Ch 26 - Capacitance

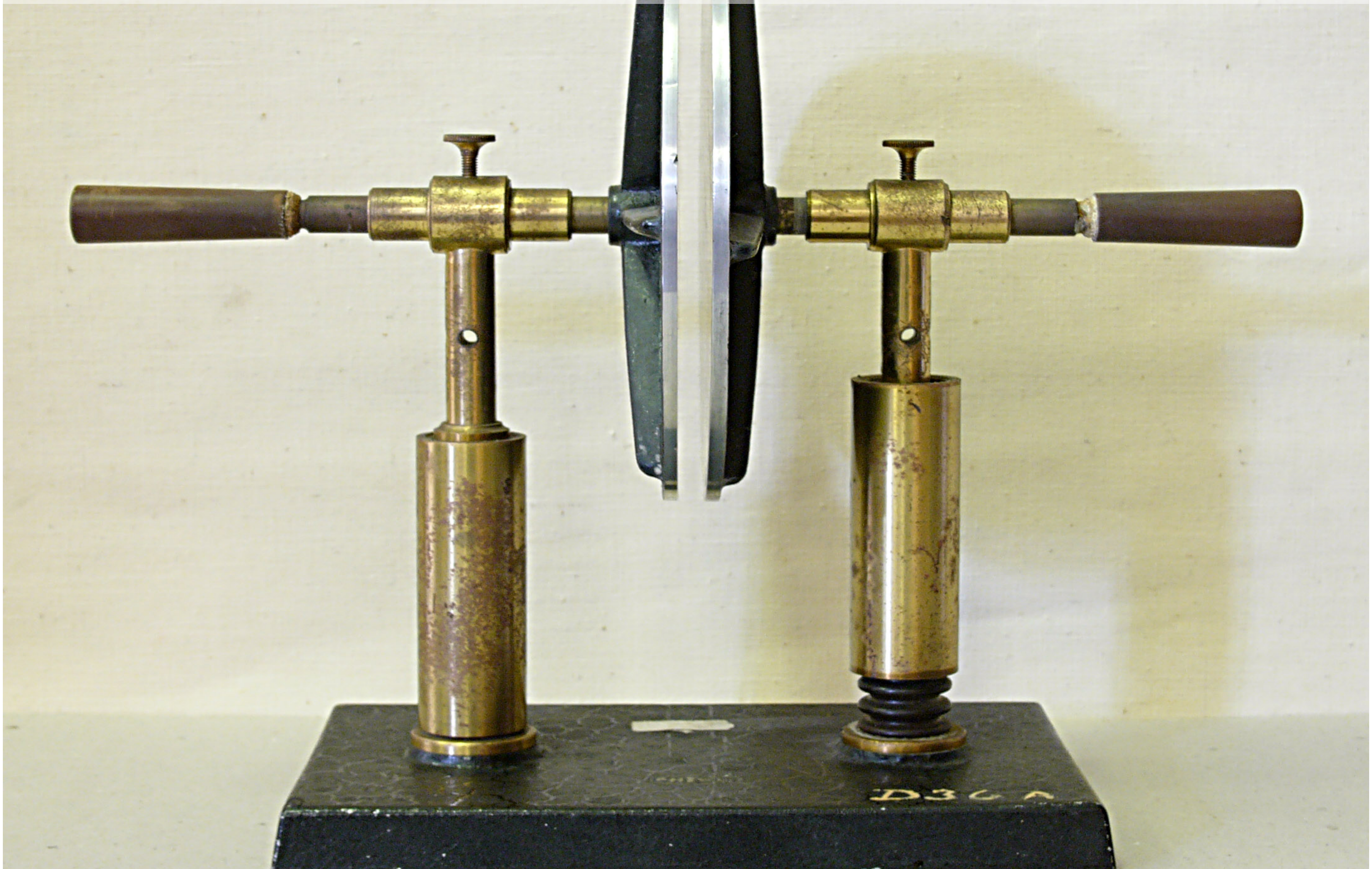


Photo: Hannes Grobe, Wikimedia

Review

- *Electric Fields*
  Exist in the presence of charged particles, apply forces to other charges
- *Gauss's Law*
  Can be used to determine electric field in certain situations
- *Electric Energy*
  Measure of how much work it takes to move charges through electric fields
- *Electric Potential*
  Measure of how much *energy per unit charge* it takes to move through fields

$$F = k\frac{q_1 q_2}{r^2}$$

$$E = \frac{F}{q_0}$$

$$\Phi = \oint E \bullet dA = \frac{q_{in}}{\varepsilon_0}$$

$$U = -\int F \bullet dr = -\int qE \bullet dr$$

$$\Delta V = \frac{\Delta U}{q}$$

$$V = \int k\frac{dq}{r}$$

$$V = -\int E \bullet ds$$

$$E = -\frac{dV}{dr}$$

What’s Next

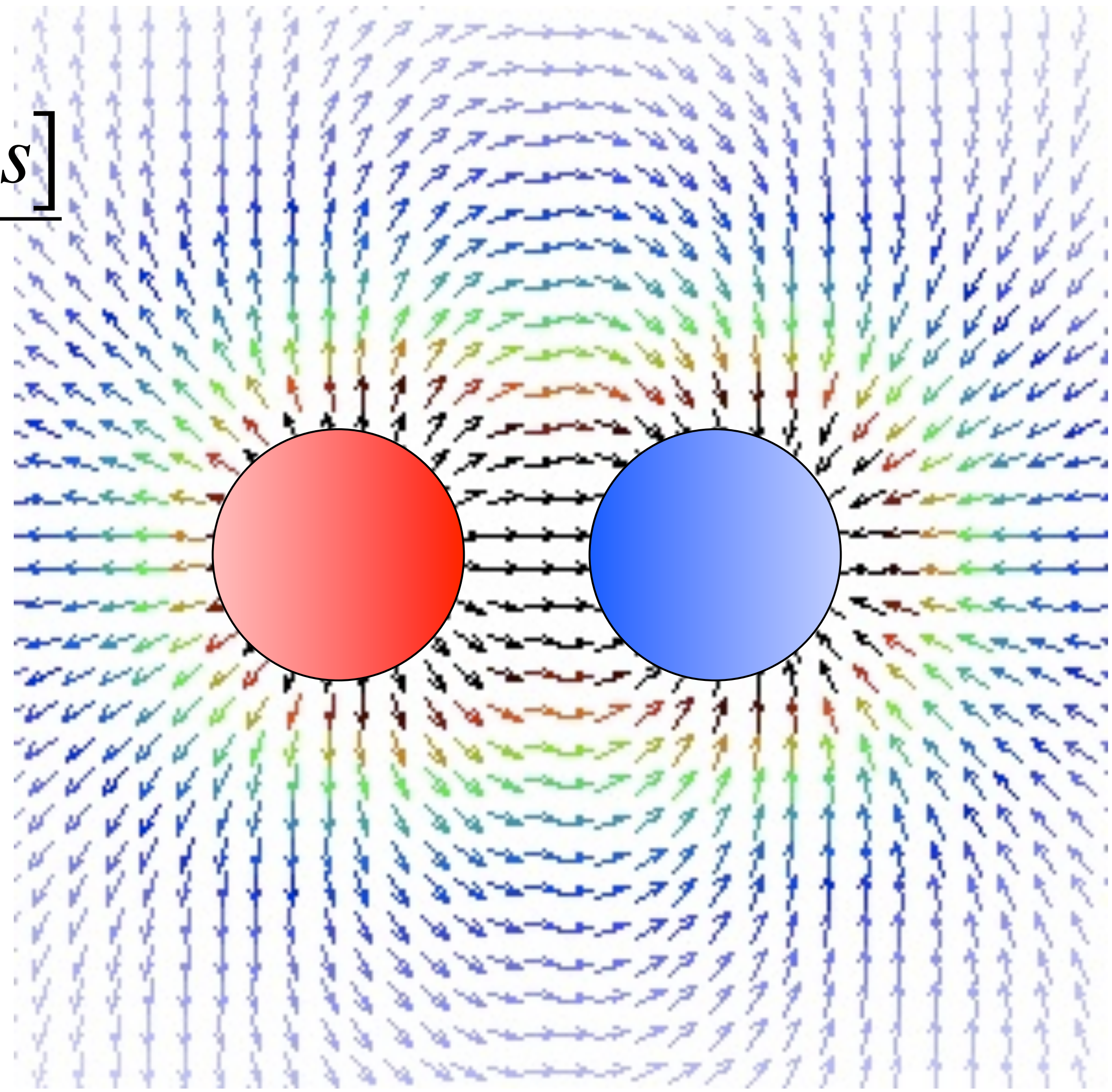
- *Capacitance*
  One week unit that has both theoretical and practical applications
- *Current & Resistance*
  Moving charges, finally!
- *Direct Current Circuits*
  Practical applications of all the stuff that we’ve been talking about

