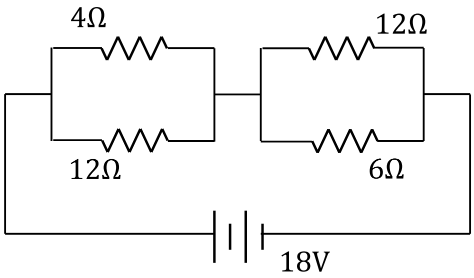
This test covers electrical current, resistance, emf, electrical power, Ohm's Law, Kirchhoff's Rules, and RC Circuits, with some problems requiring a knowledge of basic calculus.

**Part I. Multiple Choice**



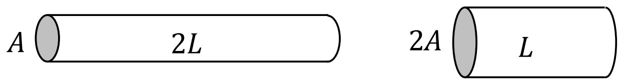

1. The equivalent resistance of the four resistors, connected as shown above, is:
   a. 7 Ω
   b. 4 Ω
   c. 2.7 Ω
   d. 8.5 Ω
   e. 14.4 Ω

2. Two conducting wires, $W_1$ and $W_2$, are made of two different materials, the first with a resistivity of $\rho_1$, the second with resistivity $\rho_2 = \frac{\rho_1}{2}$. Which of the following pairs of cylindrical wires, with indicated cross sectional area and length, will have equal resistances?

Wire $W_1$ — Wire $W_2$

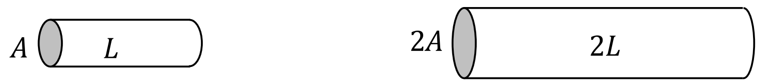
a.

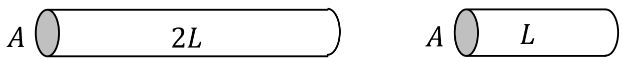


b.



c.



d.



e.
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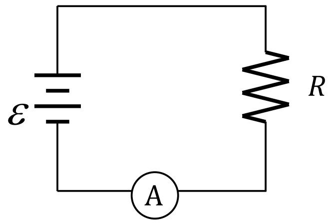
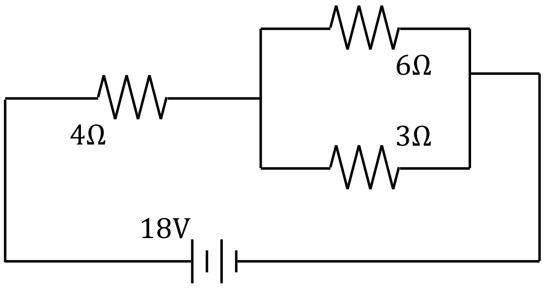



3. In the circuit above, the emf potential $\mathcal{E}$ produced by the battery is 24 Volts, and the load resistance $R$ is 10 Ω. If the ammeter reads 2.0 Amps in the circuit, the terminal voltage of the battery in Volts is:
   a. 24
   b. 26
   c. 22
   d. 20
   e. 2




4. Three resistances of 4Ω, 3Ω, and 6Ω are connected to an 18V battery as shown in the circuit above. The power dissipated by the 3Ω resistor is:
   a. 3W
   b. 12W
   c. 8W
   d. 6W
   e. 4W

5. The cost of the electricity for running a 120-Volt computer for 10 hours is a little less than 25 cents. If electricity costs 10 cents per kiloWatt-hour, what is the current running through the computer?
   a. About 2 A
   b. About 4 A
   c. About 1 A
   d. About 3 A
   e. About 0.3 A

