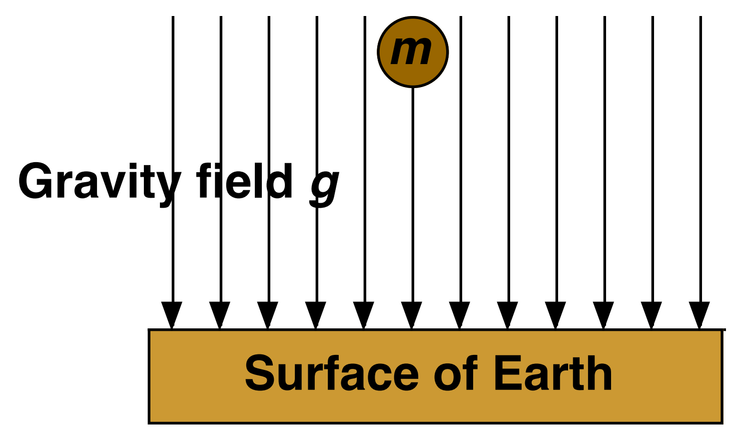
Ch 25 Electric Potential

Electric Energy, Electric Potential

Energy concepts are going to be extremely important to us as we consider the behavior of *charges* in *electric* fields.

How do energy concepts help us understand *masses* in *gravity* fields?



U_g Review

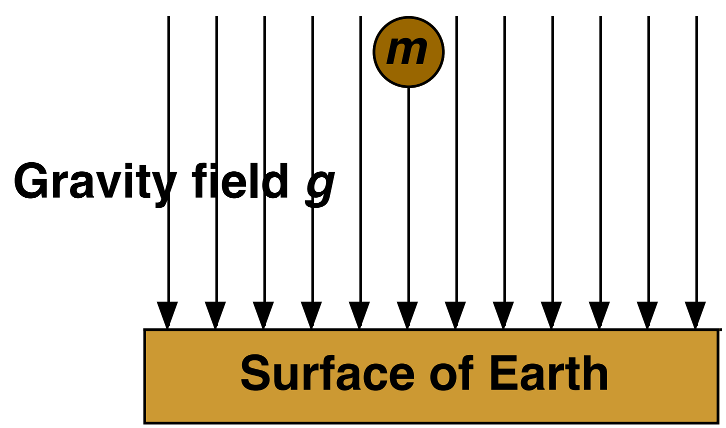
A mass *m* released near the surface of the earth begins to move, due to the force of gravity. This can be described in two ways:

$$W = \int \vec{\mathbf{F}} \bullet d\vec{\mathbf{s}}$$

$$W_{done\ \ by\ \ field} = \int \vec{\mathbf{F}}_g \bullet d\vec{\mathbf{s}}$$

$$U_f - U_i = -\int_i^f \vec{\mathbf{F}}_g \bullet d\vec{\mathbf{s}}$$

1. The gravity field will do Work on the object, $W=F_g d$, increasing its kinetic energy.
2. The object has gravitational potential energy $U_g=mgh$, which is converted to kinetic energy as the object falls.



U_e: Electric Potential Energy

The same exact thing occurs for a charge in an electrical field.

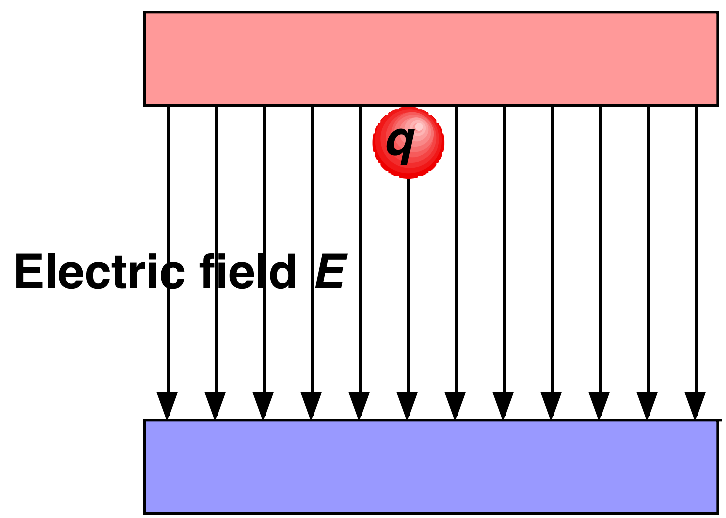
$$W = \int \vec{\mathbf{F}} \bullet d\vec{\mathbf{s}}$$

$$W_{done\ \ by\ \ field} = \int q\vec{\mathbf{E}} \bullet d\vec{\mathbf{s}}$$

$$U_f - U_i = -q\int_A^B \vec{\mathbf{E}} \bullet d\vec{\mathbf{s}}$$

Where is the electric potential *high* for this test charge?

Where is the electric potential *low* for this test charge?



Of course, a decrease in U results in an increase in K.

Example 2

A proton has a potential energy of 6.7e-15 J on the positive side of a constant 1.00 m long electric field. At the negative side, U = 0.00 J.

a. How strong is the Electric field?

$$U_f - U_i = -q\int_A^B \vec{\mathbf{E}} \bullet d\vec{\mathbf{s}}$$

$$0 - 6.7e\text{-}15 = \text{-}(1.602e\text{-}19)E(1)$$

$$E = 4.18e4 N/C$$

b. How fast is the proton traveling just before it arrives at the negative side of the field?

$$\Delta U_i = \Delta K$$

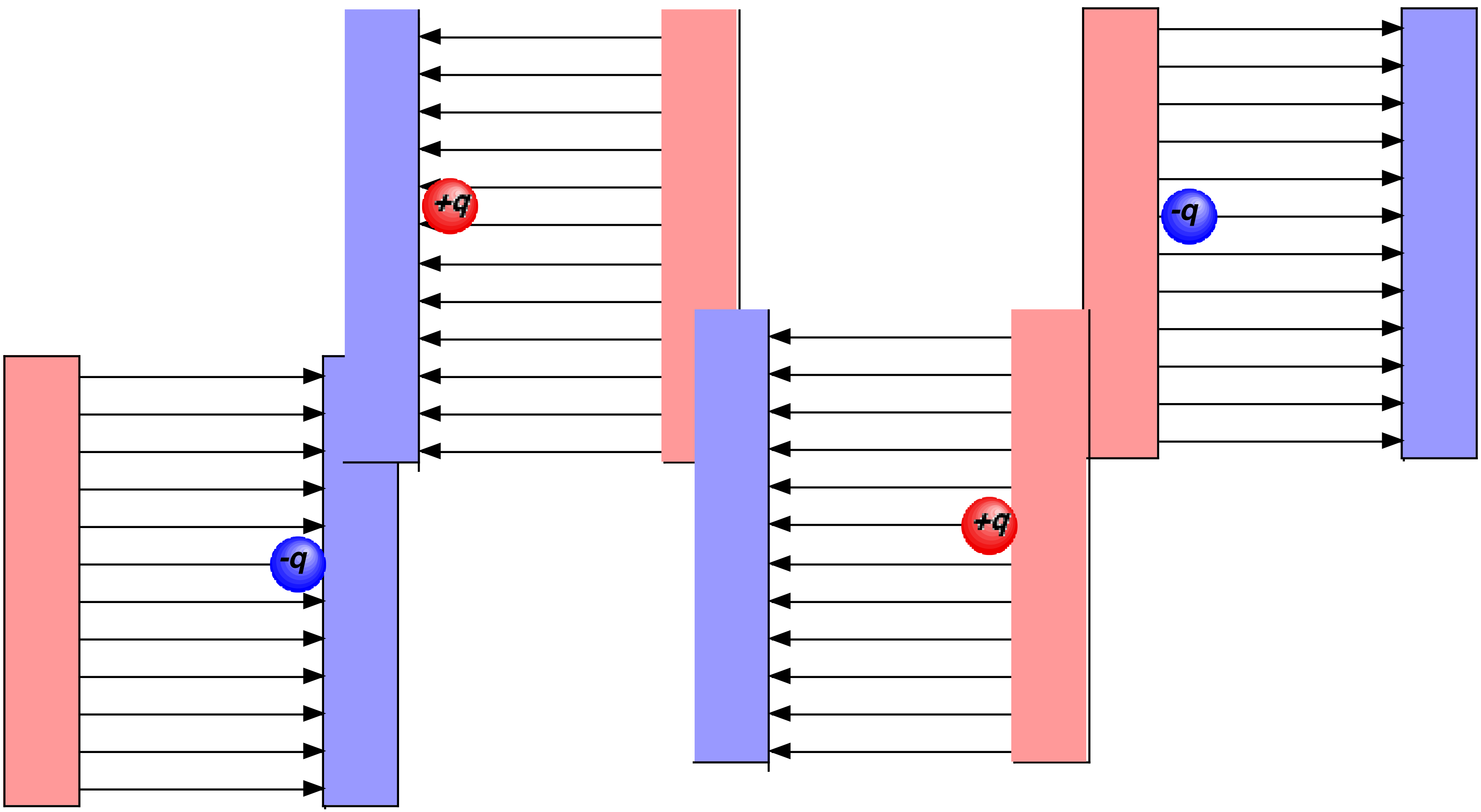
$$6.7e-15 = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{6.7e-15 \bullet 2}{1.67e-27}}$$

$$v = 2.83e6 m/s$$

Example 1

Is the electrical charge in each of these situations in a position of *high* U_e or *low* U_e?

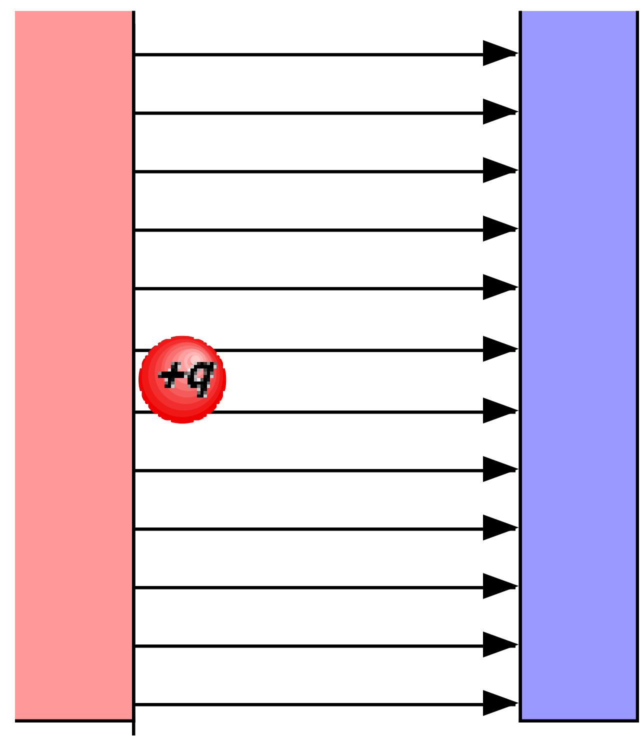


What is Electric *Potential*?