Capacitance

Capacitance = "the ability to store charge."

$$C \equiv \frac{Q}{\Delta V}$$

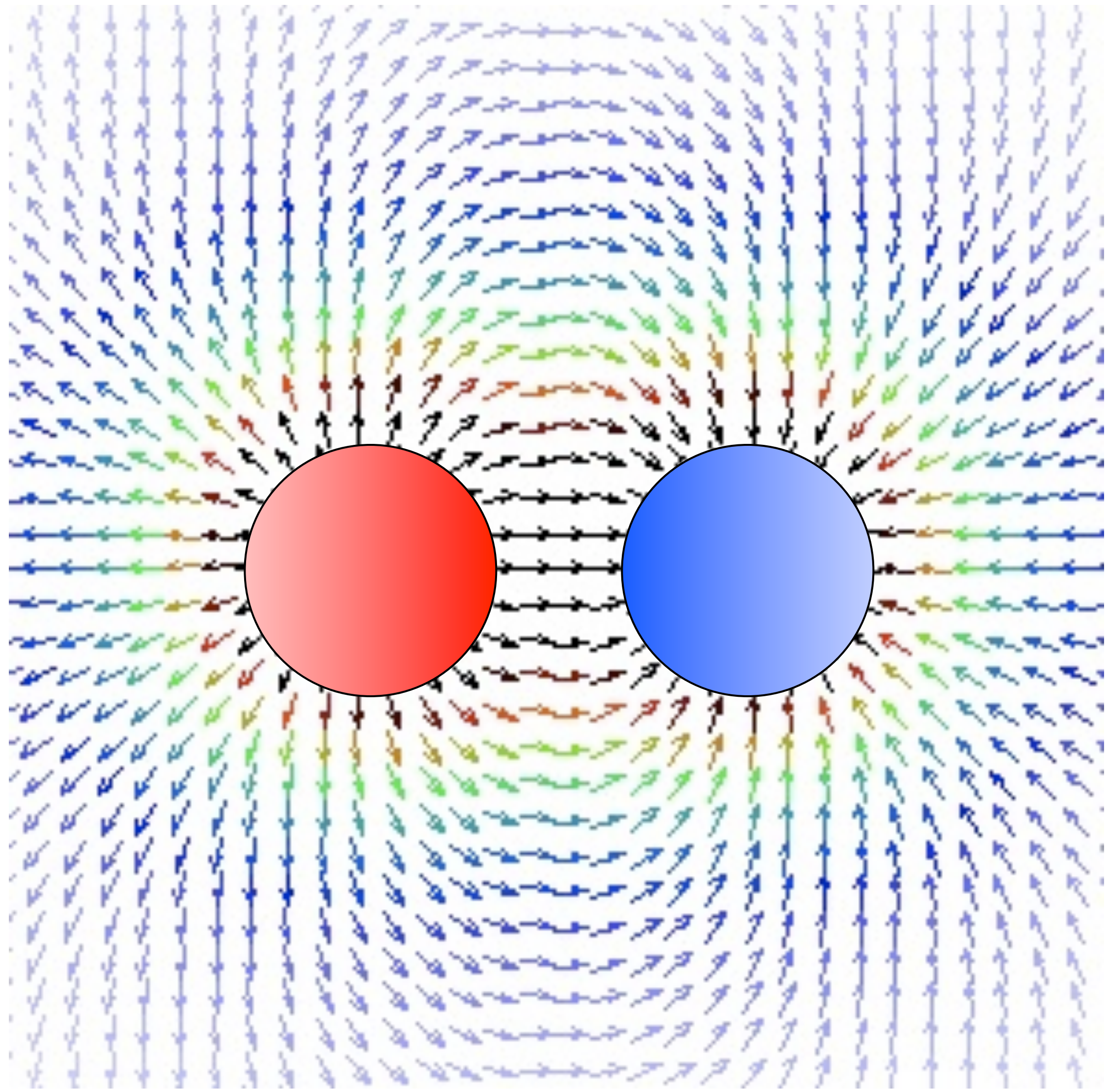
$$\left[Farads\right] \equiv \frac{\left[Coulombs\right]}{\left[Volts\right]}$$



Capacitance is always positive, and is a measure of *how much charge can be stored at a given electric potential.*

