o."

$$\Phi_c = \oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_o}$$

Magnetic Flux

$$\Phi_B = \vec{\mathbf{B}} \bullet \vec{\mathbf{A}}$$

$$\Phi_B = BA\cos\theta$$



Example 7

Find the total magnetic flux through the loop shown here.

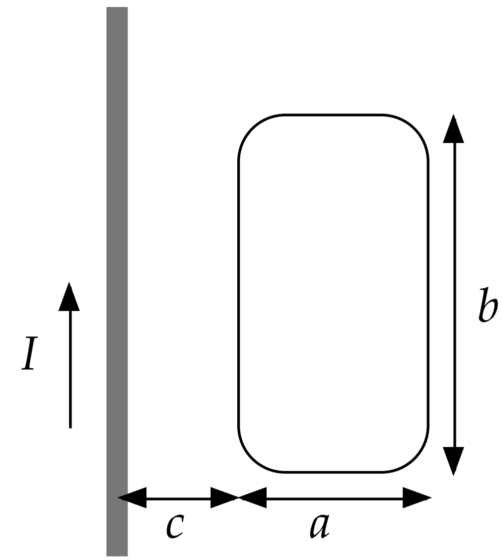
$$\Phi_B = \int B \bullet dA$$

$$\Phi_B = \int \frac{\mu_o I}{2\pi r} \bullet dA$$

$$\Phi_B = \int \frac{\mu_o I}{2\pi r} \bullet b \bullet dr$$

$$\Phi_B = \frac{\mu_o I b}{2\pi} \int_c^{a+c} \frac{1}{r} dr$$

$$\Phi_B = \frac{\mu_o I b}{2\pi} \ln\left(\frac{a+c}{c}\right)$$



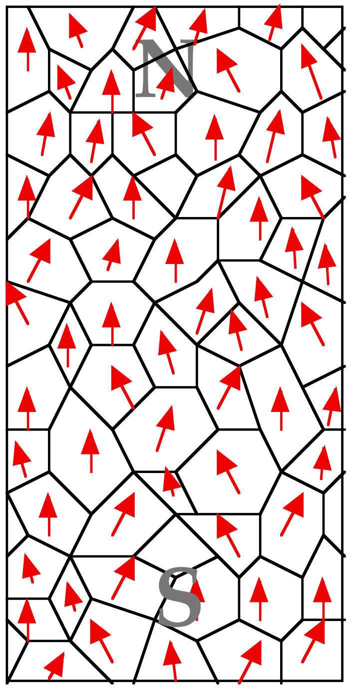
Gauss' s Laws

Electric field intensity depends only on the net internal charge.

$$\Phi_c = \oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_o}$$

Magnetic field lines are continuous, and form closed loops. Magnetic field lines created by currents don' t begin or end at any point. Therefore, the net magnetic flux through any closed surface is always zero!

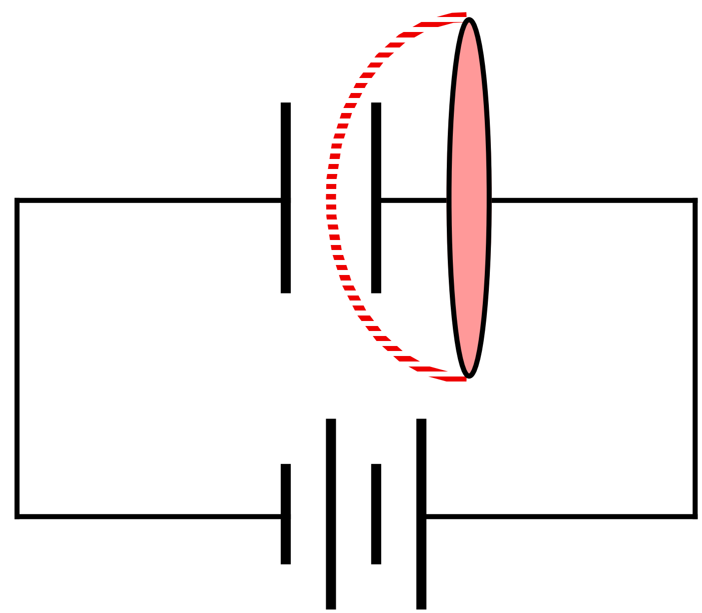
$$\Phi_B = \oint \vec{\mathbf{B}} \bullet d\vec{\mathbf{A}} = 0$$



Difficulty w/Ampère's Law

$$\oint \vec{\mathbf{B}} \bullet d\vec{\mathbf{s}} = \mu_o I$$

Two problems:
What if current is changing?
What if Amperian path *doesn't* enclose current?

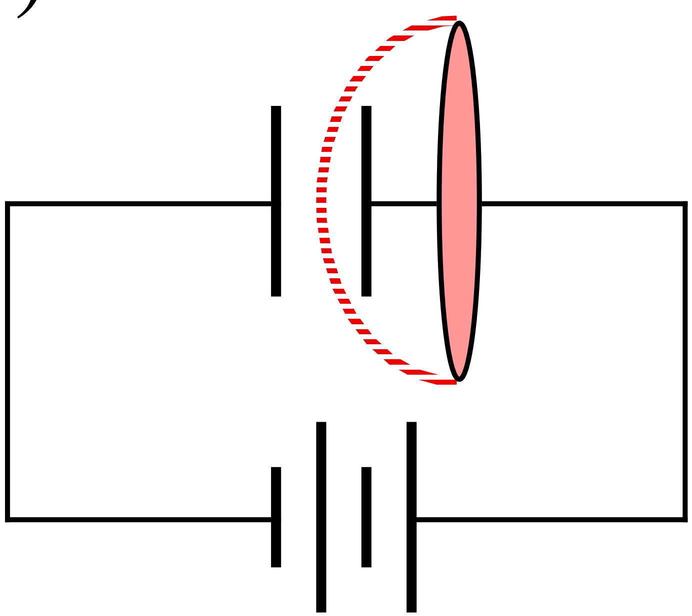


Ampère's Law (redux)

$$I_d \equiv \varepsilon_o \frac{d\Phi_E}{dt}$$

$$\oint \vec{B} \bullet d\vec{\mathbf{s}} = \mu_o (I + I_d)$$

$$\oint \vec{B} \bullet d\vec{\mathbf{s}} = \mu_o (I + \varepsilon_o \frac{d\Phi_E}{dt})$$



Ampère-Maxwell Law

$$\oint \vec{B} \bullet d\vec{\mathbf{s}} = \mu_o (I + \varepsilon_o \frac{d\Phi_E}{dt})$$