26.1

By definition

$$C = Q/V$$

The V defines the ~~[crossed out]~~ voltage difference across the plates, & Q is the magnitude of the charge on one plate.

As such

a)

$$C = Q/V$$

$$\Rightarrow (4 \times 10^{-6} f) = Q/(12 \text{ volts})$$

$$\Rightarrow Q = 48 \times 10^{-6} \, C \quad (\text{or } 48 \mu C)$$

b.) Again

$$C = Q/V$$

$$\Rightarrow (4 \times 10^{-6} f) = Q/(1.5 \text{ volts})$$

$$\Rightarrow Q = 6 \mu C$$

26.2

As usual, $C = Q/V_c$

a) $$C = \frac{(10\times10^{-6}\,C)}{12}$$

$\Rightarrow$ =

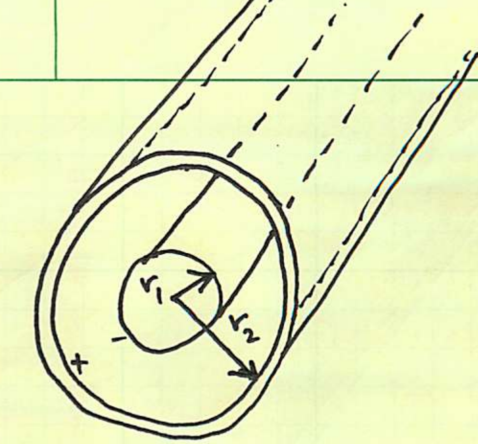
b) $C = Q/V_c \Rightarrow V_c = Q/C$

$$= \underline{(100\times10^{-6}\,C)}$$

26.5
pg 1

26.5 (Great problem)

We want the capacitance for the system shown.



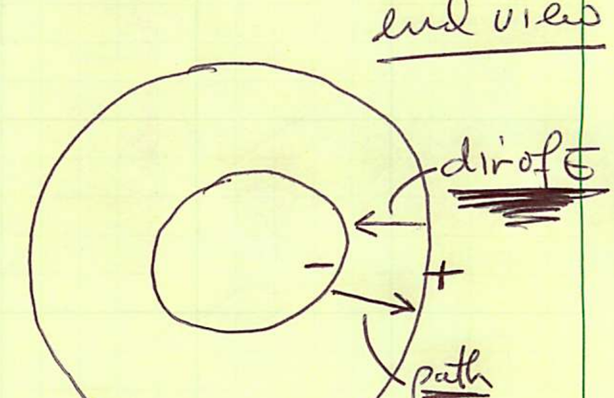
As

$$C = Q/V_c ,$$

that means we need to generate a function for the voltage V_c across the plates. To do that, we need to use the relationship

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

So what, exactly, does this relation say to us? It says:



Pick a path through the electric field. If we want the voltage change between the start point & the end point to be positive (as it needs to be in this case because V_c is defined as positive), must start at the V_- plate & end at the V_+ plate (that is $\Delta V = (V_+ - V_0) \Rightarrow$ positive). The path is shown in the sketch above.

Unfortunately, the electric field is opposite that direction, so $\vec{E} \cdot d\vec{r}$ will have a 180° angle between them.

26.15 page 2

In other words

$$V_c = (V_+ - V_-) = -\int E \cdot dr$$

$$= -\int |E||dr| \cos 180^\circ \quad (\cos 180^\circ = -1)$$

$$= \int E\,dr$$

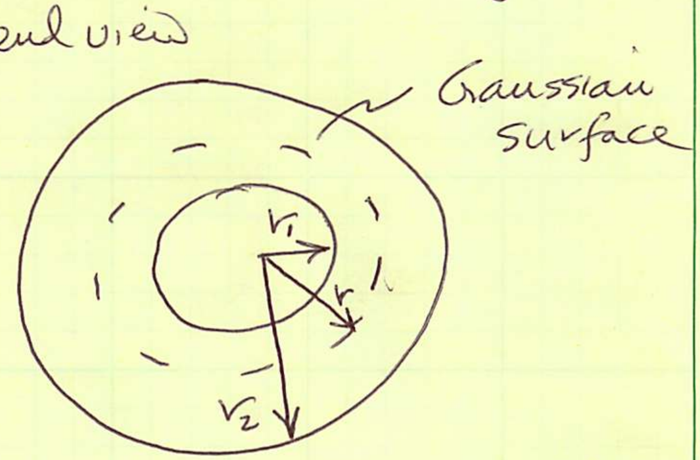
To finish this, we need to use Gauss's Law to get E. Doing so, we get:

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0}$$

for cylindrical symmetry:

$$(E(2\pi r L)) = \frac{\lambda L}{\epsilon_0}$$

$$\Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r}$$



So

$$V_c = \int E\,dr$$

$$= \int_{r_1}^{r_2} \frac{\lambda}{2\pi\epsilon_0 r}\,dr$$

$$= \frac{\lambda}{2\pi\epsilon_0}(\ln r_2 - \ln r_1)$$

$$= \frac{\lambda}{2\pi\epsilon_0}\left(\ln\frac{r_2}{r_1}\right)$$

$(V_+ - V_-)$: $V_+ \leftarrow r_2$, $V_- \leftarrow r_1$

26.5
pg 3

λ is charge per unit length, or

Q/L

So

$$C = \frac{Q}{V_c}$$

$$= \frac{Q}{\left(\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right)\right)}$$

$$= \frac{Q}{\left(\frac{Q/L}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right)\right)}$$

$$= \frac{2\pi\epsilon_0 L}{\ln(r_2/r_1)}$$

$$= \frac{2\pi(8.85\times10^{-12})(50\,\text{m})}{\ln\left(\frac{7.27/2}{2.58/2}\right)}$$

$$\boxed{C = 2.68\times10^{-9}\,f}$$

26.5 voltage difference between plates?

b.) $\Delta V = Q/C$

$$= \frac{8.1\times10^{-6}\,C}{2.68\times10^{-9}\,F}$$

$$\Rightarrow \boxed{\Delta V = 3.02\times10^{3}\ \text{volts}}$$

26.7

$$C = Q/\Delta V \underset{\text{parallel plate cap}}{=} \epsilon_0 A/d$$

$$\Rightarrow d = \epsilon_0 \Delta V (A/Q)$$

but $\sigma = Q/A$

$$\Rightarrow d = \frac{\epsilon_0 \Delta V}{\sigma}$$

$$= \frac{(8.85\times10^{-9}\ C^2/N\cdot m^2)(150\ V)}{(3\times10^{-8}\ C/cm^2)(10^4\ cm^2/m^2)}$$

$$\Rightarrow \boxed{d = 4.43\times10^{-6}\ m}$$

26.11

a) air filled parallel plate cap:

as $\Delta V = Ed$,

$E = \Delta V / d$

for E

$= \frac{20\,V}{1.8 \times 10^{-3}\,m}$

$\Rightarrow$ $\boxed{E = 1.11 \times 10^{4}\ V/m}$ toward negative plate

b) as $E = \frac{\sigma}{\epsilon_0}$ for a conductor!

for σ

$\sigma = \epsilon_0 E$

$= (8.85 \times 10^{-12})(1.11 \times 10^{4})$

$\Rightarrow$ $\boxed{\sigma = 9.8\cancel{3} \times 10^{-9}\ C/m^2}$

c) for parallel plate cap:

for C

$C = \epsilon_0 A / d$

$= \frac{(8.85 \times 10^{-12})(7.6 \times 10^{-4}\,m^2)}{1.8 \times 10^{-3}}$

$\Rightarrow$ $\boxed{C = 3.74 \times 10^{-12}\,F}$ or 3.74 pf

d) as $\Delta V = Q/C$

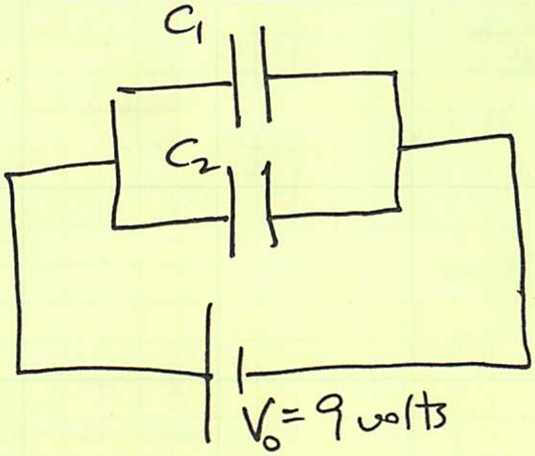
for Q on plates

$Q = C \Delta V$

$= (3.74 \times 10^{-9}\,F)(20\,V)$

$\Rightarrow$ $\boxed{Q = 74.7 \times 10^{-12}\,C}$

26.13



a.) equivalent cap:

for a parallel combo,

$$C_{eq} = C_1 + C_2 + \ldots$$

$$\Rightarrow C_{eq} = 5\mu f + 12\mu f$$

$$\boxed{C_{eq} = 17\mu f}$$

b) each element of a parallel ckt will have the same voltage across it, so both C_1 & C_2 will have 9 volts across them.

c.) as $C = Q/\Delta V$:

$$Q_5 = \frac{5\times10^{-6}}{9}$$

$$\Rightarrow \boxed{Q_5 = 4.5\times10^{-5}\,C}$$

4.5×10^{-5}

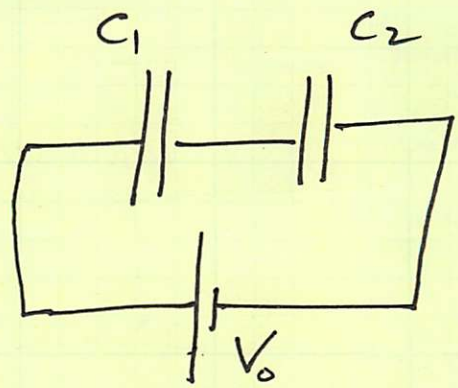
and

$$Q_{12} = \frac{12\times10^{-6}}{9}$$

$$\Rightarrow \boxed{Q_{12} = 1.08\times10^{-4}\,c}$$

26.16

a) in series, the am't of chg on each cap will be the same AND will equal the charge on the combination's equivalent cap.



Sooo

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{(2.5\times10^{-6}\,f)(6.25\times10^{-6}\,f)}{(2.5\times10^{-6} + 6.25\times10^{-6})}$$

$$= 1.79\times10^{-6}$$

as $C_{eq} = Q/\Delta V$,

$$Q = C_{eq}\,\Delta V$$

$$= (1.79\times10^{-6}\,F)(6\,V)$$

$$\Rightarrow \boxed{Q = 10.7\times10^{-6}\,C}$$

b.) As was the case in 26.13 for its parallel combo:

$$Q_{2.5} = C_{2.5}\,\Delta V = (2.5\times10^{-6})(6V) = 1.5\times10^{-5}\,C$$

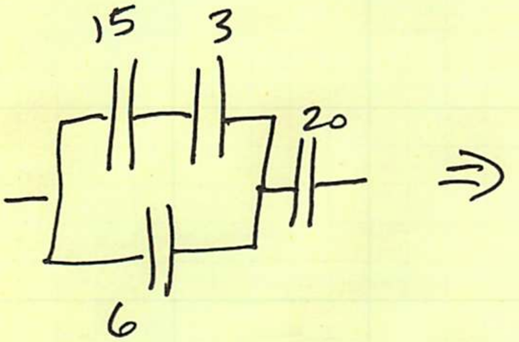
$$Q_{6.25} = C_{6.25}\,\Delta V = (6.25\times10^{-6})(6\,V) = 3.75\times10^{-5}\,C$$

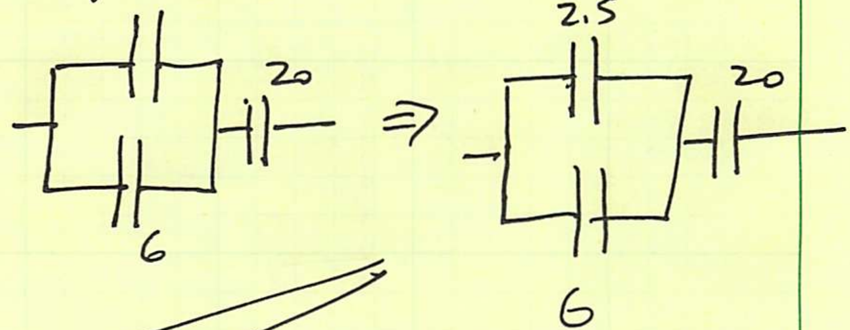
26.23

a.) C_{eq} ?