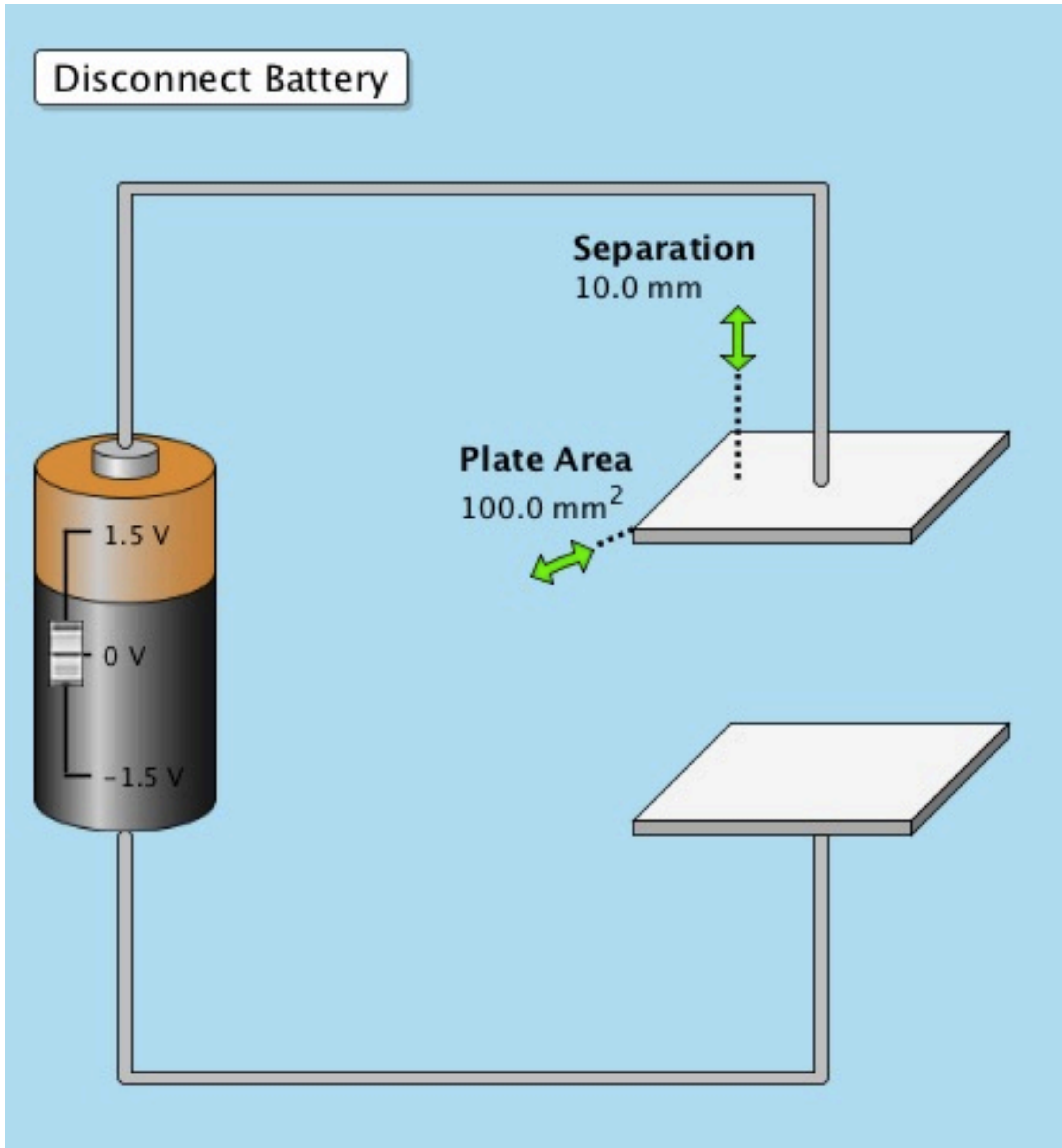
Example 1

What is the capacitance of this system, where each conductor has a charge of +/- 3 Coulombs, and a 9-Volt potential exists between the two conductors?



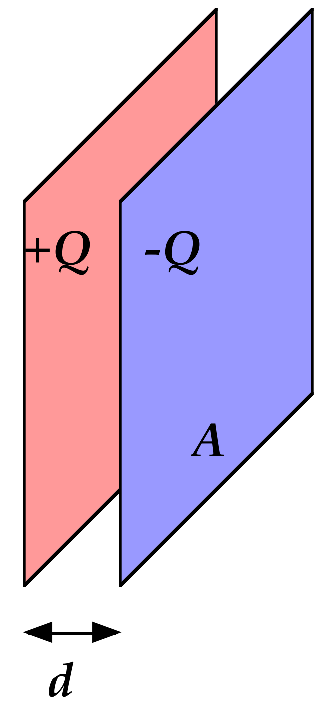
$$C = \frac{Q}{V} = \frac{3C}{9V} = 0.33C/V = 0.33F$$

Demo 1 Parallel Plate Capacitor



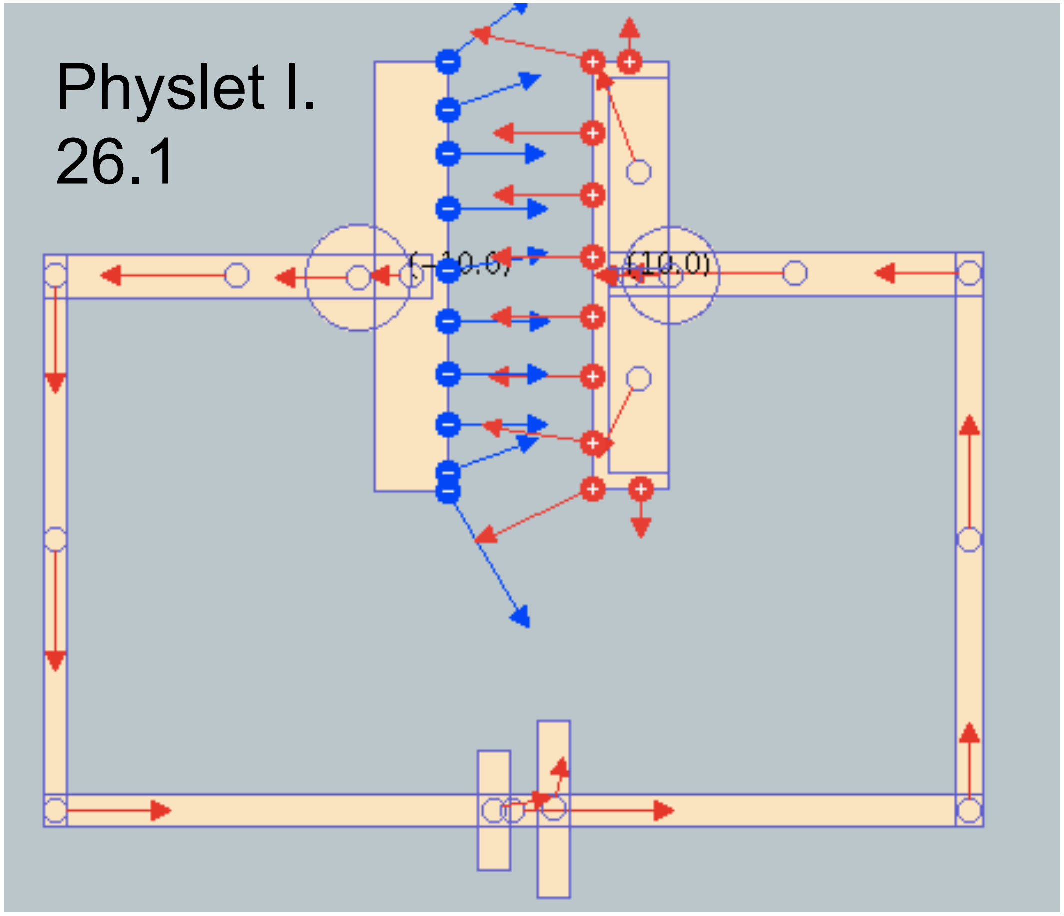
Example 2

2 conducting plates have a charge of 1.2mC on each, with a 6.00-V potential difference between the two of them. What is the capacitance of this system?



$$C = \frac{Q}{V} = \frac{1.2e-3C}{6.00V} = 2.0e-4F = 200\mu F$$

Demo 2
Parallel Plate Capacitor

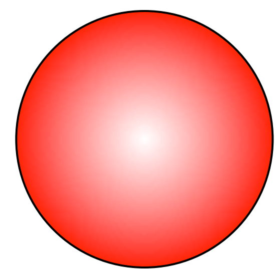


Theoretical Capacitance

Calculate the capacitance of an isolated conductor with a charge Q and radius *R*. (Assume that a second, concentric, hollow conductor exists with *r*=∞.)