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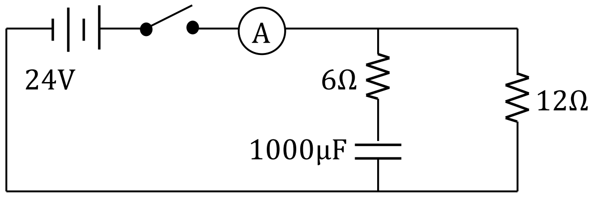
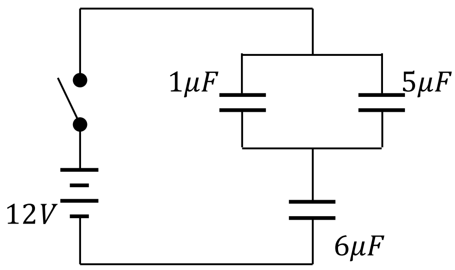
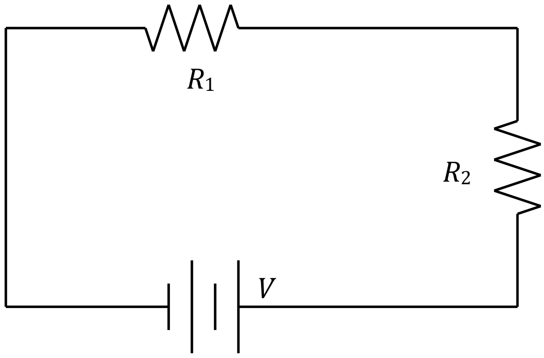



6. An 24-Volt battery is connected to a switch, an ammeter, a 6-Ω resistor, a 1000μF capacitor, and a 12-Ω resistor as shown above. The switch is closed at time $t = 0$. What is the current indicated by the ammeter immediately after the switch is closed?
   a. 0 A
   b. 1 A
   c. 2 A
   d. 4 A
   e. 6 A




1. Three capacitors, of capacitance $1\mu F$, $5\mu F$, and $6\mu F$, are arranged in a circuit with a switch and a 12-V battery as shown above. The switch is closed and the capacitors are allowed to fully charge. What is the magnitude of the charge on each plate of the $5\mu F$ capacitor?
   a. $1\ \mu C$
   b. $5\ \mu C$
   c. $6\ \mu C$
   d. $5/36\ \mu C$
   e. $30\ \mu C$

**Part II. Free Response**




2. Two resistors are placed in series with a battery of potential $V$ as shown above.
   a. On the diagram, draw wires and a (V) to show where you would connect a voltmeter to measure the potential across $R_1$. If any existing wires need to be disconnected in the circuit, draw an X over those wires in the diagram.

   b. On the diagram, draw wires and a (A) to show where you would connect an ammeter to measure the current traveling through $R_2$. If any existing wires need to be disconnected in the circuit, draw an X over those wires in the diagram.

Resistance $R_2$ has a value of $1.00 \times 10^3 \Omega$. The voltmeter shows the potential across $R_1$ to be 3.00V and the ammeter shows the current to be 5.00 mA through $R_2$.

   c. Calculate the value of $R_1$.

   d. Calculate the power dissipated by $R_2$.

   e. Calculate the terminal voltage $V$ of the battery.

   f. The battery is disconnected from the circuit, and its potential measured to be 8.20 V. Determine the internal resistance of the battery when it was connected in the circuit.