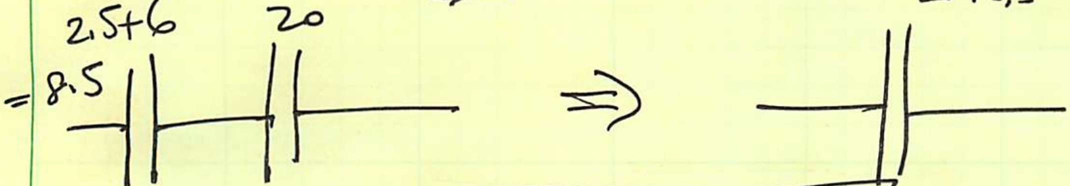
note that $\left(\frac{1}{C_{eq}}\right)^{-1} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{C_1}{C_1C_2} + \frac{C_2}{C_1C_2}\right)^{-1} = \frac{C_1C_2}{C_1+C_2}$

$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{45}{18} = 2.5$



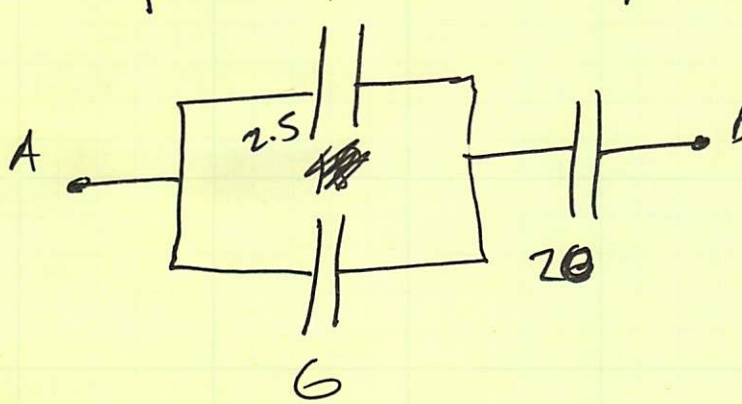


$C = \frac{20(8.5)}{20+8.5} = 5.96\times10^{-6} f$



$$\boxed{C_{eq} = 5.96\times10^{-6} f}$$

b) the charge on the ~~top~~ top caps will be the same, so combining those we get: $C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{15(3)}{18} = 2.5\mu f$



$V_{AB} = 15$ volts

26.23 b)

Think about what's actually happening. Q's worth of chg enters the parallel combo whereupon it splits into the chg on the 18μC cap (combo) & the 6μf cap. As that charge accumulates on the cap plates, it electrostatically repulses an equal amt. of chg off the other plates. That charge moves on, accumulating on the 20μf cap. In other words, the "total charge" in the system (determined by using ↘

$$C_{eq} = Q_{total}/\Delta V$$

$$\Rightarrow Q_{total} = C_{eq}\,\Delta V$$

$$= (5.96\times10^{-6}\,f)(15\,v)$$

$$= 89.5\times10^{-6}\,C$$

ends up on the 20μf cap

that means the voltage across that cap must be

$$\Delta V_{20} = Q_{20}/C_{20}$$

$$= \frac{89.5\times10^{-6}}{20\times10^{-6}}$$

$$= 4.475 \text{ volts}$$

$$\Rightarrow \Delta V_{\substack{parallel\\combo}} = 15 - 4.475$$

$$= 10.525 \text{ volts}$$

26-23 (cont)