$$V = k\frac{Q}{R} \text{ (for this sphere, so...)}$$

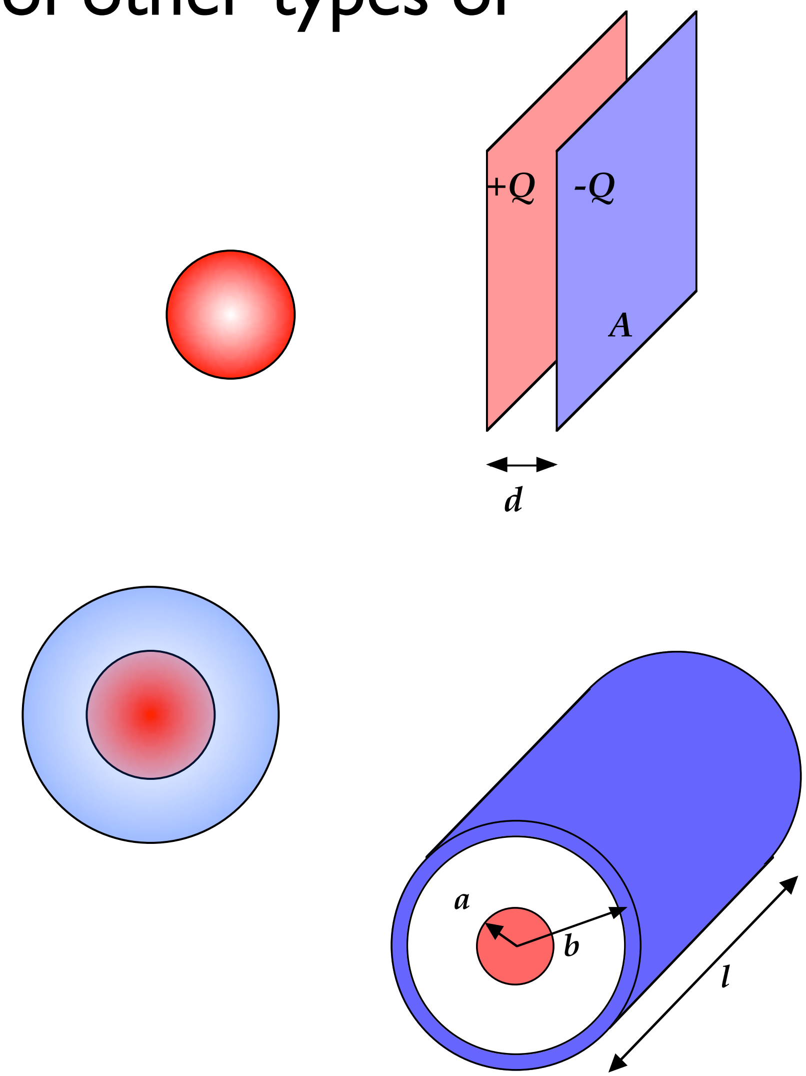
$$C = \frac{Q}{V} = \frac{Q}{k\frac{Q}{R}} = \frac{R}{k} = 4\pi\varepsilon_o R$$



Strategy

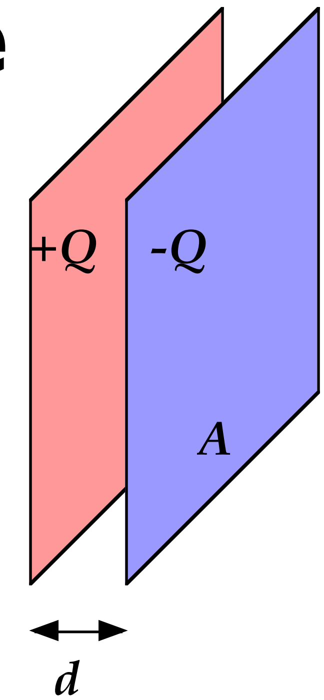
We can calculate the capacitance of other types of systems, using this basic strategy:

1. Assume a charge of magnitude Q
2. Calculate the potential V of the system (using techniques from last chapter)
3. Use C=Q/V to evaluate the capacitance



Example 3 - Parallel Plate Capacitor

Consider two parallel plates, each with area *A*, separated by a distance *d* and with equal and opposite charges on them. Assume that plates are close together compared to their area so that we can neglect the edge effects and assume that E is constant between them. Calculate the capacitance of this system.



Example 3 - Parallel Plate Capacitor

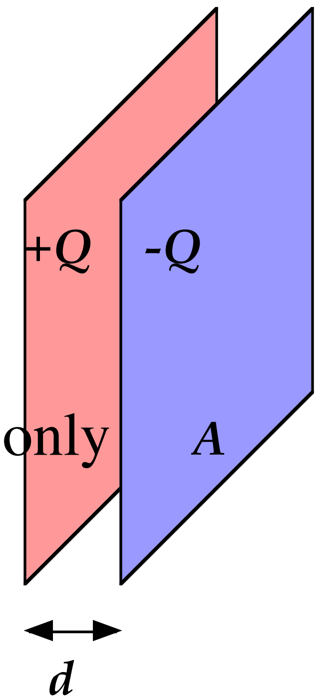
$V = -Ed$ (between parallel plates)

E for a plane is...

(Using Gaussian surface):

$$\oint E \bullet dA = \frac{q_{in}}{\varepsilon_o}$$

Because electric field is only on one side (why?):



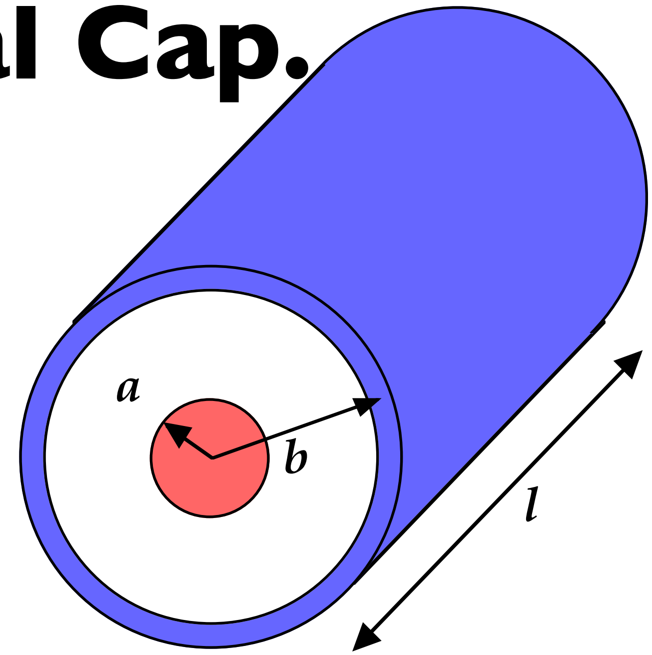
$$E \bullet A = \frac{\sigma A}{\varepsilon_o}$$

$$E = \frac{\sigma}{\varepsilon_o} = \frac{Q}{\varepsilon_o A}$$

$$V = \frac{Qd}{\varepsilon_o A}$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{\varepsilon_o A}} = \frac{\varepsilon_o A}{d}$$

Example 4 - Cylindrical Cap.



A cylindrical conductor of radius *a* and charge *Q* is coaxial with a larger cylindrical shell of radius *b* and charge -*Q*. Find the capacitance of the system if its length is *l*.

$$E = \frac{2k\lambda}{r} \text{ (developed using Gauss's Law)}$$

$$V_a - V_b = -\int_b^a E \bullet dr$$

$$V_a - V_b = -\int_b^a \frac{2k\lambda}{r} \bullet dr$$

$$V_a - V_b = 2k\lambda \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{V} = \frac{l}{2k\ln\left(b/a\right)}$$

Example 5 - Spherical Cap.