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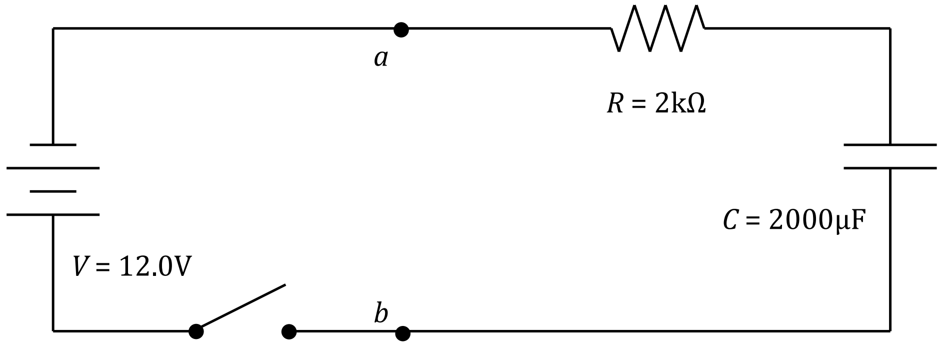
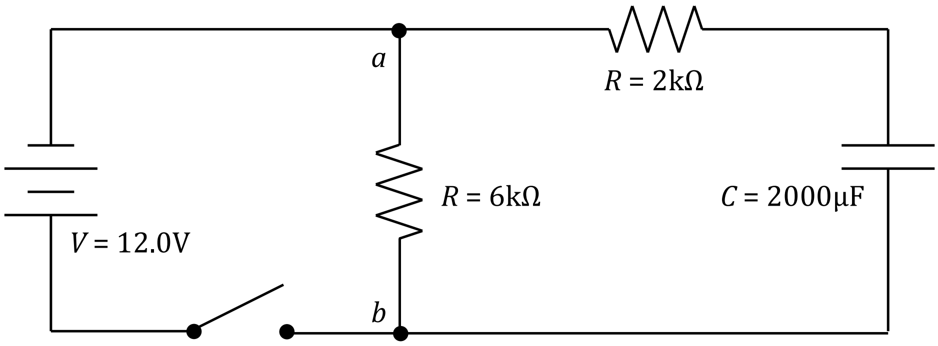



3. The circuit shown above is constructed with a 12.0-Volt power supply, a $2.00 \times 10^{3}\Omega$ resistor, and a $2.00 \times 10^{-3} F$ capacitor, initially uncharged. The switch is open, but then closed at time $t = 0$.
   a. Calculate the time constant for the circuit.

   b. Calculate the initial flow of current, both magnitude and direction, when the switch is first closed.

   c. Write and solve a differential equation to find the charge $Q$ on the plates of the capacitor as a function of time $t$.

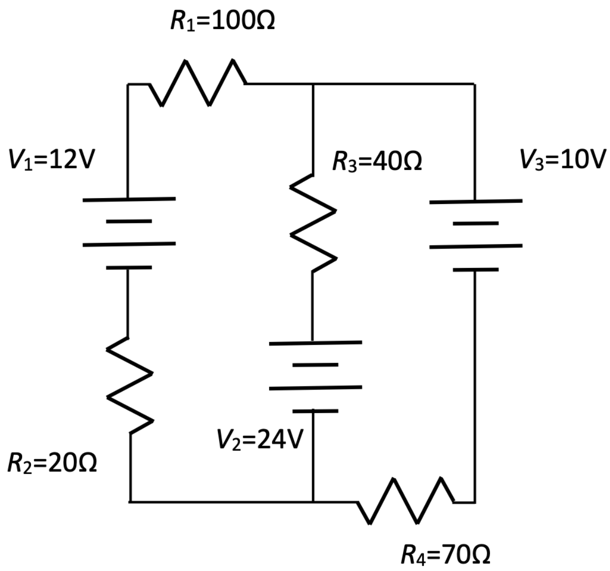



The switch is opened again, and a 6000Ω resistor placed across the circuit from point *a* to point *b* at a new time $t = 0$.

d. What is the time constant of this new circuit?

e. At what time is the potential across the capacitor 6.00 V?

f. Calculate the total energy dissipated by the 6000Ω resistor. (Assume that any heating effects of the wire are negligible.)




4. Examine the complex circuit shown here.
   a. Use a Kirchhoff's analysis to write, *but do not solve*, a series of non-redundant equations that would allow you to solve for the currents in each branch of the circuit.

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b. Solve the independent equations that you've come up with to identify the currents in each branch of the circuit.

c. Identify which colored bands would correctly identify resistor $R_4$ in the circuit, assuming it has a tolerance of 10%.