With that voltage across each of the parallels, we can write

$$Q_{2.5} = C_{2.5} \Delta V_{par}$$

$$= (2.5\times10^{-6})(10.525\,V)$$

$$\boxed{Q_{2.5} = 26.3\,\mu C}$$

As $Q_{Total} = 89.5\,\mu C$, $Q_6 = Q_{tot} - Q_{2.5}$ or

$$Q_6 = 89.5 - 26.3$$

$$\boxed{Q_6 = 63.2\,\mu C}$$

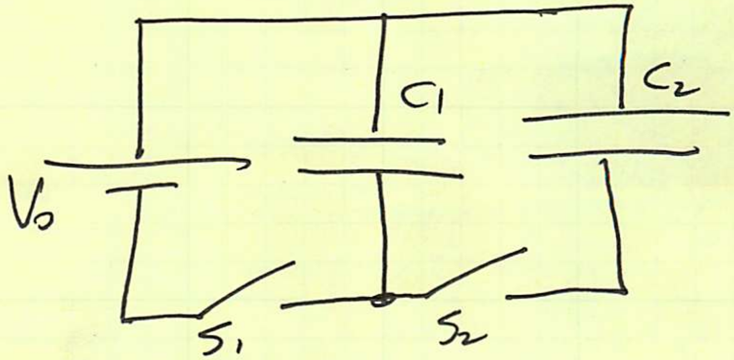
26.24

initially, $V_{C_1} = V_0$

$\Rightarrow Q_{C_1} = C_{C_1} V_0$

$= (6\times10^{-6})(20)$

$\boxed{Q_{C_1} = 1.2\times10^{-4}\,C}$



This is the amount of charge to start with in the ckt.

b.) By ~~[illegible]~~ throwing both switches, the battery is removed from the circuit and the initial charge on C_1 is redistributed between C_1 & C_2 until their voltages are the same. If we call the final charge on C_1 "Q_1," we know the charge on C_2 will be $Q_2 = 1.2\times10^{-4} - Q_1$, & as the voltage will be equal, we can write

$$V_{C_1} = \frac{Q_1}{C_1} = V_{C_2} = \frac{1.2\times10^{-4} - Q_1}{C_2}$$

&

$$\frac{Q_1}{C_1} = \frac{1.2\times10^{-4} - Q_1}{C_2}$$

$$\Rightarrow Q_1 C_2 = (1.2\times10^{-4})(C_1) - C_1 Q_1$$

$\Rightarrow$ ~~[illegible]~~

$$(C_1 + C_2)Q_1 = 1.2\times10^{-4} C_1$$

so

$$Q_1 = \frac{(1.2\times10^{-4})(6\times10^{-6})}{(3\times10^{-6} + 6\times10^{-6})}$$

$$= 8\times10^{-5}\,C$$

26.24
(cont)

$$Q_1 = 8\times10^{-5}C$$

$$\Rightarrow \quad Q_2 = 1.2\times10^{-4} - Q_1$$

$$= 1.2\times10^{-4} - 8\times10^{-5}$$

$$\Rightarrow \quad \boxed{Q_2 = 4\times10^{-5}\,C}$$

42-381 50 SHEETS EYE-EASE® - 5 SQUARES
42-382 100 SHEETS EYE-EASE® - 5 SQUARES
42-389 200 SHEETS EYE-EASE® - 5 SQUARES
National® Brand

26.27

There is no rhyme or reason to a problem like this. You just have to take what you know & play with it until something pops. Here goes

We have $C_1, C_2, \ldots C_n$

We know

$$C_p = C_1 + C_2 + \ldots C_n = nC$$

as each cap has the same value

Also

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots \frac{1}{C_n}$$

$$\Rightarrow C_s = \frac{1}{\frac{1}{C} + \frac{1}{C} + \ldots \frac{1}{C}} = \frac{1}{\left(\frac{1}{C}\right)n}$$

$$= C/n$$

We were told

$$C_p = 100 C_s$$

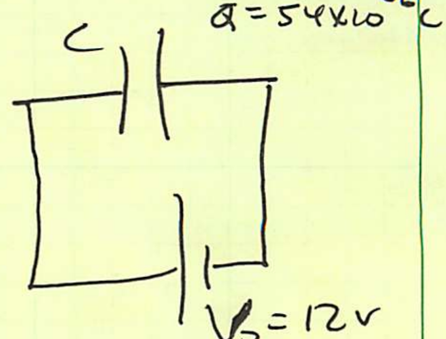
$$\Rightarrow \quad n\cancel{C} = 100\left(\cancel{C}/n\right)$$

$$\Rightarrow n^2 = 100$$

$$\Rightarrow n = 10$$

Apparently, we have 10 caps.