We need to be able to talk about the change in an electric field without knowing about a charged particle that might be in it.

$$\Delta V = \frac{\Delta U}{q_o}$$

To do this, we' ll refer to a *change in electric potential*, rather than a *change in electric potential energy*.

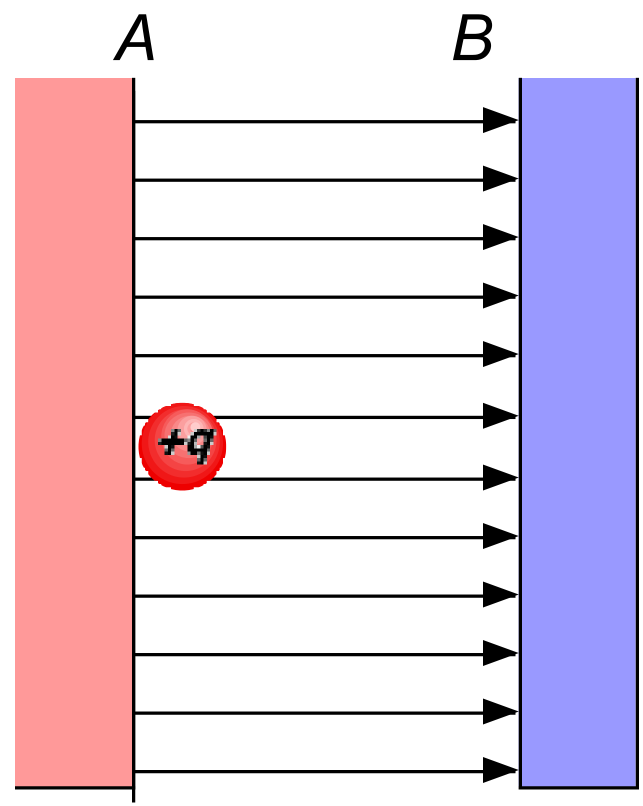


Difference in Electric Pot.

$$\Delta V = V_B - V_A = \frac{\Delta U}{q} = \frac{-W_{BA}}{q} = \frac{-q\int E \bullet ds}{q} - \int_A^B \vec{\mathbf{E}} \bullet d\vec{\mathbf{s}}$$

The units for *electrical potential* are **Volts**, where 1 Volt = 1 Joule/Coulomb.

We' ve talked about high and low *U*, but where are the high and low *electric potentials*? Most of the time, the potential is set at 0 at the ground (negative plate).



Example 2

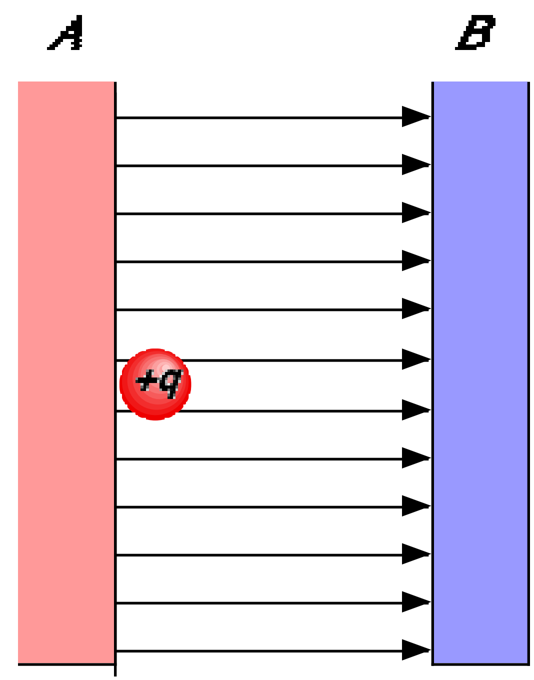
A potential difference of 9.0-Volts exists between two plates.

How much Work is required to move a 2.0-Coulomb charge across that potential?

$$\Delta V = \frac{W_{BA}}{q}$$

$$W = \Delta V q = (9V)(2C)$$

$$W = 18VC = 18 Joules$$

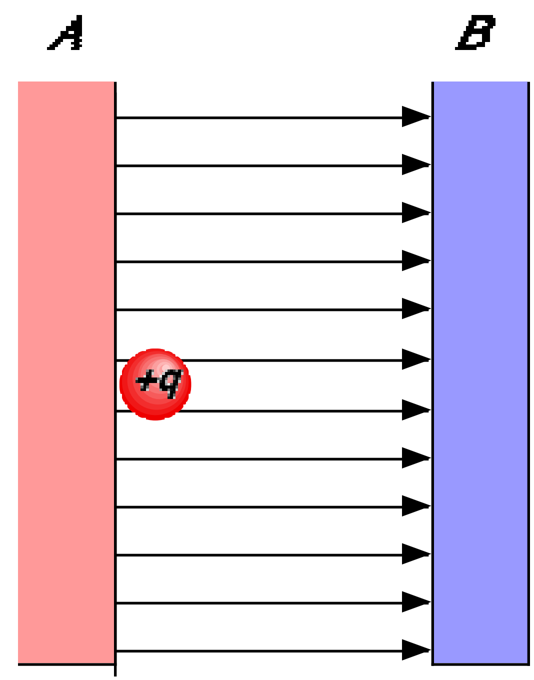


E Potential ≠ E Potential E!

Electric potential is a measure of how much energy per unit charge a particle will acquire as it travels a given distance through a given electric field.

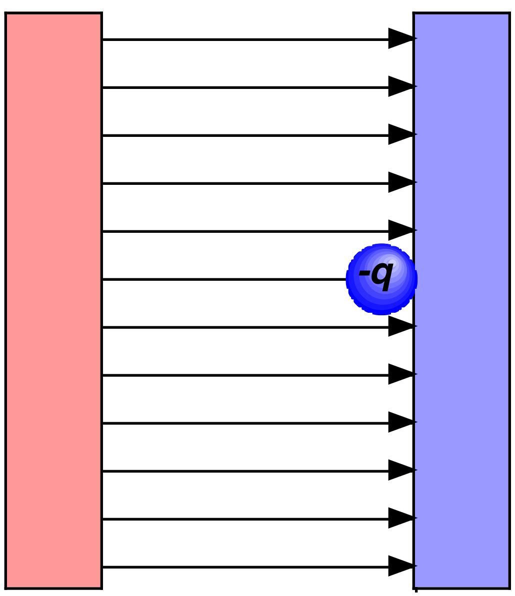
$$\Delta U = q\Delta V$$

$$\Delta V = \frac{\Delta U_e}{q} = \frac{-W_{BA}}{q}$$



Example 3

An electron (e^-) in a TV picture tube is accelerated from rest through a potential difference of 5000 V.



a. What is the change in the *U* of the electron?

$$\Delta U = q\Delta V$$
$$= (1.602e-19C)(5000V)$$
$$= 8e-16J = 5000eV$$

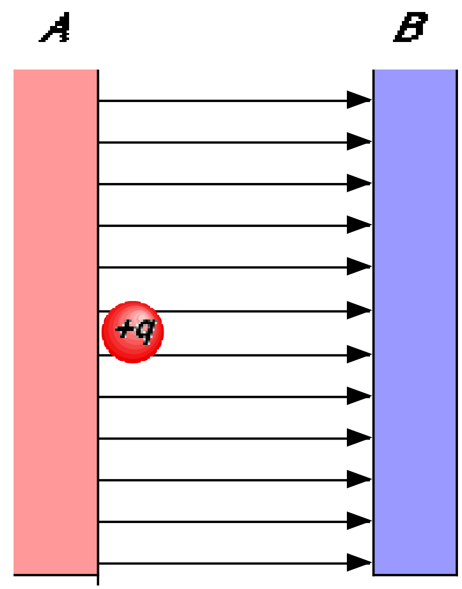
b. What is the final speed of the electron?

$$\Delta U = \Delta K$$
$$8e-16J = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2 \bullet 8e-16}{9.11e-31}} = 4.05e7m/s$$

ΔV in constant E field

A positive test charge experiences a decrease in potential energy when moving from A to B.

A charge experiences a decrease in potential when moving from A to B.



We've been using constant E fields in our examples. What does the math look like for that situation?

$$\Delta U = q\Delta V$$

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \bullet d\vec{\mathbf{s}}$$

$$V_B - V_A = -Ed$$

The negative sign here indicates that V_A is at a higher potential.

Example 4

Two parallel plates are charged to a voltage of 50V.

a. If the separation between the plates is 0.050m, calculate the strength of the E field between them.

$$V = -Ed$$

$$E = \frac{V}{d} = \frac{50V}{0.050m} = 1000N/C$$

b. What is the acceleration of an electron in this field?

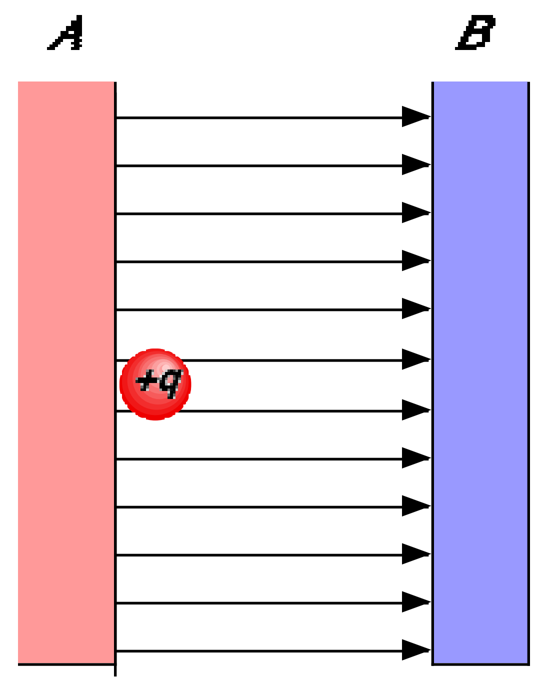
$$E = 1000N/C$$

$$F = qE = ma$$

$$a = \frac{qE}{m} = \frac{(1.602e-19C)(1000N/C)}{9.11e-31kg} = 1.8e14m/s^2$$

What we know so far

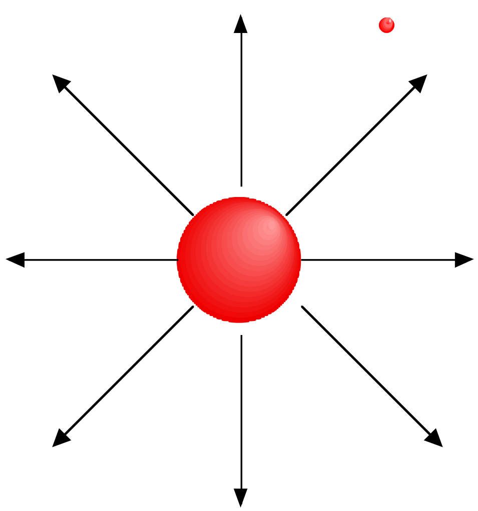
- It takes a force *F*(=*qE*) to move a charge through an electric field
- We can calculate the work *W* (=*Fd*) it takes to move the charge through that field.
- We have a quantity called *electric potential V* (=*W*/*q* = -*Ed*), which is related to how much energy/work per unit charge it takes to move through a field.