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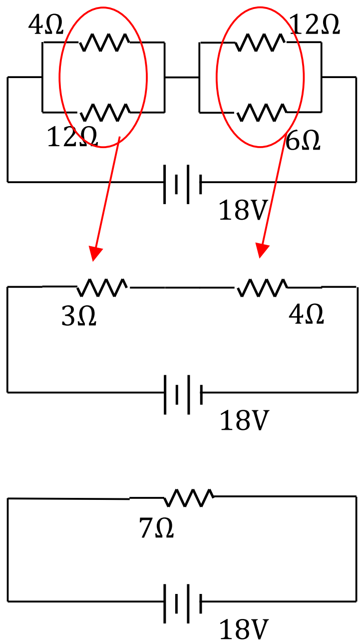
1. The correct answer is *a*. Solve first for the pairs of resistors in parallel, then use those equivalent resistances to get the effective, or equivalent, resistance for the circuit as a whole.

$$\frac{1}{R_1}+\frac{1}{R_2}=\frac{1}{R_{equivalent}}$$
$$\frac{1}{4}+\frac{1}{12}=\frac{1}{R_{equivalent}} \rightarrow R_{equivalent}=3\Omega$$
$$\frac{1}{12}+\frac{1}{6}=\frac{1}{R_{equivalent}} \rightarrow R_{equivalent}=4\Omega$$

Now use these two equivalent resistances in series to determine the overall resistance.

$$R_1+R_2=R_{equivalent}$$
$$R_{equivalent}=3\Omega+4\Omega=7\Omega$$




2. The correct answer is *a*. The resistance of a wire can be calculated using the formula $R=\frac{\rho L}{A}$. With half as big a resistivity in wire $W_2$, wire $W_1$ needs to either be half as long as $W_2$, or have twice as large a cross-sectional area. Choice *a* satisfies this requirement.

3. The correct answer is *d*. We can determine the terminal voltage of the battery—its emf $\mathcal{E}$ less the potential drop due to internal resistance *r*—in a couple of ways. The easiest is to use Ohm's Law with the stated *I* and R to calculate *V*:

$$V=IR=(2.0A)(10\Omega)=20V$$

A slightly more complex analysis involves solving for the unknown internal resistance first, then using this information to solve for *V*.

$$\mathcal{E}=IR=I(R+r)$$
$$24V=(2A)(10+r)$$
$$r=2\Omega$$

$$V=\mathcal{E}-Ir$$
$$V=24V-(2A)2\Omega=20V$$

4. The correct answer is ***b***. Analysis of the effective resistance of the resistor shows that the current coming from the battery is 3 Amps, resulting in a potential decrease of 12V over the 4Ω resistor. Thus, there is a 6V decrease over the 3Ω and 6Ω resistors.

The Power dissipated by that resistor can then be calculated using $P = \frac{V^2}{R} = \frac{6^2}{3} = 12W$.

A similar Power analysis may be performed by using $P = IV$ or $P = I^2R$, with appropriate values substituted in.

5. The correct answer is ***a***. Based on the total cost of the energy, the time, and the price per kWh, we can determine about how much Power the computer is using:

$$\frac{\sim 25cents}{1} \times \frac{kW \bullet hr}{10cents} \times \frac{1}{10hrs} = 0.25kW = \sim 250W$$

Now, using $P=IV$, we can determine the current being drawn by the computer:

$$P = IV, \; so \; I = \frac{P}{V}$$

$$I = \frac{\sim 250W}{120V} = \sim 2A$$

6. The correct answer is *e*. Immediately after the switch is closed, the capacitor doesn't yet have any charge built up on its plates, so there is no potential there. Current flows through both resistors, which are effectively connected in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \bullet 12}{6 + 12} = 4\Omega$$

$$I = \frac{V}{R} = \frac{24V}{4\Omega} = 6A$$

7. The correct answer is *e*. The equivalent capacitance of the three capacitors is $3\mu F$, which allows us to determine the amount of charge stored on each plate of the equivalent capacitor:

$$Q = VC$$

$$Q = (12V)(3\mu F) = 36\mu C$$

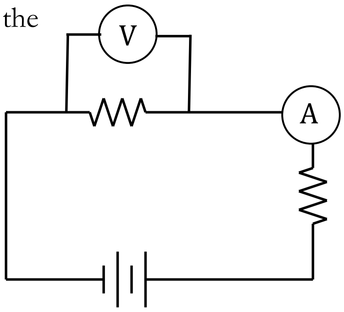
We can see that there is a 6V potential drop over the $6\mu F$ effective capacitance of the parallel capacitors (the $12V$ potential of the battery is reduced equally by each $6\mu F$ capacitance), so

$$Q = VC$$

$$Q = (6V)(5\mu F) = 30\mu C$$

8.