26.31



We know Q & V_c but not C. We also know $C = Q/V_c$ & $E = \frac{1}{2} C V_c^2$. From there

$$E = \frac{1}{2} C V_c^2$$

$$= \frac{1}{2}\left(\frac{Q}{V_c}\right) V_c^2$$

$$= \frac{1}{2} Q V_c$$

$$= \frac{1}{2}(54\times10^{-6}\,C)(12\,V)$$

$$\boxed{E = 3.24\times10^{-4}\,J}$$

26.33

a) what Q?

$$Q = C\Delta V$$

$$= (1.5\times10^{-10}\,f)(10^4\,V)$$

$$\boxed{Q = 1.5\times10^{-6}\,C}$$

b.) 250μJ corresponds to what V?

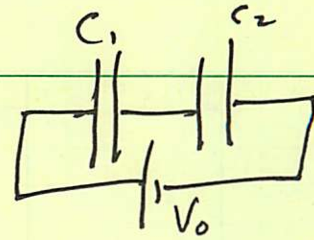
$$E = \frac{1}{2}CV^2$$

$$\Rightarrow V = \sqrt{2E/C}$$

$$= \left(\frac{2(250\times10^{-6}\,J)}{1.5\times10^{-10}\,F}\right)^{1/2}$$

$$= 1.83\times10^{3}\ V$$

Apparently, if you can discharge a spark, your elect. devices (or, at least, their inner workings) are in jeopardy



26.34

a) C_{eq}?

$$C_{eq} = \frac{1}{1/C_1 + 1/C_2}$$

$$= \frac{1}{\frac{1}{1.8\times10^{-5}} + \frac{1}{3.6\times10^{-6}}}$$

$$\Rightarrow \boxed{C_{eq} = 12\times10^{-6}\,F}$$

b.) energy stored

$$E = \frac{1}{2} C_{eq} V_0^2$$

$$= \frac{1}{2}(12\times10^{-6})(12\,V)^2$$

$$\boxed{U_{eq} = 8.64\times10^{-4}\,J}$$

c.) In series, each cap has the same chg. That means

$$Q_{total} = C_{eq} V_c$$

$$= (12\times10^{-6})(12)$$

$$= 1.44\times10^{-4}\,C \qquad (= Q_1 = Q_2)$$

This is the charge on each cap, so as $V = Q/C$ &

$$E = \frac{1}{2} C V^2$$

$$= \frac{1}{2} C \left(\frac{Q^2}{C^2}\right)$$

$$= \frac{1}{2} \frac{Q^2}{C}$$

26.34
c) (cont)

We can write

$$E_{18} = \frac{1}{2}\frac{(1.44\times10^{-4}C)^2}{(1.8\times10^{-5}F)}$$
$$= 5.76\times10^{-4}J$$

$$E_{36} = \frac{1}{2}\frac{(1.44\times10^{-4}C)^2}{(3.6\times10^{-5}F)}$$
$$= 2.88\times10^{-4}J$$

d.) Do the individual energies correspond to the energy of the equiv. cap?

$$E_{18} + E_{36} \stackrel{?}{=} E_{eq}$$
$$5.78\times10^{-4} + 2.88\times10^{-4} \stackrel{?}{=} 8.64\times10^{-4}$$

YES!

e.) The total energy wrapped up in the equivalent cap will always equal the sum of the energies of the individual caps involved (that's what it <u>means</u> to be an equivalent cap).