Determine the capacitance of a spherical capacitor, with an inner sphere of radius *a* charged to +*Q*, and an outer sphere of radius *b* charged to -*Q*.

$$E = \frac{kq}{r^2} \text{ (developed using Gauss's Law)}$$

$$V_b - V_a = -\int_a^b E \bullet dr$$

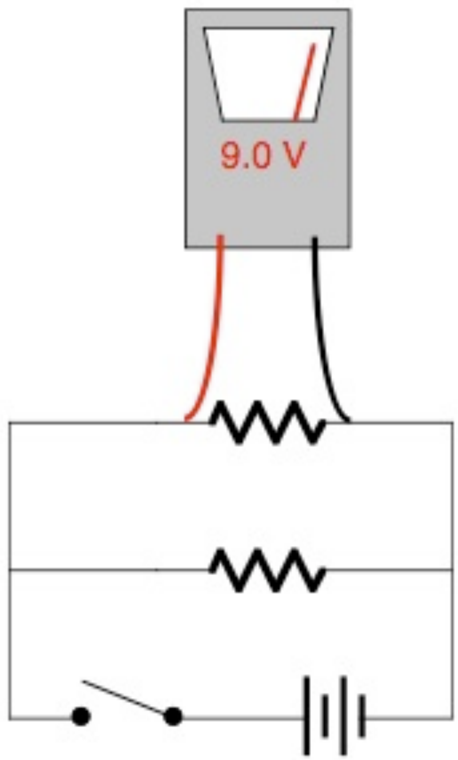
$$V_b - V_a = -\int_a^b \frac{kq}{r^2} \bullet dr$$

$$V = kQ\left[\frac{a-b}{ab}\right]$$

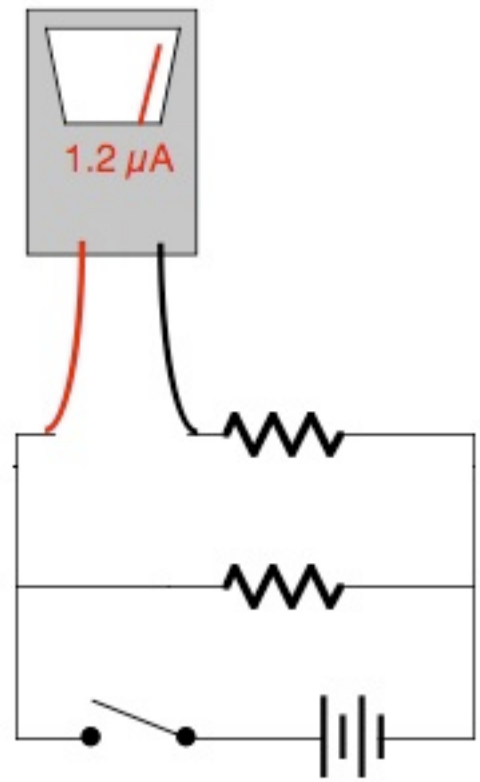
$$C = \frac{Q}{V} = \frac{ab}{k(b-a)}$$

(Have to change sign, because C is always positive.)