*a.* The voltmeter must be placed *over* the resistor to measure the potential difference before and after.




*b.* The ammeter must be inserted *into* the circuit so that there is only one path for current to follow. The ammeter may be placed anywhere in the circuit to measure current because it's a series circuit and the current is the same everywhere. But it make s most sense intuitively to place the meter near the resistor.

c. The two resistors are in series, so the current $I$ is the same throughout the circuit. Using Ohm's Law and the information we have for $R_1$:

$$V_1 = IR_1$$
$$3V = (0.005A)R_1$$
$$R_1 = \frac{3}{0.005} = 600\Omega$$

d. Using the stated current and resistance:

$$P = I^2R$$
$$P = (0.005A)^2(1000\Omega) = 2.5e-2W = 25mW$$

e. The terminal voltage of the battery can be calculated using the total $I$ and $R$ for the circuit, along with Ohm's Law:

$$V = IR$$
$$V = (0.005A)(600\Omega + 1000\Omega) = 8.00V$$

f. Terminal voltage is emf less the potential drop due to the internal resistance:

$$V = \varepsilon - Ir$$
$$8 = 8.2 - (0.005)r$$
$$r = 40\Omega$$

9.

a. $\tau = RC = (2000e-6F)(2000\Omega) = 4.00s = 4.00\Omega F$

b. Initial current flow based on capacitor initially being uncharged:

$$I_0 = \frac{V}{R} = \frac{12.0V}{2000\Omega} = 0006A = 6.00mA$$

c. Using the changes in potential over the loop:

$$V_{battery} - V_{resistor} - V_{capacitor} = 0$$
$$V - IR - \frac{q}{C} = 0$$
$$V - \frac{dq}{dt}R - \frac{q}{C} = 0$$

This is the initial set-up. Now rearrange to get *dq* and *dt* on opposite sides:

$$V - \frac{dq}{dt}R - \frac{q}{C} = 0$$
$$\frac{dq}{dt}R = V - \frac{q}{C}$$
$$\frac{dq}{dt} = \frac{CV}{RC} - \frac{q}{RC}$$
$$\frac{dq}{dt} = \frac{q - CV}{-RC}$$
$$\frac{dq}{q - CV} = \frac{1}{-RC}dt$$

It helps to know where you're going in that derivation! Now integrate both sides, the left for the charge on the plates, from 0 to *Q*, and the right for time *t* = 0 to *t*.

$$\int_0^Q \frac{dq}{q - CV} = \int_0^t \frac{1}{-RC}dt$$
$$\ln(q - CV)_0^Q = \frac{-t}{RC}$$
$$\ln(Q - CV) - \ln(0 - CV) = \frac{-t}{RC}$$
$$\ln\left(\frac{Q - CV}{-CV}\right) = \frac{-t}{RC}$$
$$\ln\left(1 - \frac{Q}{CV}\right) = \frac{-t}{RC}$$
$$1 - \frac{Q}{CV} = e^{-t/RC}$$
$$\frac{Q}{CV} = 1 - e^{-t/RC}$$
$$Q = CV\left(1 - e^{-t/RC}\right)$$

d. Now the capacitor is in series with two resistors, and is discharging through both of them. The time constant for this circuit is calculated just as before:
$$\tau = RC = (2000\Omega + 6000\Omega)(2000e - 6F) = 16s$$

e. This question is a bit complex. We don't have a way of determining *V* across the capacitor as a function of time, although we can get current flow *I* as a function of time if we calculate or know that $I = I_0 e^{-t/RC}$. Using this fact with Ohm's Law allows us to determine the time at which *V* = 6.00V:

$$I_{inst} = \frac{V_0}{R} e^{-t/RC}$$
$$\frac{V_{inst}}{R} = \frac{V_0}{R} e^{-t/RC}$$
$$\frac{V_{inst}}{V_0} = e^{-t/RC}$$
$$\ln\left(\frac{V_{inst}}{V_0}\right) = -\frac{t}{RC}$$
$$t = -RC\ln\left(\frac{V_{inst}}{V_0}\right) = -16s\ln\left(\frac{6.00V}{12.00V}\right) = 11.1s$$