Absolute Electric Potential

When we discussed gravitational potential energy, it eventually became useful for us to define $U_g = 0$ at a distance infinitely far from the source of gravity; all large-scale gravitational energy calculations were done using

$$U_g = -G\frac{Mm}{r}$$



In the same way, we're going to define an absolute electric potential at a position in space, which will be the amount of *Work per unit charge* necessary to bring a positive test charge from infinity to that point in space.

Absolute Electric Potential

For a point charge in space:

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \bullet d\vec{\mathbf{s}}$$

$$\vec{\mathbf{E}} = k\frac{q}{r^2}\hat{\mathbf{r}}$$

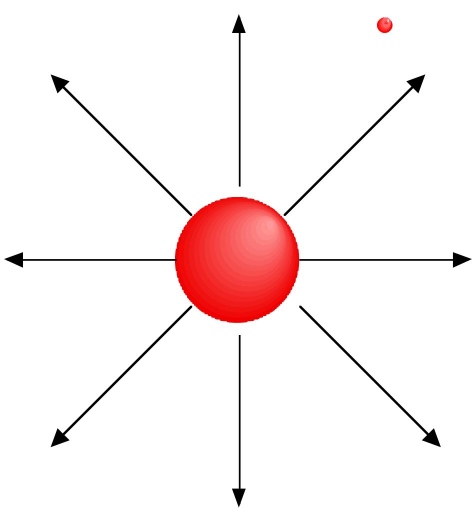
$$V_B - V_A = -\int_A^B k\frac{q}{r^2}\hat{\mathbf{r}} \bullet d\vec{\mathbf{s}}$$

$$V_f - V_i = -\int_{r\ initial}^{r\ final} k\frac{q}{r^2}\ dr$$

$$V_f - V_i = kq\left(\frac{1}{r_{final}} - \frac{1}{r_{initial}}\right)$$

If we set $V_i = 0$ at $r = \infty$,

$$V = k\frac{q}{r}$$

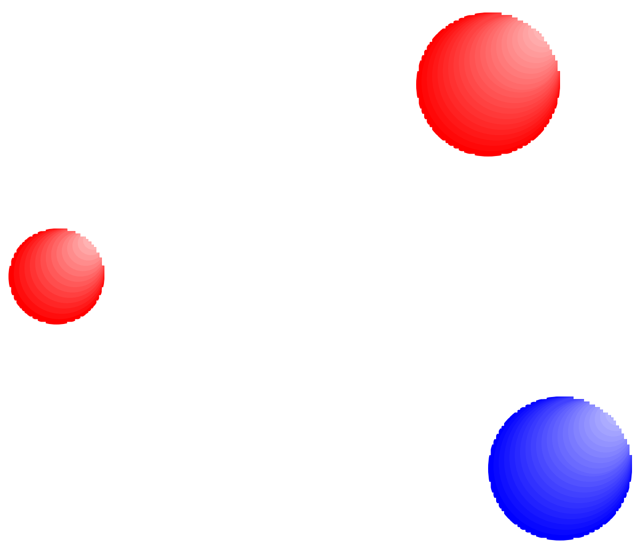


Electric Potential

For many point charges in space:

$$V_{net} = \sum_i k \frac{q_i}{r_i}$$

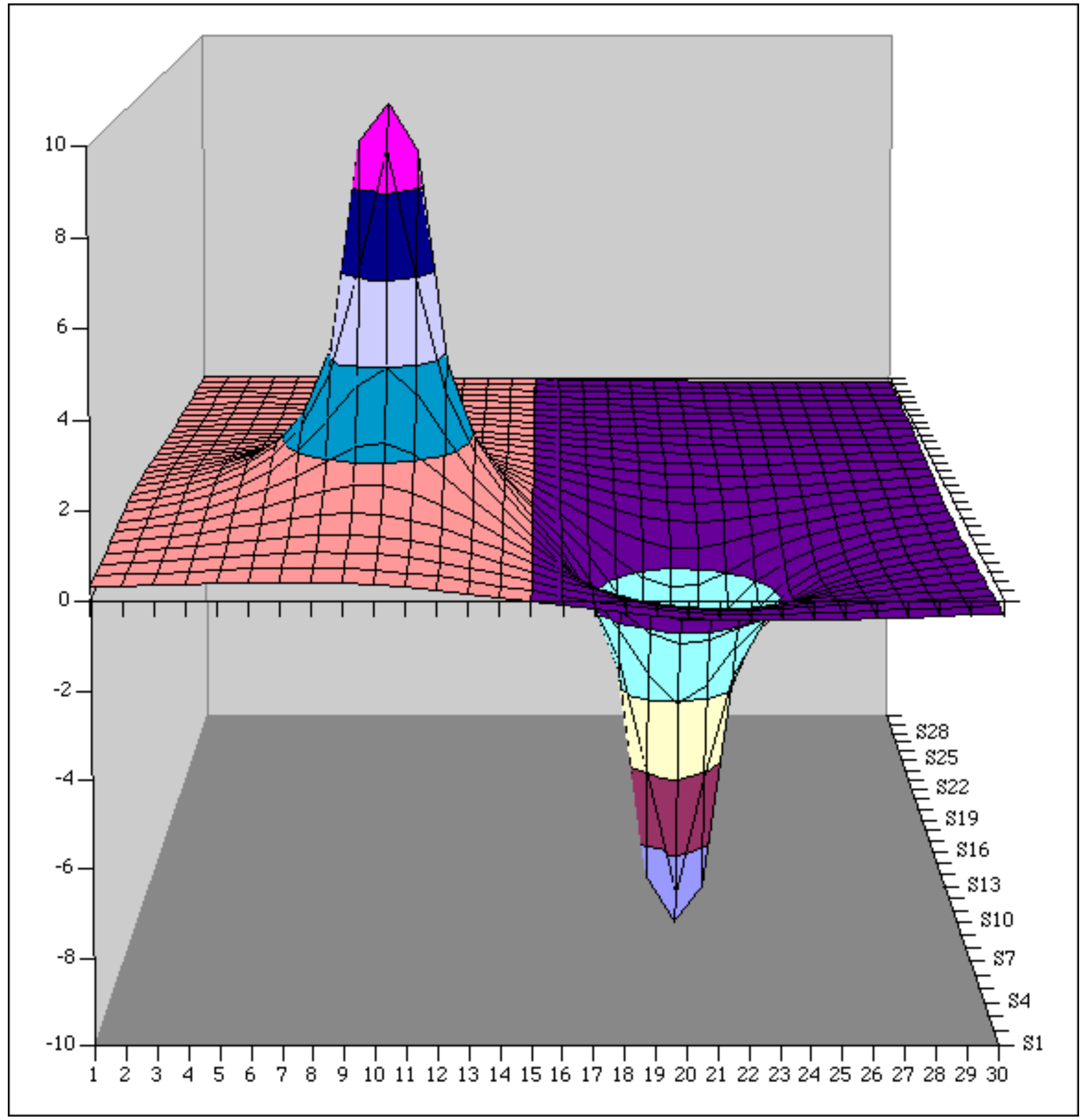
If there are numerous point charges in a given area, then the potential *V* at any given nearby point will be the sum of the potentials. (Cool point to keep in mind: potentials are not vectors, so you don't have to worry about *x* and *y* components, or directions--just add the + and - potentials and that's your answer!)



Electric Potential

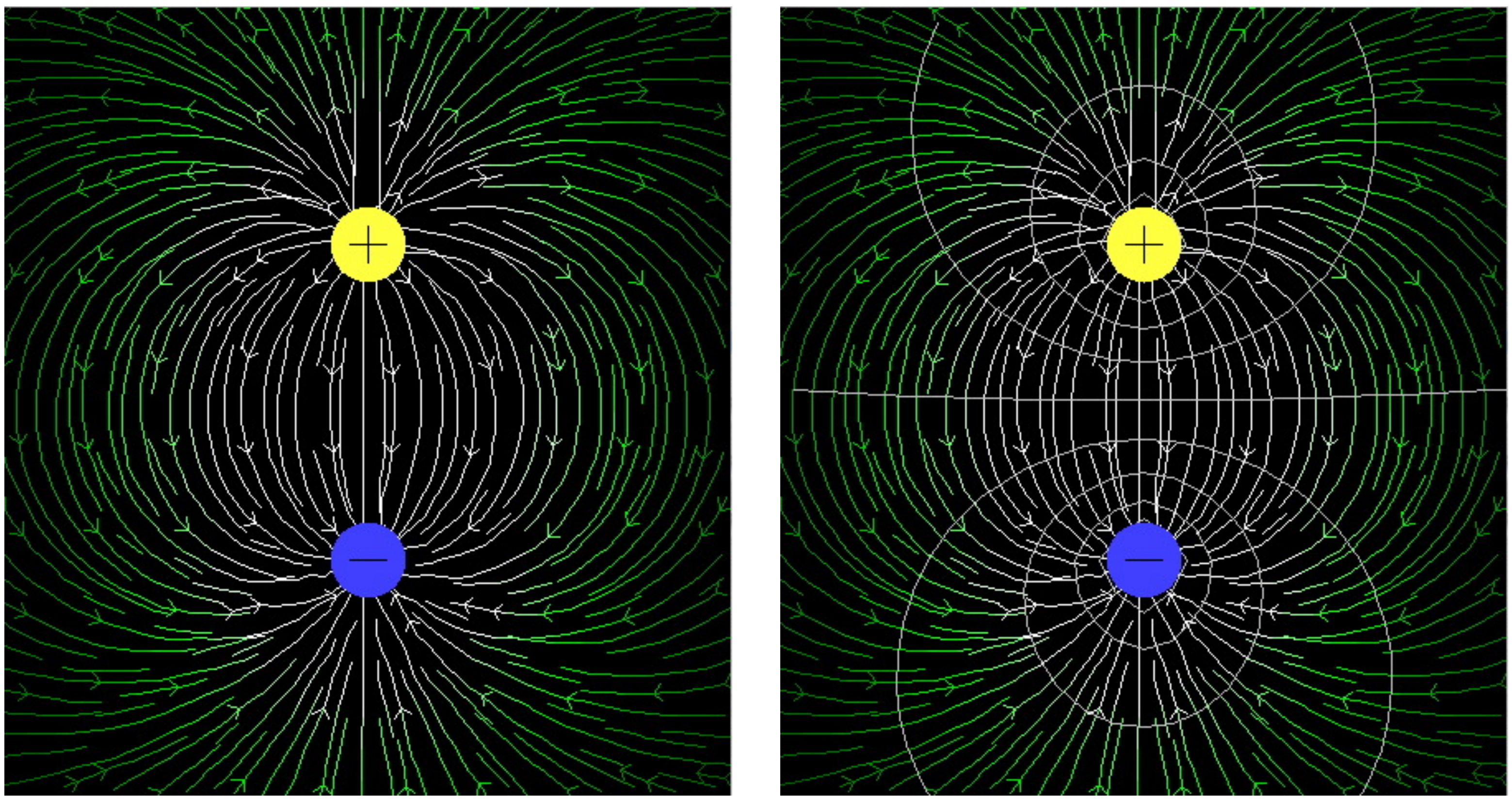
What are the potentials of each of the charges shown here? What do these potential values represent?