f.) The C_{eq} for a parallel combo would be

$$C_{eq} = C_1 + C_2$$
$$= 18\mu f + 36\mu f$$
$$= 54\mu f$$

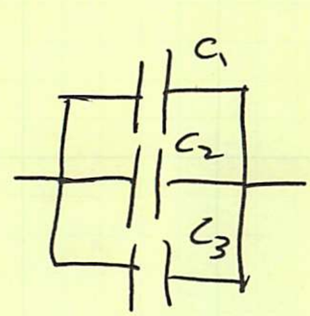
page 2 of 3

2634

f. (cont)

For $E_{given\,par} = E_{given\,series}$

$$\Rightarrow 8.64\times10^{-4} = \frac{1}{2} C_{eq} V^2$$
$$= \frac{1}{2}(54\times10^{-6})V^2$$
$$\Rightarrow \boxed{V = 5.66 \text{ volts}}$$



g.) In a parallel combo (which this part is associated with), V_c is the same for each cap. As

$$U = \frac{1}{2} C V^2,$$

a larger C (for a constant V) means the larger cap will have the most energy

Note: If the caps had been in series, Q would have been common to all elements. As

$$U = \frac{1}{2} C V^2$$
$$= \frac{1}{2} C \left(Q/C\right)^2$$
$$= \frac{1}{2} \frac{Q^2}{C},$$

big C (with Q constant) means small U & vice versa

26.36

Determine the force one plate of a parallel plate cap exerts on the other

We know:

$$C = Q/V_c$$

$$\Rightarrow V_c = \frac{Q}{C}$$

$$= \frac{Q}{\left(\epsilon_0 A/x\right)}$$

$$\Rightarrow V_c = \left(\frac{Q}{\epsilon_0 A}\right) x$$

We also know this voltage difference across the plates is DEFINED as positive.

We can relate the voltage difference between any two points in an electric field with the electric field vector by

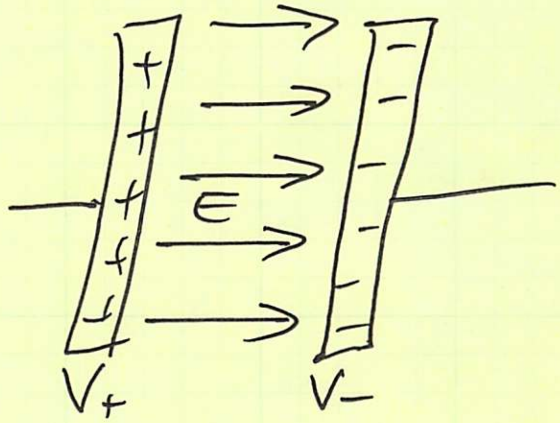
$$E_x = - \frac{dV_x}{dt}$$

There is a subtlety to this relationship we need to note before continuing. Specifically, why the negative sign?

To understand this, think about a charge parallel plate capacitor.

26.36

If we follow along the electric field lines (in the positive direction), we must traverse from higher voltage to lower voltage. In other words,



$$\Delta V = (V_{final} - V_{initial})$$
$$= [(V_-) - (V_+)]$$

$\Rightarrow$ ΔV will be NEGATIVE

Because the signs have to match up in an equation, we must in this case write

$$E = - \frac{dV}{dx}$$

The problem with voltage differences and capacitors is that the voltage difference V_c is <u>defined</u> as positive. To use the electric field relationship quoted above in that situation, with the voltage difference STARTING OUT AS POSITIVE, we have to write

$$\underset{\text{between plates}}{E} = + \frac{dV_c}{dx}$$

page 2 of 3

2636 (cont)

In other words, in this case we can write

$$\mathcal{E} = + \frac{dV_c}{dx}$$

$$= \frac{d\left(\frac{Q}{\epsilon_0 A} x\right)}{dx}$$

$$= \frac{Q}{\epsilon_0 A}\left(\frac{x^2}{2}\right)$$

$$= \frac{Q}{2\epsilon_0 A} x^2$$

As the force on a charge in an E field is $F = q\mathcal{E}$, we can additionally write