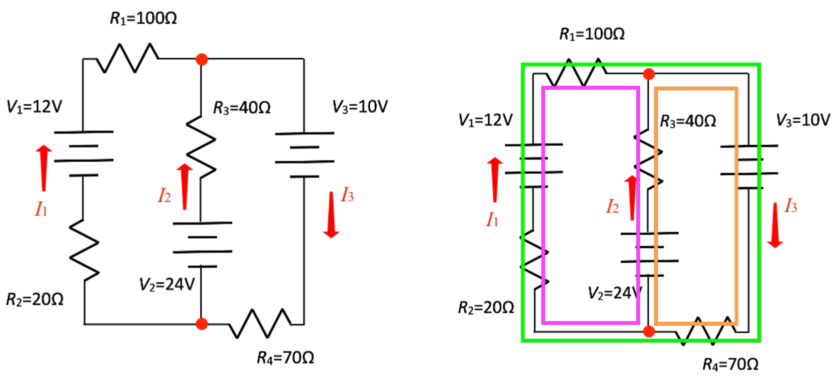
f. The total amount of energy stored in the capacitor is calculated based on its capacitance and the total potential across its plates, which is 12 V when fully charged:

$$U = \frac{1}{2}CV^2$$
$$U = \frac{1}{2}(2000e-6F)(12V)^2 = 0.144J$$

That energy is dissipated through the two resistors in the circuit according to $P = I^2R$ for each resistor. Because the current flow through the resistors in series is identical, the Power dissipated by each resistor is proportional to its resistance. The $6000\Omega$ resistor is 75% of the total $8000\Omega$ resistance, so it dissipates 75% of the 0.144J, or $0.75 \times 0.144J = 0.108J$ .

10.

a. To solve a complex circuit like this, we'll use Kirchhoff's rules, both the *Junction Rule* and the *Loop Rule*. We start by identifying the *branches* in a circuit—the single paths that run between *nodes*—and labeling them with a current label and a direction (which may or may not be correctly oriented). The set-up shown here is just one example.




We have three unknown currents—$I_1$, $I_2$, and $I_3$—so we'll need three equations to be able to solve simultaneously.

Looking at the top junction, we can apply the Junction Rule to get the equation
$I_1 + I_2 = I_3$
For additional equations, we'll turn to the Loop Rule. There are three loops to choose from, colored magenta, orange, and green in the diagram. The solution here is going to follow the magenta and orange loops clockwise to identify two additional equations.
$+12V - 100I_1 + 40I_2 - 24V - 20I_1 = 0$
$+24V - 40I_2 - 10V - 70I_3 = 0$
These three equations are what can be used to solve for the three currents.

b. There are a variety of strategies for solving these simultaneous equations, including solving by hand or using matrix manipulation on a calculator. Regardless of the technique used, the final solutions to these three equations are:
$I_1 = -0.0475$ A
$I_2 = 0.1575$ A
$I_3 = 0.11$ A
The fact that the first current is negative indicates that the presumed upward direction of current in that branch was incorrect, and that conventional current is actually flowing in the opposite direction in that branch. There is no need to re-do any of the calculations, however—the current magnitudes are correct.

c. Resistor R4 has a value of 70Ω, or $70 \times 10^0$ (note the lack of a decimal point). According to the resistor code, this would be indicated by color bands *Violet, Black, Black, Silver* (where Silver indicates the tolerance of +/- 10%).