$$V_{net} = \sum_i k \frac{q_i}{r_i}$$



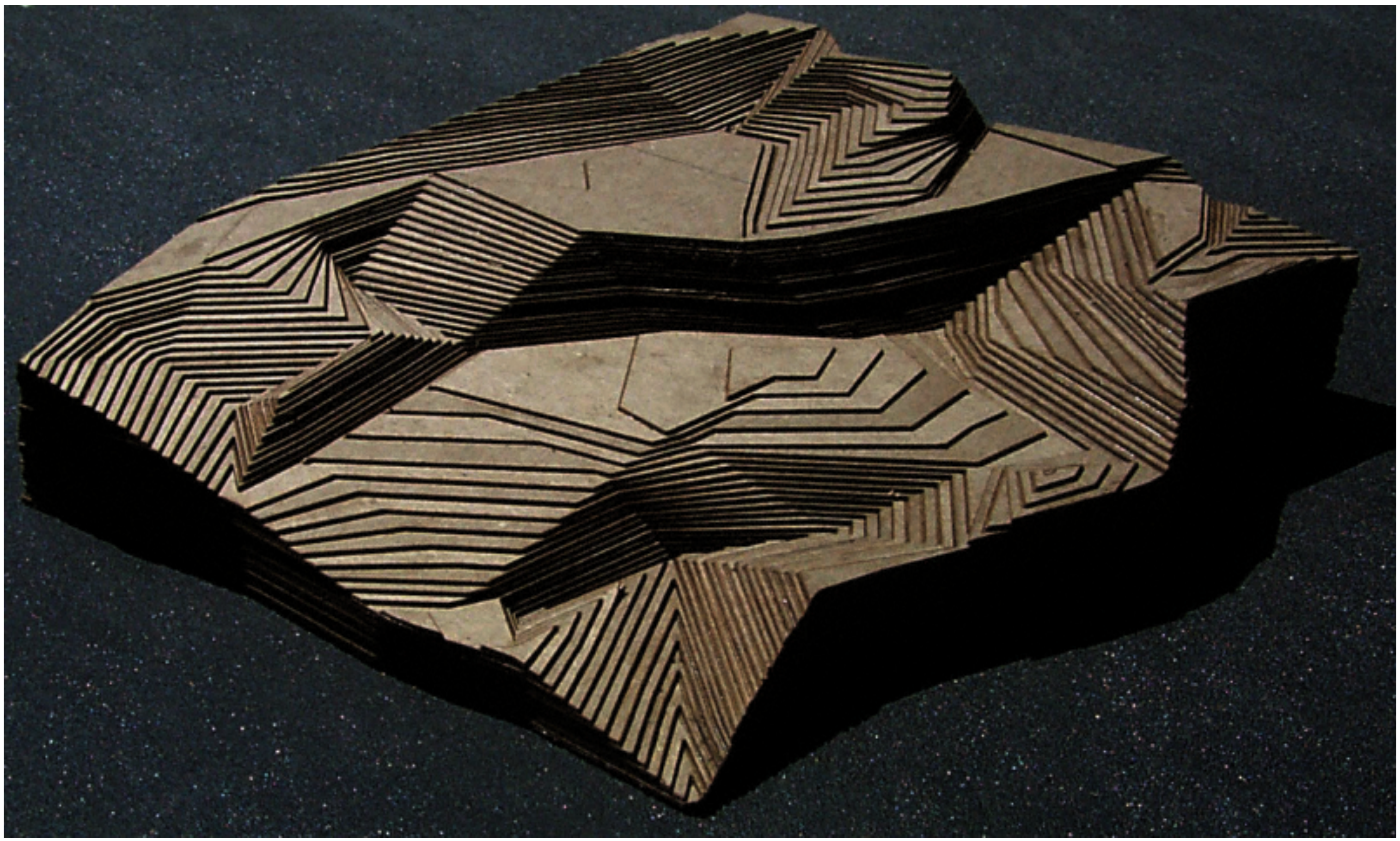
Equipotential Lines

... are related to Electric Field lines. How?



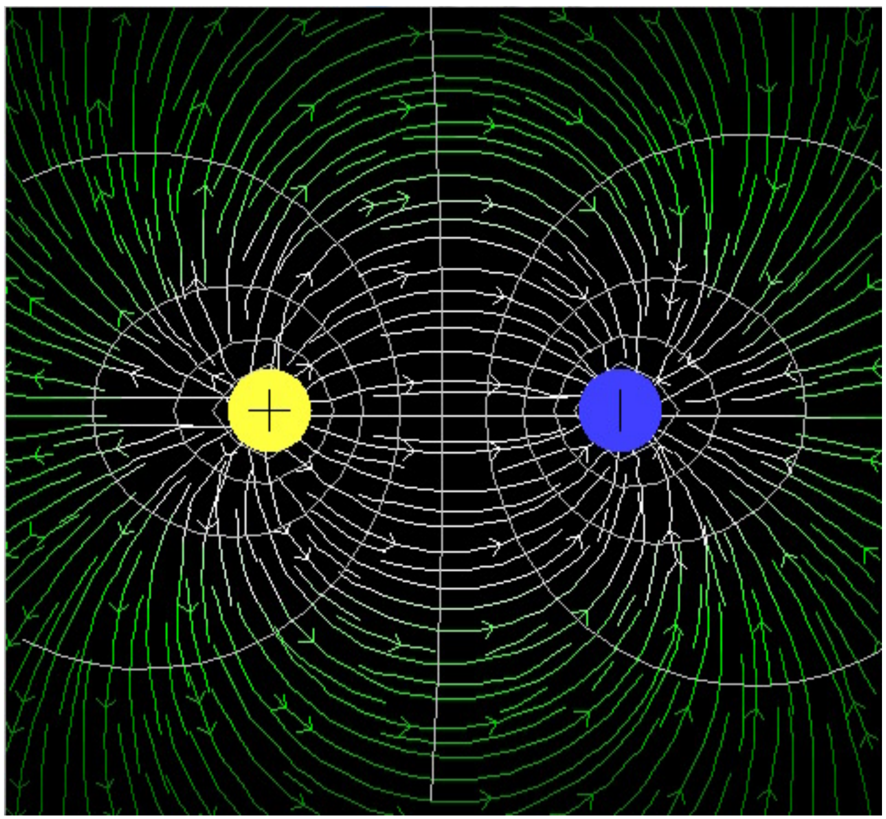
Equipotential Lines

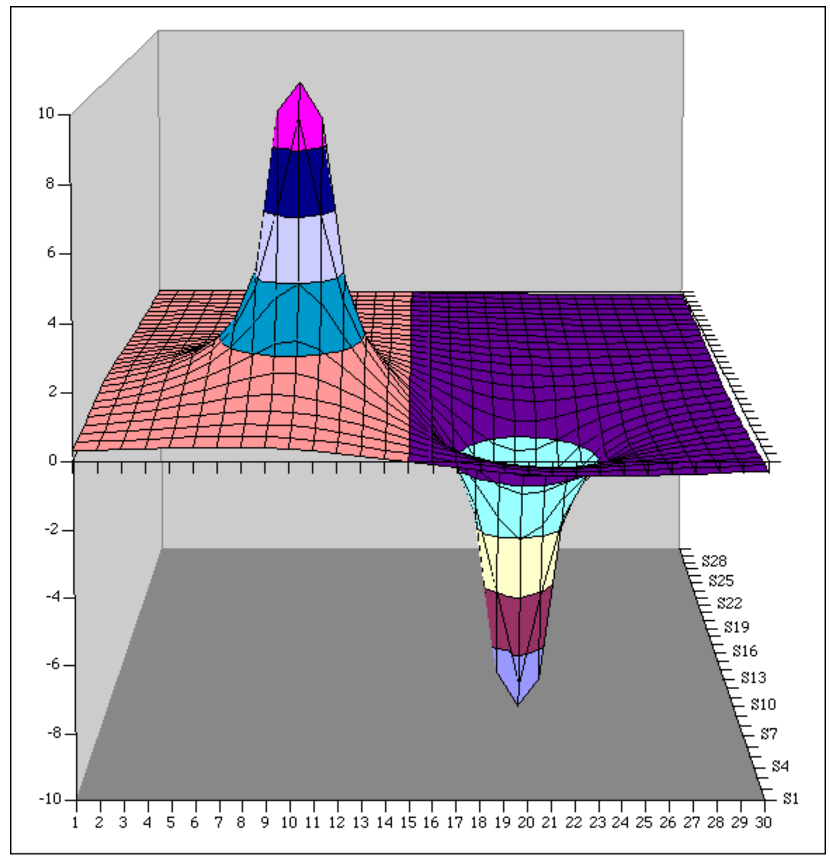
... can be visualized as a topographic map.



Equipotential Lines

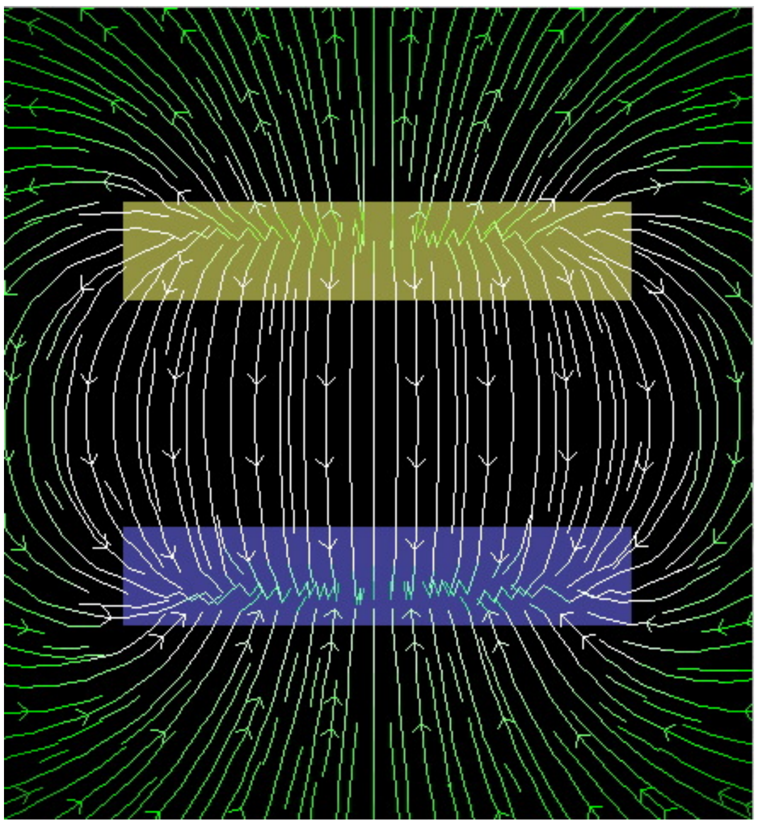
... reveal areas of higher vs. lower electric potential.