Lab this week



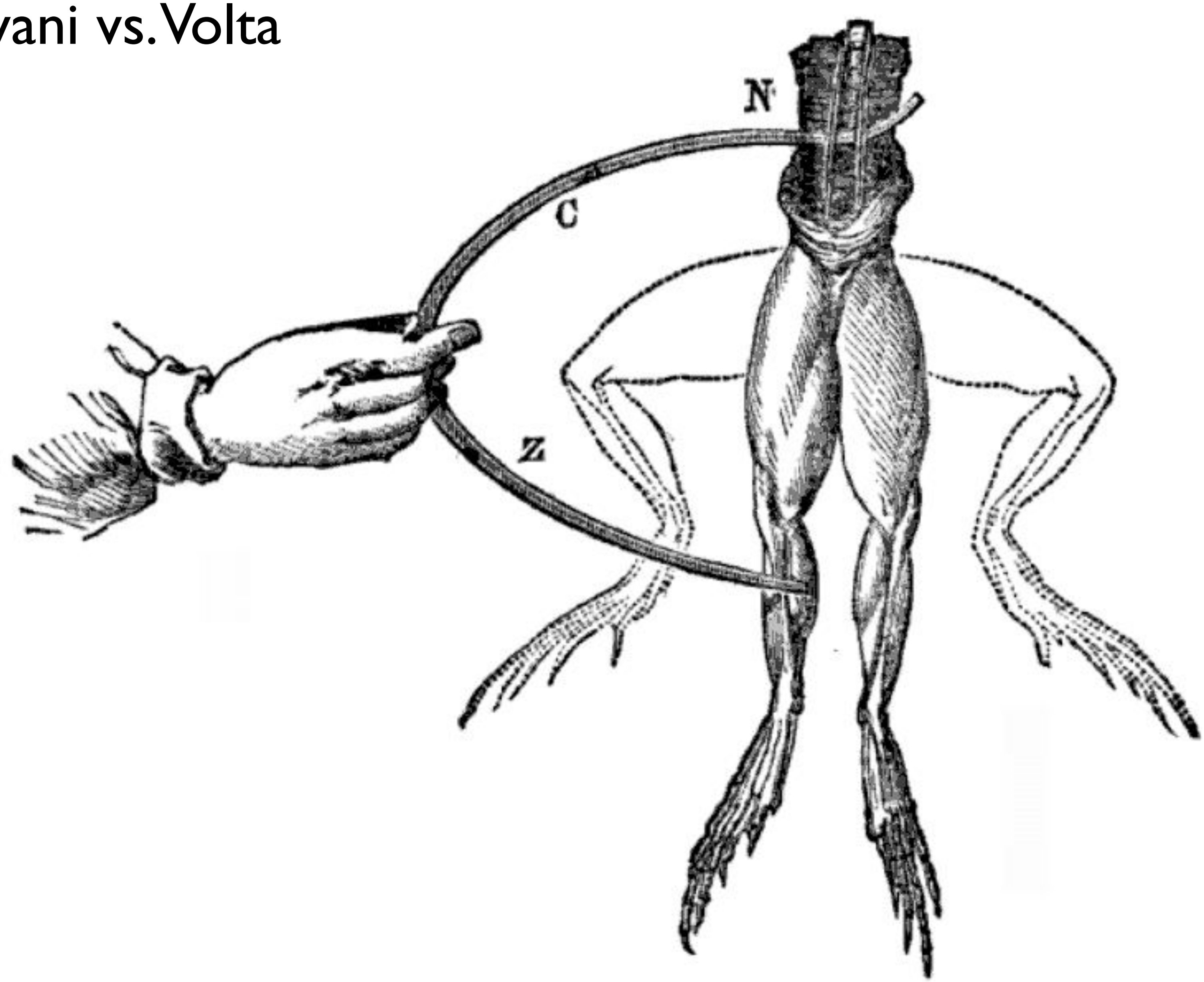
Measuring voltage



Measuring current

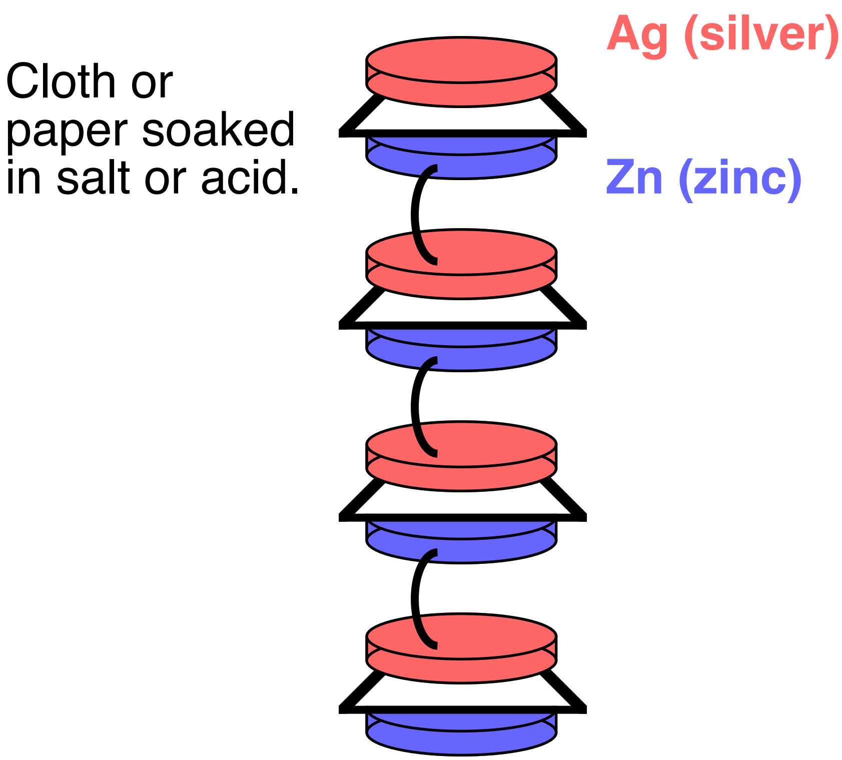
The Electric Battery

1780s - Galvani vs. Volta



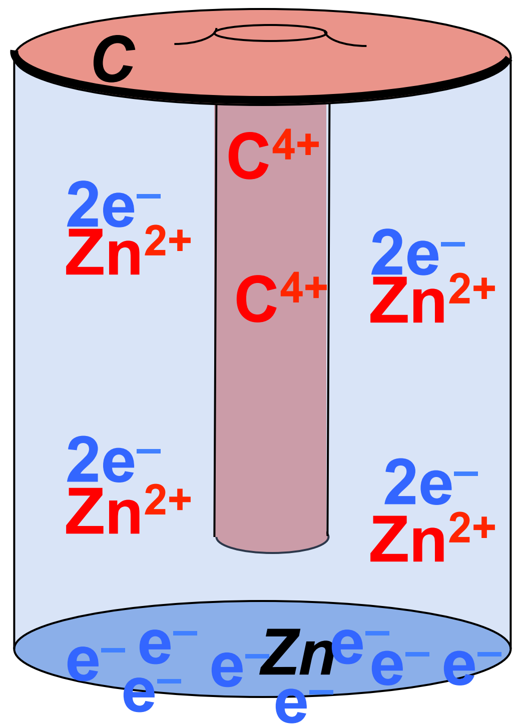
The Electric Battery

1780s - Galvani vs. Volta



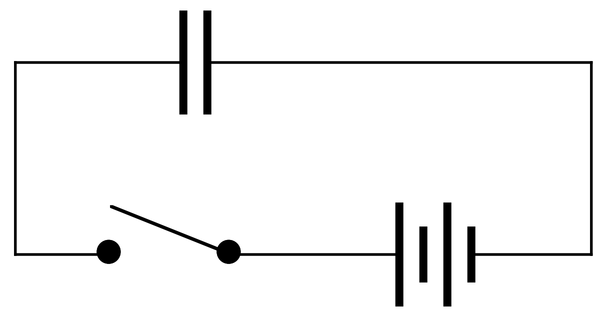
How It Works

Two electrodes (carbon and zinc) are immersed in a dilute acid (the *electrolyte*).



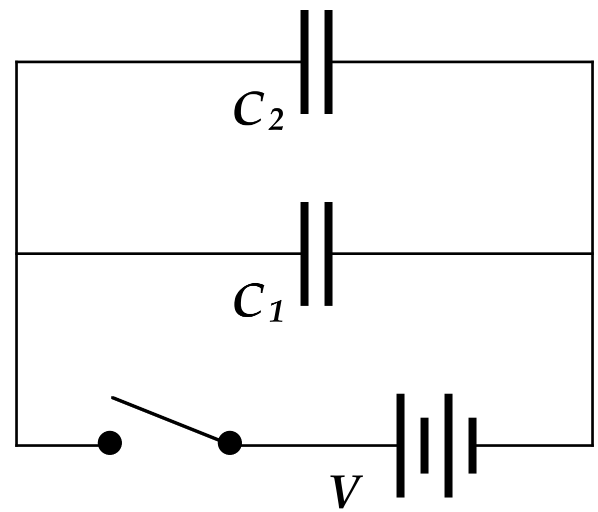
Charging a Capacitor

If a 9V battery is connected to a 300 μF capacitor, how much charge builds up on the plates?



Capacitors in Parallel

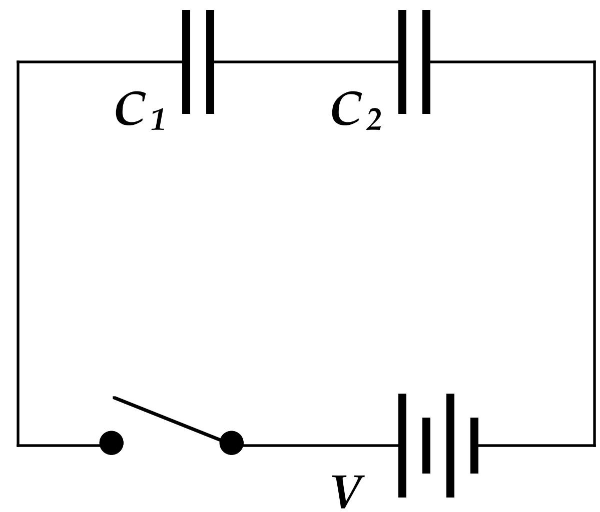
Capacitors with both of their ends connected as shown are said to be connected *in parallel*.



$$C_{effective} = C_1 + C_2$$

Capacitors in Series

Capacitors connected in a row as shown are said to be connected *in series*.

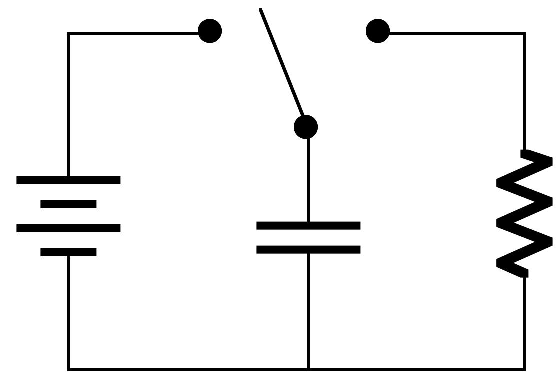


$$\frac{1}{C_{effective}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors store Energy

A capacitor can store charge, and then discharge through a device, doing Work.