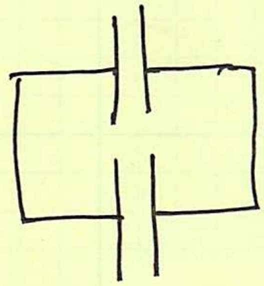
$$F = Q\mathcal{E}$$

$$= Q\left(\frac{Q}{2\epsilon_0 A} x^2\right)$$

$$= \frac{Q^2}{2\epsilon_0 A} x^2$$

26.37

a) Because the caps are at the same voltage, & as they are the same size, no charge flow will occur when the two are connected in parallel. The energy involved is



$$U = 2\left[\tfrac{1}{2} C V_c^2\right]$$

$$= 2\left[\tfrac{1}{2}(10^{-5}\,F)(50\,V)^2\right]$$

$$\boxed{U = 2.5 \times 10^{-2}\,J}$$

b) Doubling the plate distance will halve the capacitance. Additionally, the charge will redistribute until the voltage is again the same. Calling this V_{new}, & noting that the total charge in the system hasn't changed, we can write:

$$Q_{initial} = CV_c + CV_c = 2CV_c$$

$$Q_{final} = CV_{new} + \tfrac{C}{2}V_{new}$$

so as $Q_{init} = Q_{fin}$ (no charge is added to the system during the redistribution)

$$2\cancel{C}V_c = \tfrac{3}{2}\cancel{C}\,V_{new}$$

$$\Rightarrow V_{new} = \tfrac{4}{3}V_c$$

$$= \tfrac{4}{3}(50\,V)$$

$$\boxed{V_{new} = 66.7\text{ volts}}$$

2637
(cont) The new energy is

c.) $U_{new} = \left[\frac{1}{2} C V_{new}^2 + \frac{1}{2}\left(\frac{C}{2}\right) V_{new}^2 \right]$

$= \frac{3}{4} C V_{new}^2$

$= \frac{3}{4}\left(10^{-5} F\right)\left(66.7 V\right)^2$

$\boxed{U = 3.3 \times 10^{-2} J}$

d) $U_{new} = 3.3 \times 10^{-2} J$

$U_{old} = 2.5 \times 10^{-2} J$

Where'd the extra energy come from? It came from the work required to pull the plates apart.

26.42

The dielectric constant is such that

$$C_w = K\, C_{w/o}$$

$$\Rightarrow K = \frac{C_w}{C_{w/o}}$$

$$= \frac{Q/V_w}{Q/V_{w/o}}$$ (the charge on the plates doesn't change)

$$= \frac{V_{w/o}}{V_w}$$

$$= \frac{85}{25}$$

$$\boxed{K = 3.4}$$

b) Kind of material?

From Table 26.1 in the book, it is probably nylon.

c.) The dielectric weakens the E-fld between the plates. As $E \propto \Delta V$, that means V_c goes down &, $C\ (=Q/V_c)$ goes up. If the dielectric only partially fills the space, E ~~won't go down as much~~ will go in the dielectric region with an appropriate ΔV associated with it, but it will not go down in the air & the ΔV with that gap won't change from before. In other words, the net ΔV will be smaller, but not as small as if all the space was filled.

26.45

a) Teflon's $K = 2.1$, so the capacitance is

$$C = K \frac{\epsilon_0 A}{d}$$

$$= \frac{2.1\,(8.85\times10^{-12}\,F/m)(1.75\times10^{-4}\,m)}{4\times10^{-5}\,m}$$

$$\boxed{C = 8.13\times10^{-11}\,F}$$

b) The voltage?

This is a little convoluted [illegible] unless you realize that the "DIELECTRIC STRENGTH" column in Table 26.1 in your text has the units V/m, which is the unit for electric field (in fact, for Teflon this is 6×10^{7} V/m). With that, we can write

$$V = Ed$$

$$= (6\times10^{7}\,V/m)(4\times10^{-5}\,m)$$

$$\boxed{V = 2.4\times10^{3}\ \text{volts}}$$

26.46

We know

$$C = K\frac{\epsilon_0 A}{d}$$

So

$$9.5\times10^{-8}f = \frac{3.7(8.85\times10^{-12})\overbrace{(.07)\ell}^{\text{area}}}{2.5\times10^{-5}m}$$

$$\Rightarrow \quad \ell = 1.04m$$