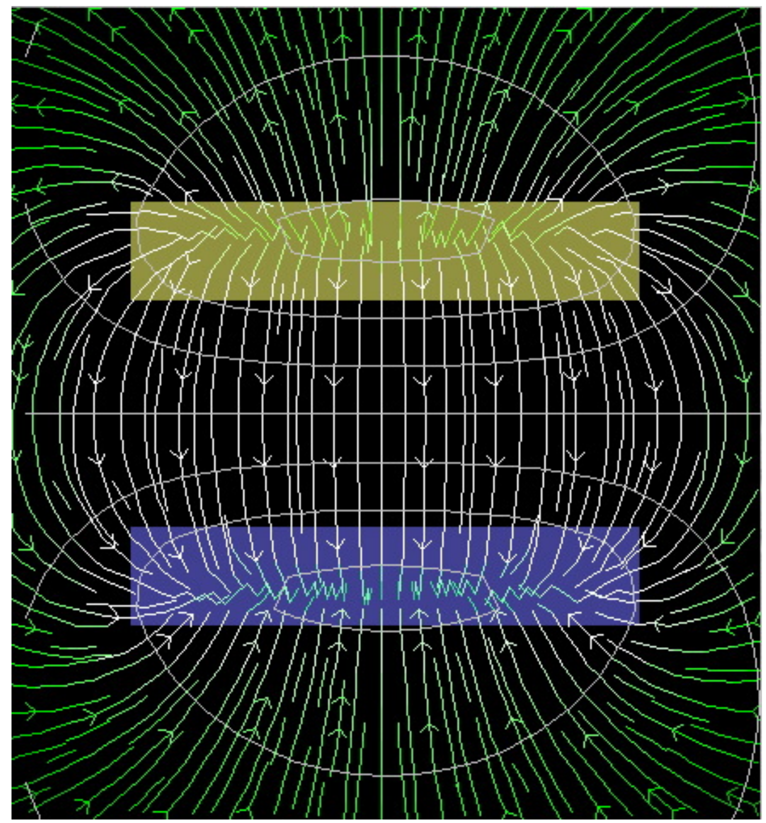




Example 5

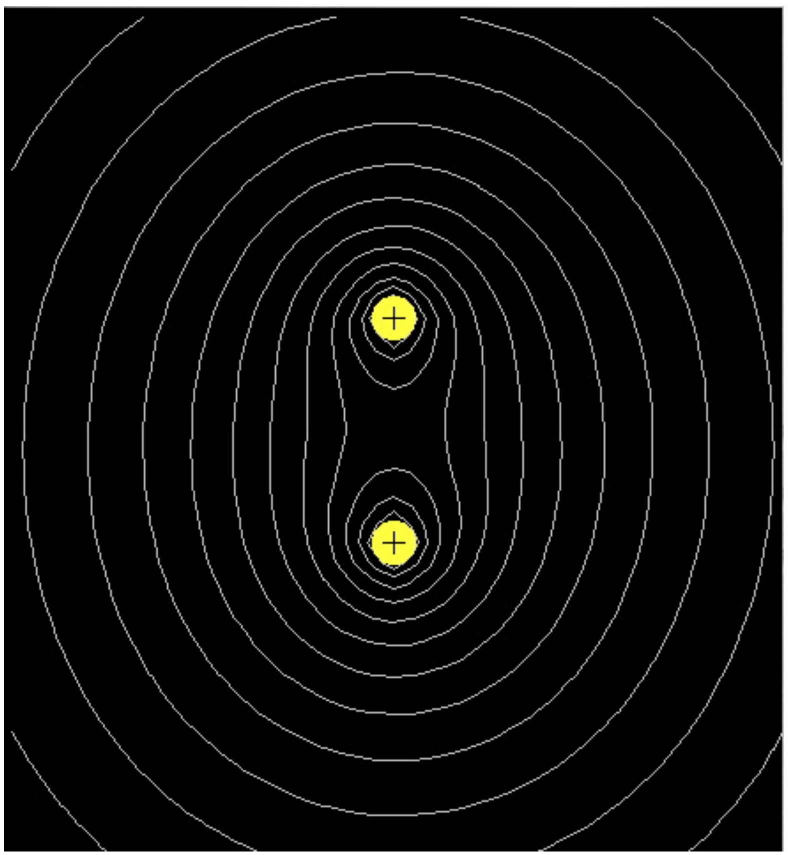
Draw appropriate equipotentials for this electric field.

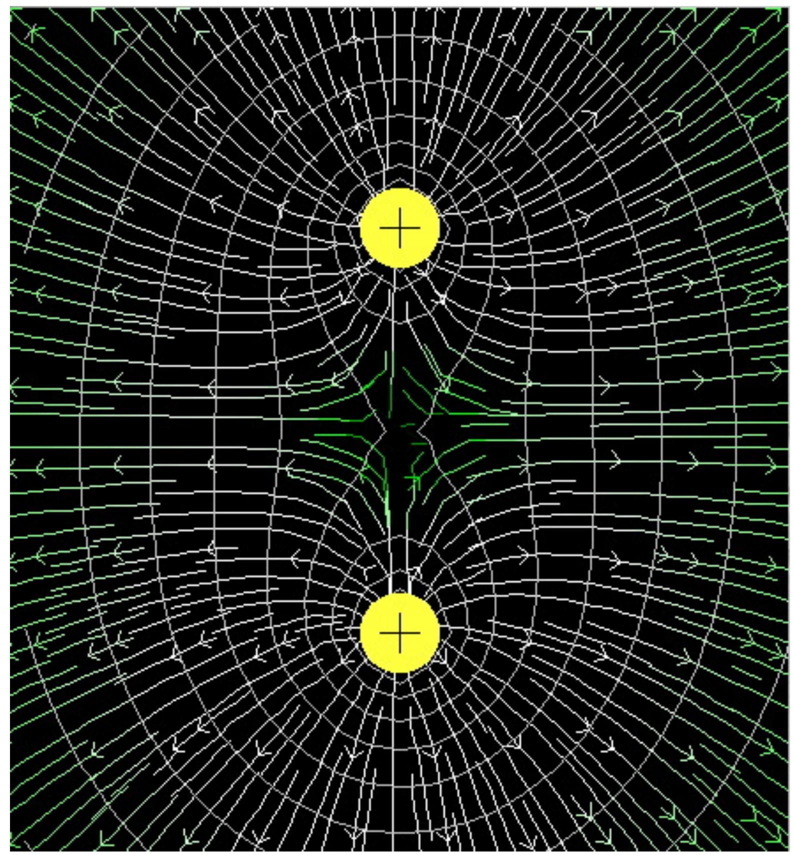




Example 6

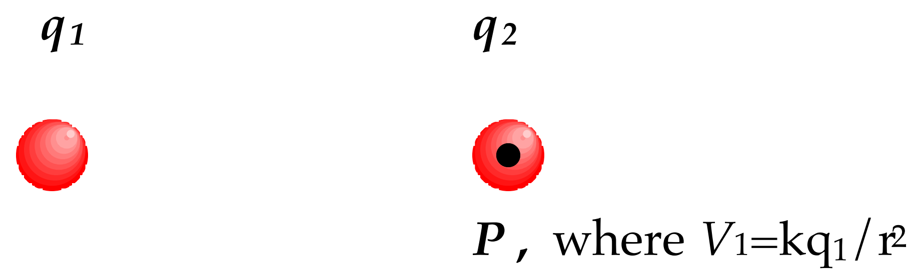
Draw appropriate field lines for these equipotentials.





U for a system of charges

In a system of charged particles, the forces between the particles themselves give the system a potential energy. We can calculate that U by considering what kind of work would be required to bring the charges together.



$$U = q_2 V_1$$

$$U = q_2\left(\frac{kq_1}{r}\right)$$

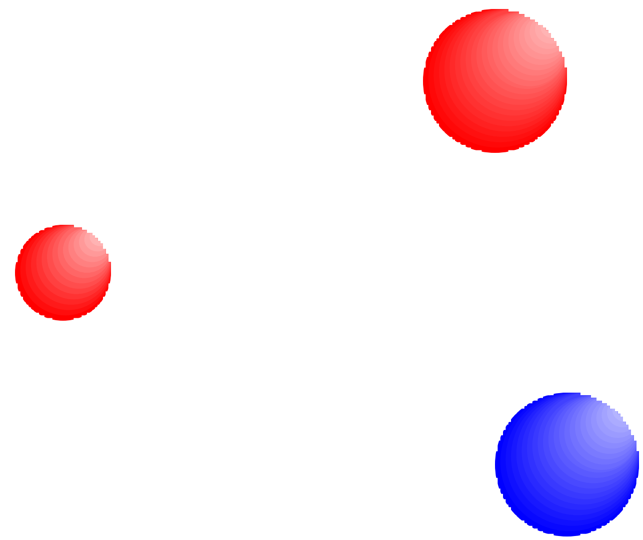
$$U = k\frac{q_1 q_2}{r}$$

U for a system of charges

For more than 2 charges, simply sum the Us.

$$U_{total} = U_1 + U_2 + U_3$$

$$U_{total} = k\frac{q_1 q_2}{r_{12}} + k\frac{q_2 q_3}{r_{23}} + k\frac{q_1 q_3}{r_{13}}$$



Electric Field from V

We know that $\Delta V = -\int_A^B \vec{\mathbf{E}} \bullet d\vec{\mathbf{s}}$

How can we go backwards to get Electric Field from Potential?

$$dV = -\vec{\mathbf{E}}_x \bullet d\vec{\mathbf{x}}$$

$$E_x = -\frac{dV}{dx}$$

E from V, radial

If we have radial symmetry, we can write

$$E_r = -\frac{dV}{dr}$$

Determine the electric field in the vicinity of a spherical charge distribution, given the electric potential function for a sphere.

$$E_r = -\frac{dV}{dr}$$

$$V = \frac{kq}{r}$$

$$E_r = -\frac{d}{dr}\left(\frac{kq}{r}\right) = \frac{kq}{r^2}\ !!!$$

Partial derivatives

What if $V=5x-3x^2y+2yz^2$? Given a 3-dimensional function, how can we find its derivative? How do we determine

$$E_r = -\frac{dV}{ds}$$

We can't differentiate all at the same time, so we use a *partial derivative* in each dimension, and combine those to get the result.