But how much Work?



Capacitor energy derivation

With q_{inst} on capacitor plates, there is a ΔV potential difference across them.

$$\Delta V_{across\ \ plates} = \frac{q_{inst\ \ on\ \ plates}}{C}$$

How much Work is required to separate those charges so they can be stored on those plates?

$$dU = \Delta V \bullet dq$$

$$dU = \frac{q_{inst}}{C} \bullet dq$$

We can solve this using a model that is different from the actual charging process, but which yields the same result: how much work to transfer a charge *dq* across the plates?

$$U = \int_0^Q \frac{q_{inst}}{C} \bullet dq$$

$$U = \frac{Q^2}{2C}$$

Another derivation

The final potential difference across the plates is?

$$V = \frac{Q}{C}$$

What is the average potential difference during this charging process?

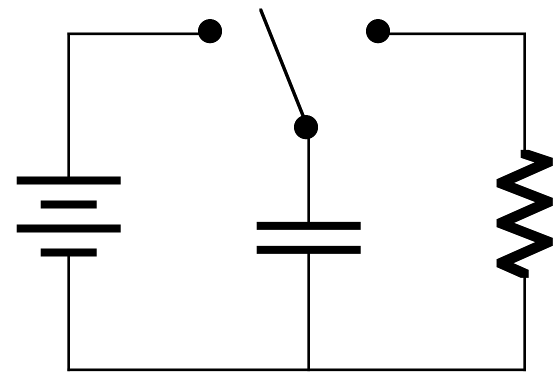
$$\frac{V_o + V_f}{2} = \frac{Q}{2C}$$

What is the work necessary to achieve that potential?

$$U = Vq = \frac{Q^2}{2C}$$

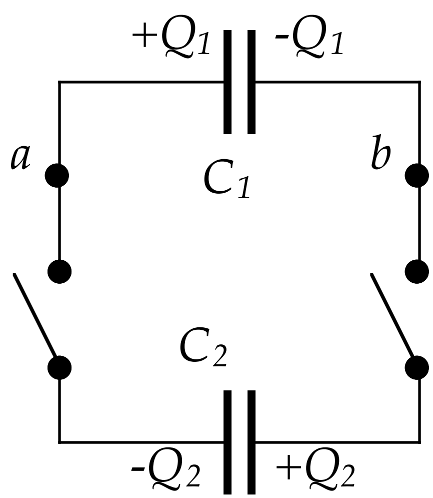
Other energy formulae

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$



Example 6

Examine the circuit here, where $C_1 > C_2$ and both have been charged to the same potential *V*.



a. What is the potential between points *a* and *b* before the switches are closed?

Q1/C1

b. What happens to charges after the switches are closed?

Q1>Q2, so excess Q1 flows until V1 = V2

c. What is the potential from *a* to *b* a long time after the switches have been closed?

Total charge Q = Q1 + -Q2 = VC1 + -VC2
Caps in parallel so $C_{effective}$ = C1 + C2
Now... V' = Q'/C' = V(C1 – C2) / (C1 + C2)

d. What is the energy stored in the system before and after the closing of the switches?

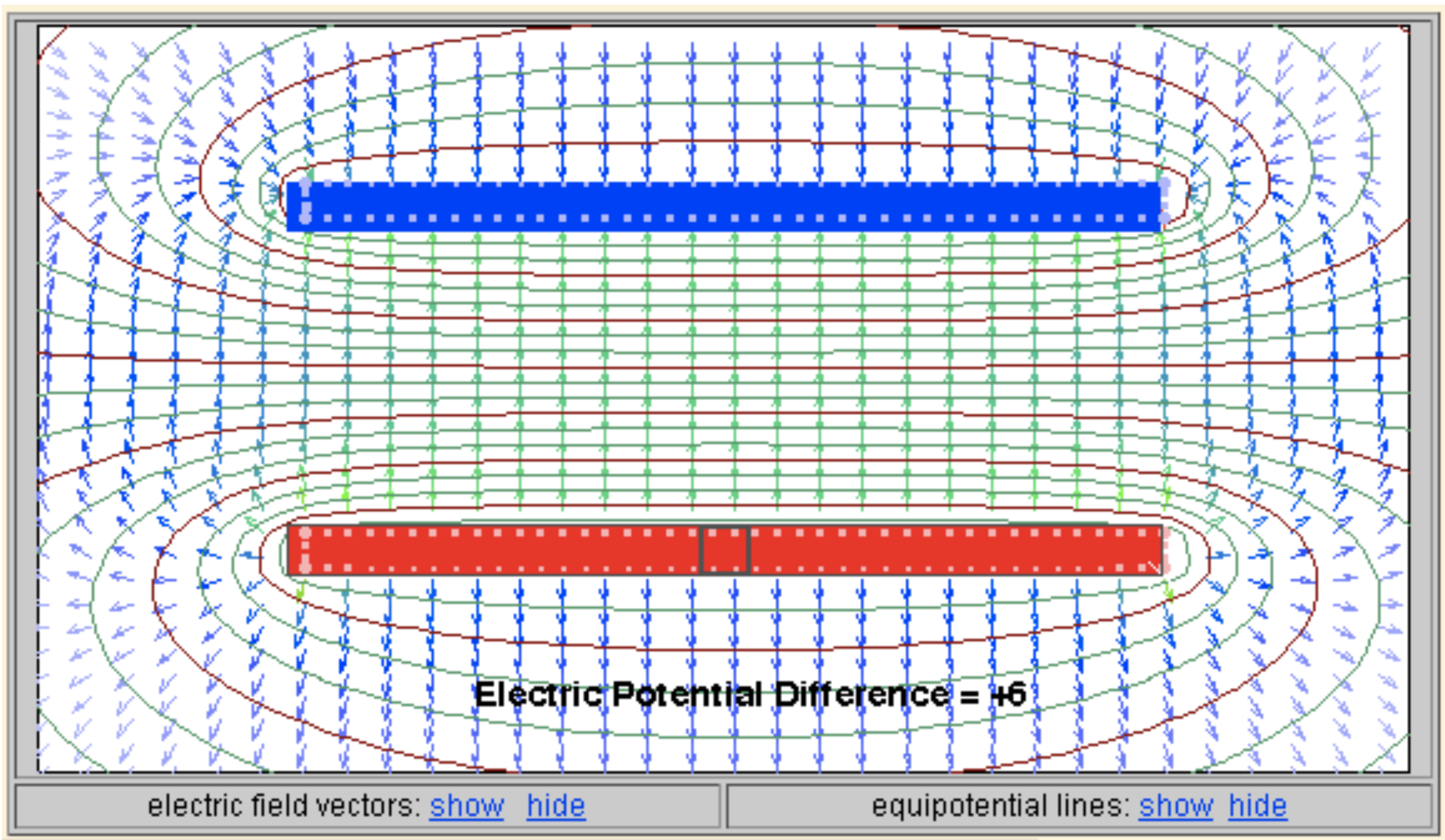
$$U_{initial} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}V^2(C_1 + C_2)$$

$$U_{final} = \frac{1}{2}C_1V'^2 + \frac{1}{2}C_2V'^2 = \frac{1}{2}\left(V\frac{C_1 - C_2}{C_1 + C_2}\right)^2(C_1 + C_2)$$

$$U_{final} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2 U_{initial}$$

Demo 2
Parallel-Plate Capacitor

Physlet I.26.2



Dielectrics

An insulator inserted between the plates of an isolated parallel-plate capacitor causes a *decrease* in the potential difference between the two plates, a decrease of a factor κ.

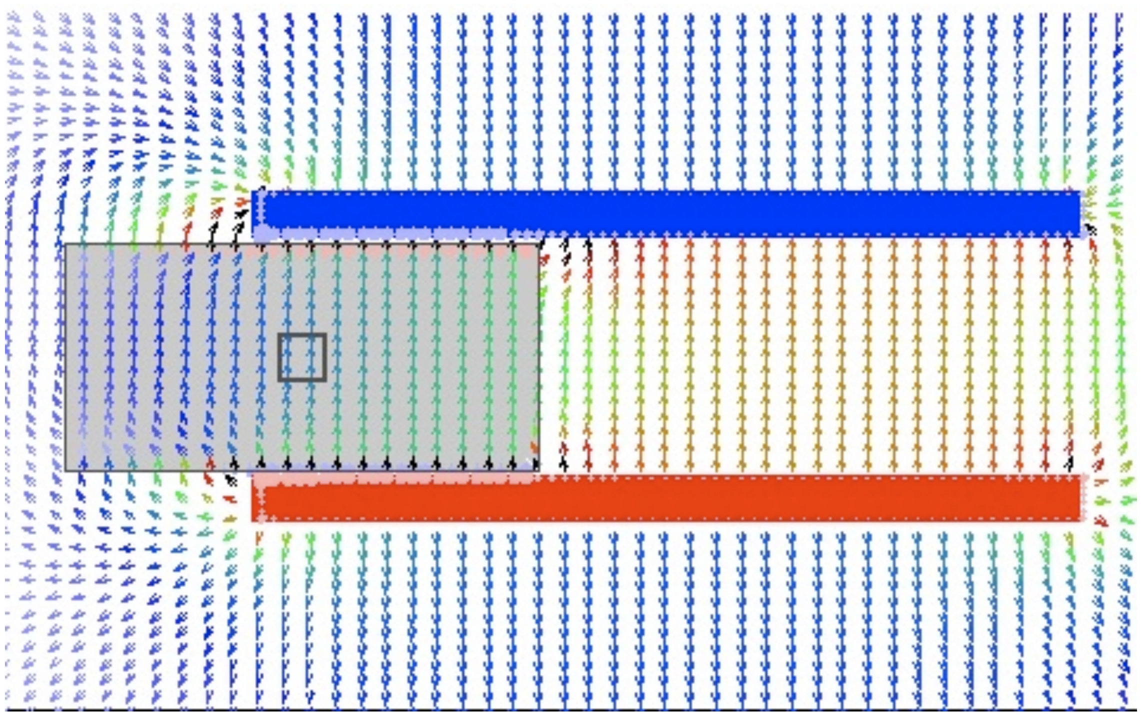
$$C_o = \frac{Q_o}{V_o}$$

$$V = \frac{V_o}{\kappa}$$

Why? What effect does this have on capacitance of the system?

$$C = \frac{Q_o}{V} = \frac{Q_o}{V_o/\kappa} = \kappa \frac{Q_o}{V_o}$$

$$\boxed{C = \kappa C_o}$$



Dielectric Values

Material	Dielectric Constant (κ)	Dielectric Strength (V/m)
Vacuum	1.00000	------
Air (dry)	1.00059	3e6
Paper	3.7	16e6
Water	80	------

$$C = \frac{\varepsilon_o A}{d}$$

$$C = \frac{\kappa \varepsilon_o A}{d}$$

Advantages to using dielectrics in capacitors include:

1. increasing capacitance (!)
2. increasing the maximum operating voltage of the capacitor (most dielectrics have a greater breakdown strength than air does)
3. the dielectric itself provides a mechanical support between the plates.

Example 7

A parallel-plate capacitor has plates 2.0cm x 3.0 cm, separated only by a 1.00mm thickness of paper.

a. Find the device's capacitance.
b. What is the maximum charge that can be placed on the capacitor?
c. What is the maximum energy that can be stored in this capacitor?

1. 19.7 pF
2. Dielectric strength=16e6 V/m. Vmax=Ed=16e6•0.001m)-16000V, so max Q=CV=(19.7pF)(16000V)=0.315μC
3. $U=CV^2/2=2.52e-3J$

Debrief test

2 years ago: avg=65, range=35-88

Last year: avg=60, range=24-96

This year: avg=70 , range=42-88

Absolute Electric Potential

For a point charge in space:

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \bullet d\vec{\mathbf{s}}$$

$$\vec{\mathbf{E}} = k\frac{q}{r^2}\hat{\mathbf{r}}$$

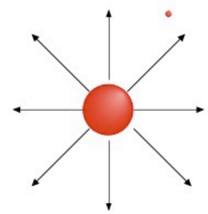
$$V_B - V_A = -\int_A^B k\frac{q}{r^2}\hat{\mathbf{r}} \bullet d\vec{\mathbf{s}}$$

$$V_f - V_i = -\int_{r\ initial}^{r\ final} k\frac{q}{r^2}\,dr$$

$$V_f - V_i = kq\left(\frac{1}{r_{final}} - \frac{1}{r_{initial}}\right)$$

If we set $V_i = 0$ at $r = \infty$,

$$V = k\frac{q}{r}$$



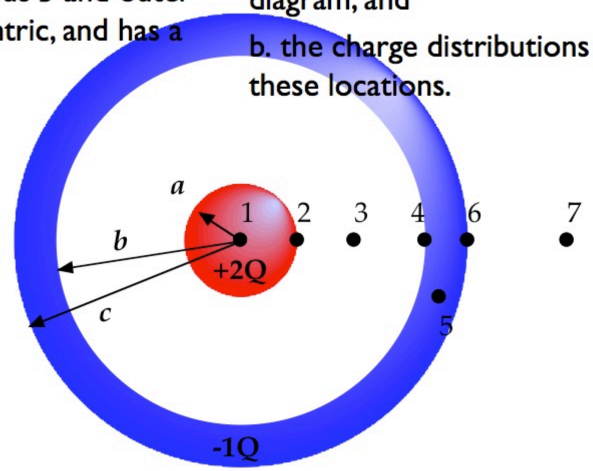
Example 15 - E for conductors

A solid conducting sphere of radius *a* has a net positive charge +2Q. A conducting spherical shell of inner radius *b* and outer radius *c* is concentric, and has a net charge -Q.

Use Gauss's Law to find

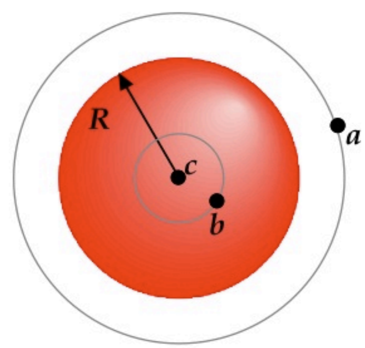
a. the electric field at the 7 regions (1-7) shown in the diagram, and

b. the charge distributions at these locations.



Example 10

An insulating solid sphere of radius *R* has a uniform positive charge density w/ total charge *Q*.



a. Find the electric potential at a point outside the sphere, $r > R$. (Take potential to be 0 at $r = \infty$.)

b. Find the potential at a point inside the charged sphere, $r < R$.

c. What are the electric field and electric potential at the center of the uniformly charged sphere?