A partial derivative is indicated $\frac{\partial V}{\partial x}$ and is determined by taking the derivative with respect to one variable while keeping the other variables constant.

Example 7

In a region of space, the electric potential is given by the function $V = 5x - 3x^2y + 2yz^2$.

a) Find functions for the electric field components in this region, and

b) determine the magnitude of the field at the point *P* that has coordinates (1, 0, -2) m.

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(5x - 3x^2y + 2yz^2) = (-5 + 6xy)\mathbf{i}$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}(5x - 3x^2y + 2yz^2) = (3x^2 - 2z^2)\mathbf{j}$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}(5x - 3x^2y + 2yz^2) = (-4yz)\mathbf{k}$$

$$E_x = (-5 + 6xy)\mathbf{i} = (-5 + 6 \bullet 1 \bullet 0) = -5$$

$$E_y = (3x^2 - 2z^2)\mathbf{j} = (3 \bullet 1^2 - 2(-2)^2) = -5$$

$$E_z = (-4yz)\mathbf{k} = (-4 \bullet 0 \bullet -2) = 0$$

$$E_{magnitude} = \sqrt{E_x^{\ 2} + E_y^{\ 2} + E_z^{\ 2}} = 7.07 N/C$$

The del operator

A convenient mathematical shorthand for this operation is the *del* operator:

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\bar{E} = \left[\ (E_x)\hat{i} + \ (E_y)\hat{j} + (E_z)\hat{k}\right]$$

$$\vec{E} = -\left[\left(\frac{\partial V}{\partial x}\right)\hat{i} + \left(\frac{\partial V}{\partial y}\right)\hat{j} + \left(\frac{\partial V}{\partial z}\right)\hat{k}\right]$$

$$\vec{E} = -\left[\left(\frac{\partial}{\partial x}\right)\hat{i} + \left(\frac{\partial}{\partial y}\right)\hat{j} + \left(\frac{\partial}{\partial z}\right)\hat{k}\right]V$$

$$\vec{E} = -\vec{\nabla}V$$

Example 8

If the electric potential is constant in some region of space, what can you conclude about the electric field in that region?

E= -dV/dr, so if the potential isn't changing, the E field must = 0. (Likewise, if E=0, then we're not seeing any change in potential there.)

V from continuous charge?

If we need to calculate electric potential (energy per unit charge) for a point charge, it's easy: $V = \frac{kq}{r}$

If we need to calculate electric potential for a continuous distribution of charge, we have two strategies:

a. For a known charge distribution:

$$dV = k\frac{dq}{r}$$

$$V = \int k\frac{dq}{r}$$

a. If Electric Field is known:

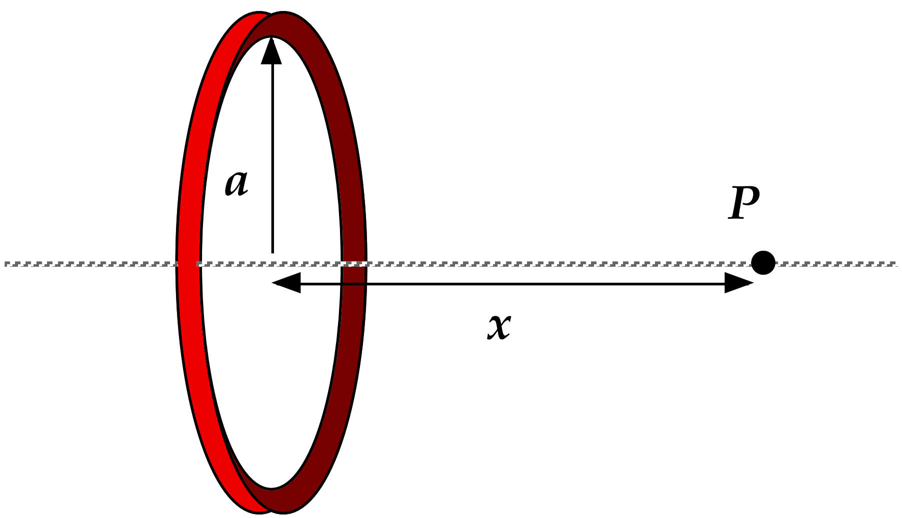
$$\Delta V = \frac{\Delta U}{q_o} = -\int_A^B \vec{\mathbf{E}} \bullet d\vec{\mathbf{s}}$$

Example 9

Determine the electric potential for a hoop of charge +Q and radius *a*, at a distance *x* along the x-axis, as shown.

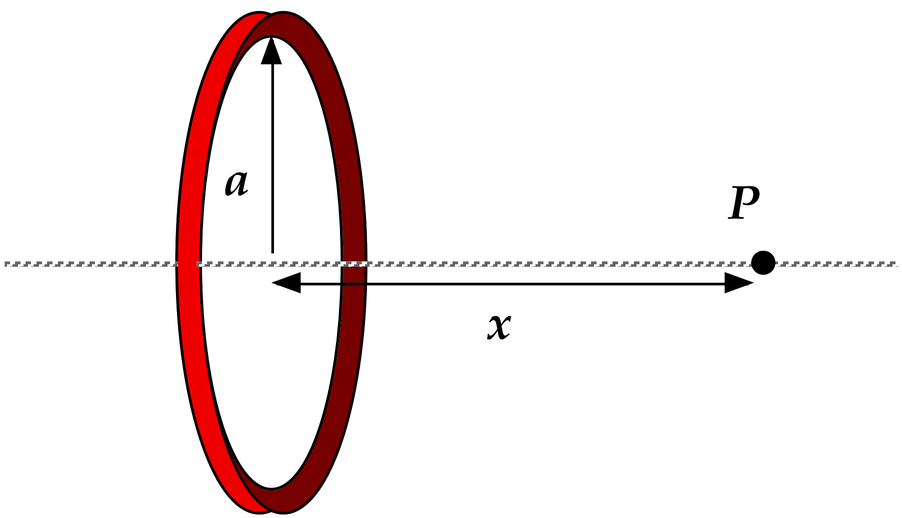
$$V = k\int \frac{dq}{r}$$

$$V = \frac{kQ}{\sqrt{x^2 + a^2}}$$



Example 9 (continued)

Find the electric field E at this same position, using *V*.



$$V = -\int E \bullet dr$$

$$dV = -E \bullet dr$$

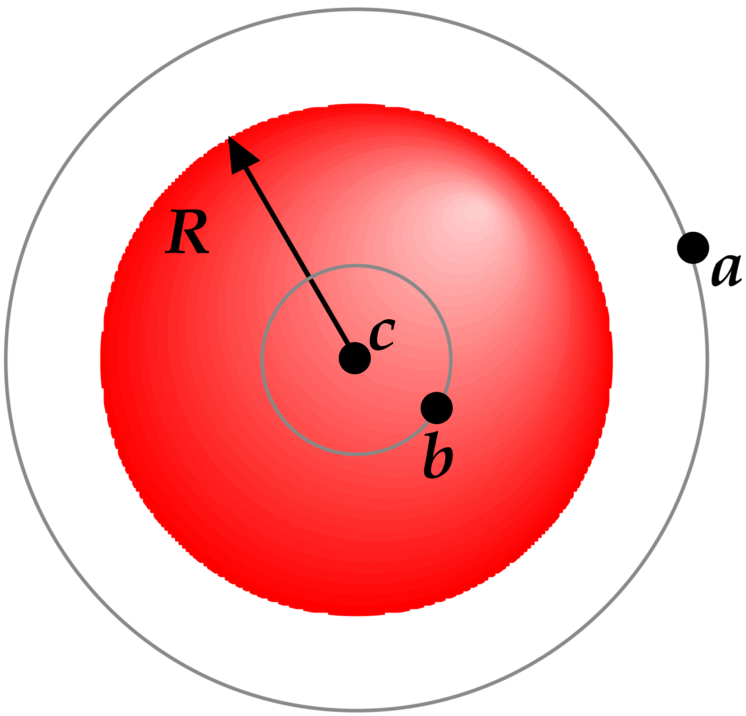
$$E_x = -\frac{dV}{dx}$$

$$E = -\frac{d}{dx}\left(k\frac{Q}{\sqrt{x^2 + a^2}}\right)$$

$$E = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

Example 10

An insulating solid sphere of radius *R* has a uniform positive charge density w/ total charge *Q*.



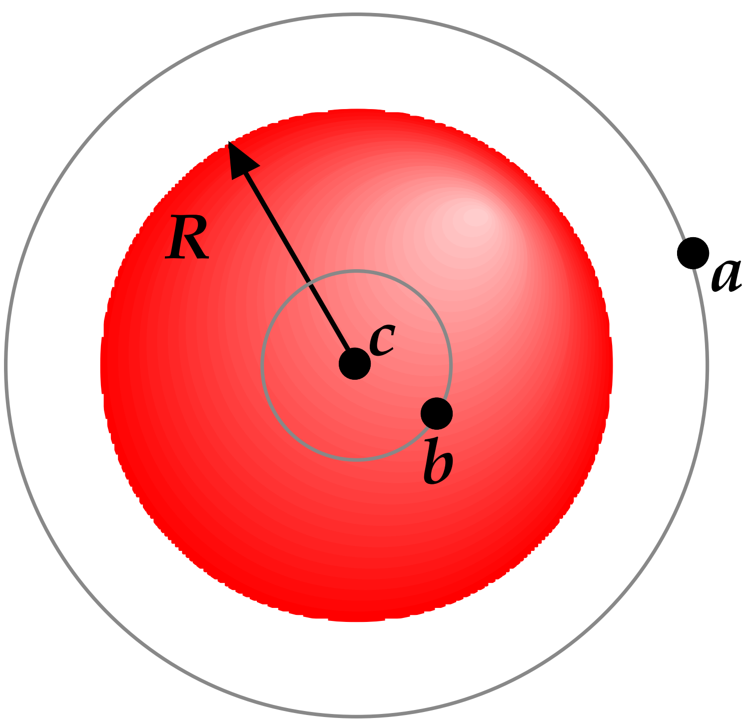
a. Find the electric potential at a point outside the sphere, $r > R$. (Take potential to be 0 at $r =$ [illegible].)

b. Find the potential at a point inside the charged sphere, $r < R$.

c. What are the electric field and electric potential at the center of the uniformly charged sphere?

Example 10

An insulating solid sphere of radius *R* has a uniform positive charge density w/ total charge *Q*.



a. Find the electric potential at a point outside the sphere, *r* > *R*. (Take potential to be 0 at *r* = ∞.)

$$E = k\frac{q}{r^2} \text{ for } r > R$$

$$\Delta V = -\int_A^B E \bullet ds =$$

$$V_B - V_A = -\int_\infty^r k\frac{q}{r^2} \bullet dr$$

$$V = k\frac{Q}{r} \text{ (for } r > R)$$

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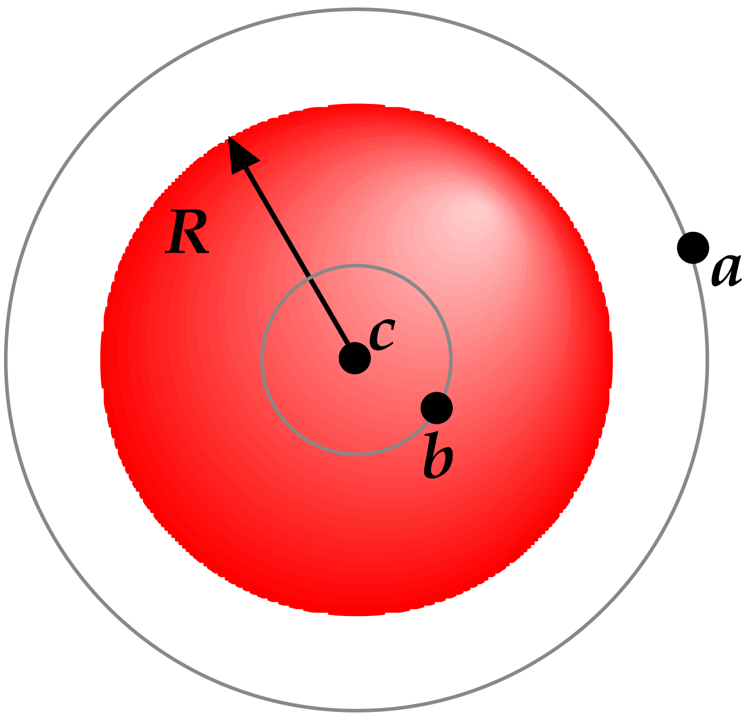


b. Find the potential at a point inside the charged sphere, $r < R$.

$$E = k\frac{Qr}{R^3} \text{ for } r < R$$

$$\Delta V = -\int_A^B E \bullet ds =$$

$$V_b - V_{surface} = -\int_R^{r_B} k\frac{Qr}{R^3} \bullet dr = -k\frac{Q}{R^3}\int_R^{r_B} r \bullet dr$$

$$V_b - V_{surface} = -k\frac{Q}{R^3}\left(\frac{1}{2}\left(r^2 - R^2\right)\right)$$

$$V_b = k\frac{Q}{2R^3}\left(R^2 - r^2\right) + k\frac{Q}{R}$$

$$V = k\frac{Q}{2R}\left(3 - \frac{r^2}{R^2}\right), \text{ for } r < R$$

Example 10

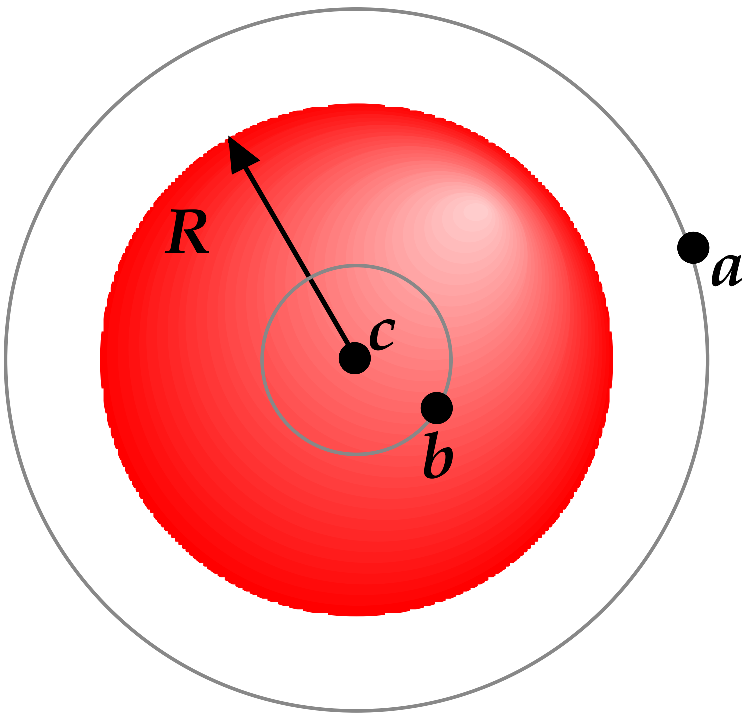
An insulating solid sphere of radius *R* has a uniform positive charge density w/ total charge *Q*.

c. What are the electric field and electric potential at the center of the uniformly charged sphere?

$E = 0$ at center (all fields cancel)

Use equation from (b) with $r = 0$ to get

$$V_o = \frac{3kQ}{2R}$$



E for Charged Conductors

What we already know about E fields and charged conductors:

a. In a conductor, charges reside on the surface.

b. Just *above* the surface of the conductor, the E field is perpendicular to the surface, and has a magnitude $E=\sigma/\epsilon_o$.

c. In the conductor, the E=0. (Otherwise, charges would be moving, and we wouldn't have a static situation).

V for Charged Conductors

a. V is constant everywhere along the surface of a conductor.

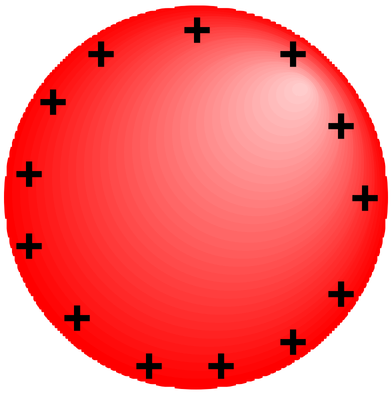
$$\Delta V = -\int \vec{\mathbf{E}} \bullet d\vec{\mathbf{s}}$$

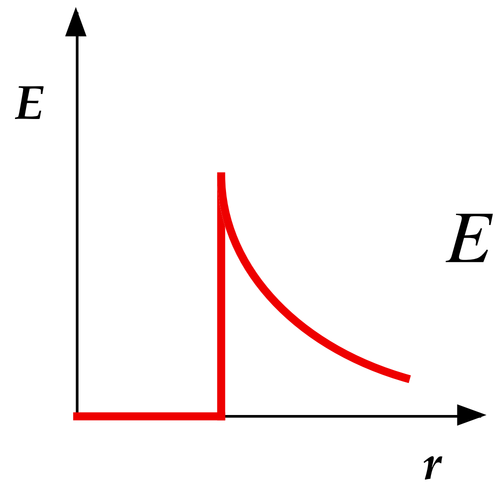
but $\mathrm{E} \perp \mathrm{ds}$, so

$$\Delta \mathrm{V} = 0$$

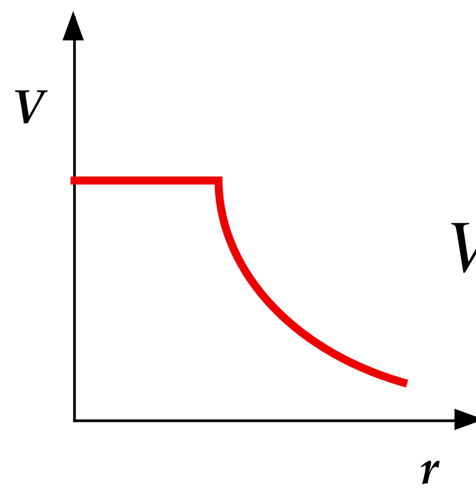
b. The surface of any charged conductor (in equilibrium) is an equipotential surface. Furthermore, since the E=0 inside the conductor, we conclude that the potential is constant everywhere inside the conductor, and equal to its value at the surface.

E & V, Charged Conductors





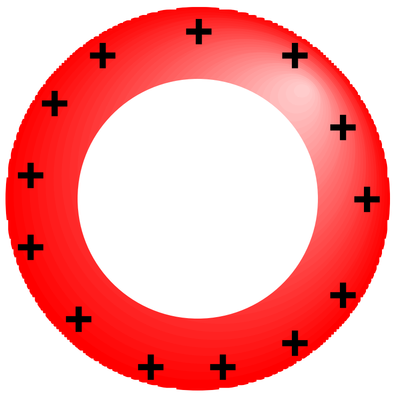
$$E = k\frac{Q}{r^2}$$



$$V = k\frac{Q}{r}$$

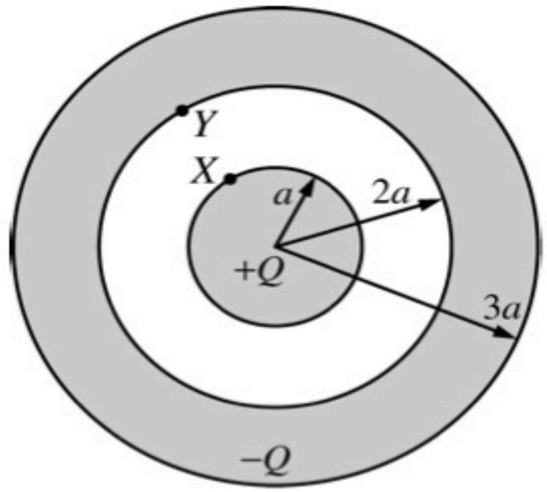
Charged Conducting Cavity

Inside a charged conducting shell, what' s going on? Any 2 points on the inside surface of the shell are at the same potential, so if $\Delta V=0$ across interior of shell , there can be no E.



Lesson: If you want to shield a circuit, or a lab, or *anything*, from electric fields, just enclose them in a conductor.

Review slides/your notes



E&M 2.

In the figure above, a nonconducting solid sphere of radius a with charge $+Q$ uniformly distributed throughout its volume is concentric with a nonconducting spherical shell of inner radius $2a$ and outer radius $3a$ that has a charge $-Q$ uniformly distributed throughout its volume. Express all answers in terms of the given quantities and fundamental constants.

(a) Using Gauss's law, derive expressions for the magnitude of the electric field as a function of radius r in the following regions.

i. Within the solid sphere ($r < a$)

ii. Between the solid sphere and the spherical shell ($a < r < 2a$)

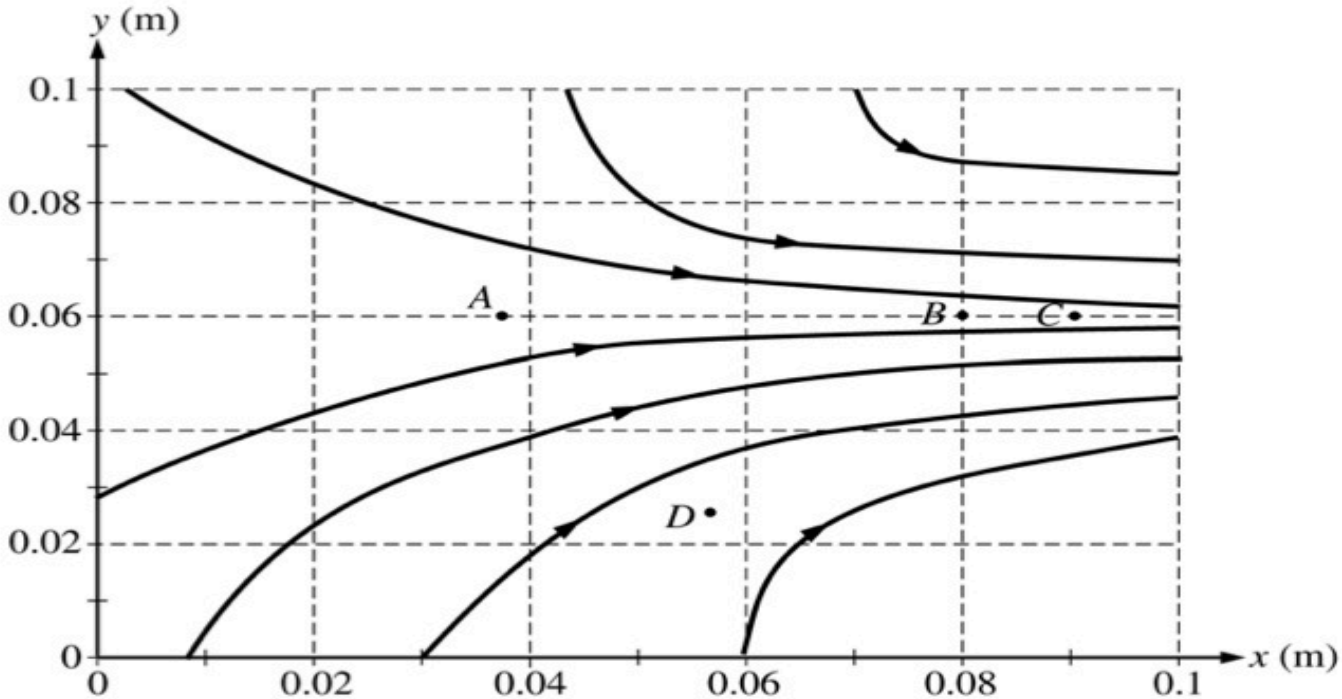
iii. Within the spherical shell ($2a < r < 3a$)

iv. Outside the spherical shell ($r > 3a$)

(b) What is the electric potential at the outer surface of the spherical shell ($r = 3a$)? Explain your reasoning.

(c) Derive an expression for the electric potential difference $V_X - V_Y$ between points X and Y shown in the figure.

Review slides/your notes



E&M. 1.

Consider the electric field diagram above.

(a) Points A, B, and C are all located at $y = 0.06$ m .

i. At which of these three points is the magnitude of the electric field the greatest? Justify your answer.

ii. At which of these three points is the electric potential the greatest? Justify your answer.

(b) An electron is released from rest at point B.

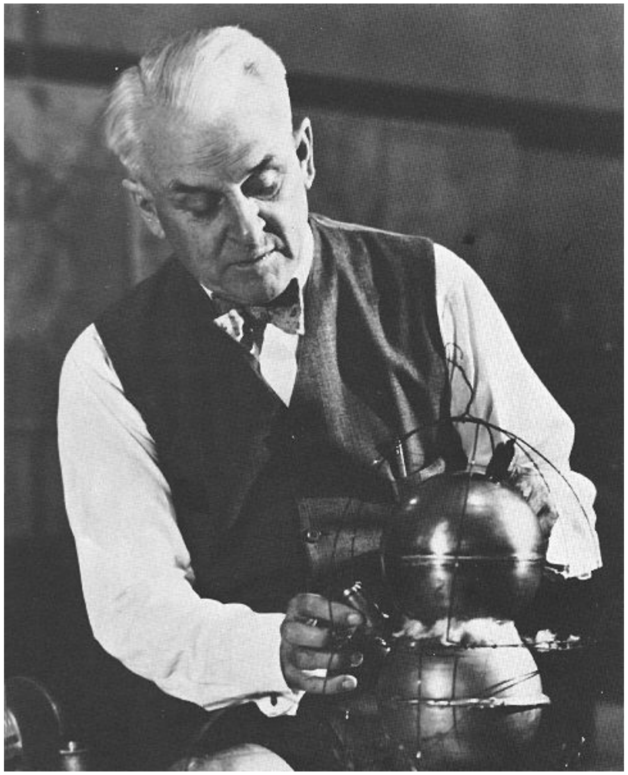
i. Qualitatively describe the electron's motion in terms of direction, speed, and acceleration.

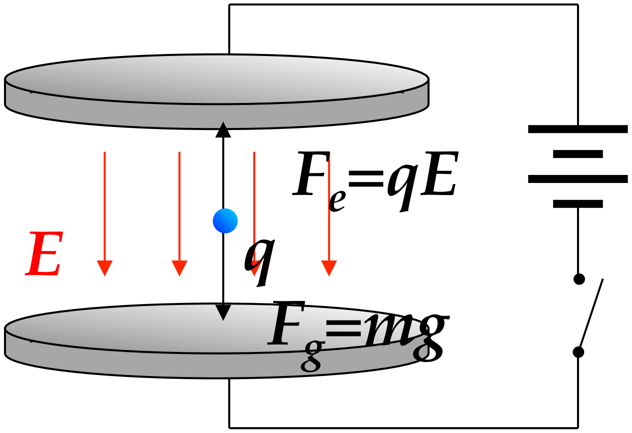
ii. Calculate the electron's speed after it has moved through a potential difference of 10 V.

(c) Points B and C are separated by a potential difference of 20 V. Estimate the magnitude of the electric field midway between them and state any assumptions that you make.

Millikan's Oil Drop Experiments

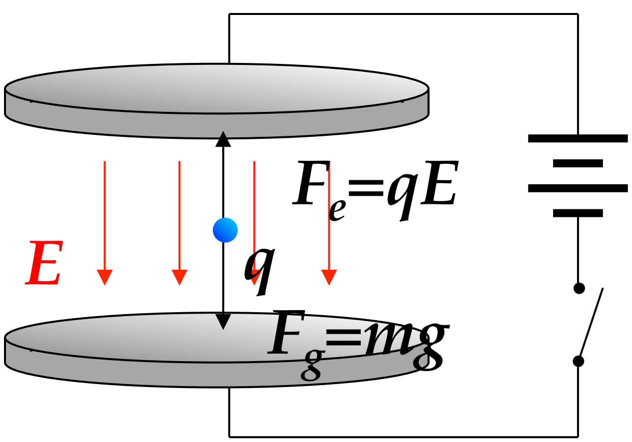
- Conducted from 1909-1913
- Determined the magnitude of electron charge
- Earned Millikan the Nobel Prize in 1923





Millikan's Oil Drop Experiments

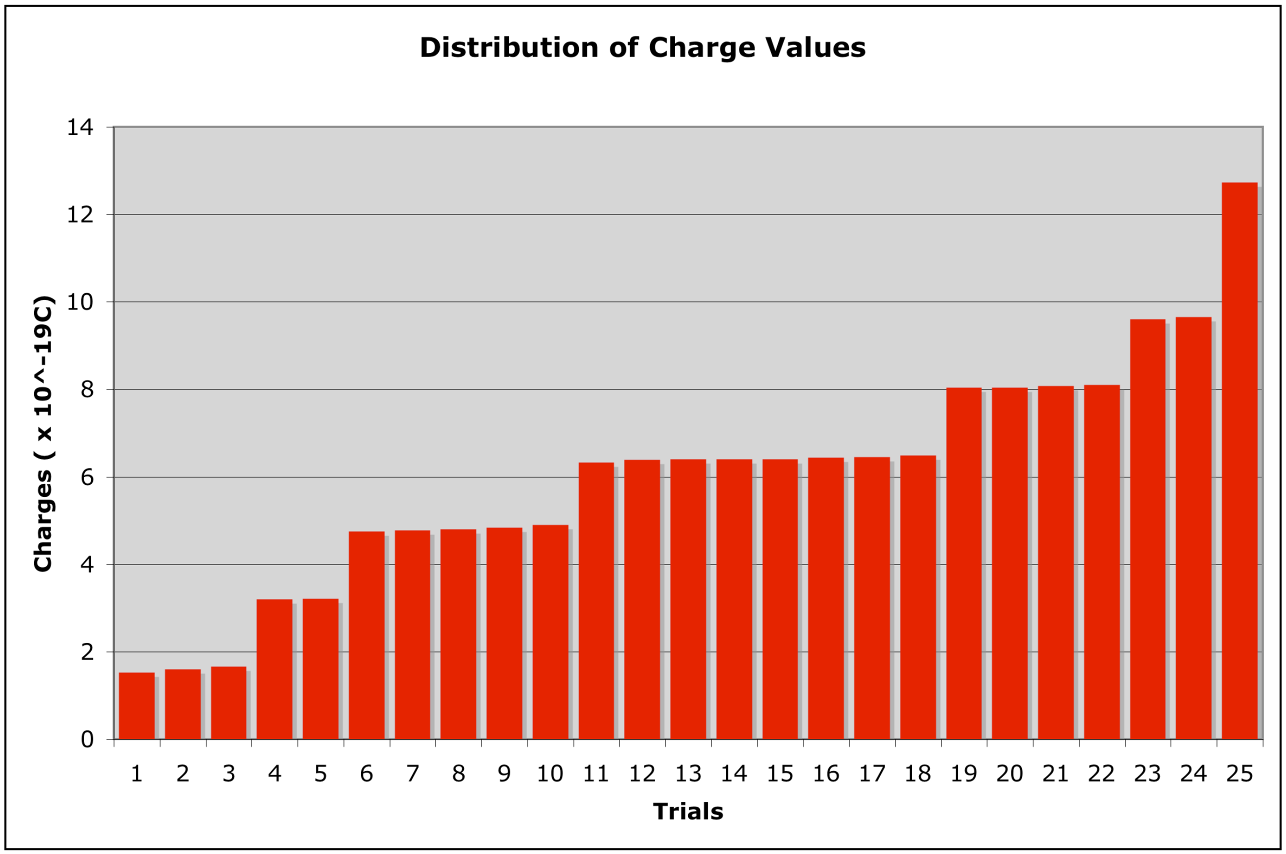
Assume charged sphere with mass m, charge q. It should be possible to set up a specific electric field E, based on a specific potential V, that will cause mass to levitate.



charge (x10^-19 C)
8.04204
4.90212
6.408
6.3279
1.602
12.7359
9.612
6.408
4.806
6.45606
6.408
3.204
1.5219
6.4881
8.04204
6.44004
4.83804
3.22002
6.39198
8.07408
9.66006
8.10612
4.75794
4.77396
1.66608

Millikan’s Oil Drop Experiments

By statistically grouping the results, it’s possible to determine the fundamental unit of charge.



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6.44004
4.83804
3.22002
6.39198
8.07408
9.66006
8.10612
4.75794
4.77396
1.66608

Debrief test

Average: 65

Range: 35-88

Absolute Electric Potential

For a point charge in space:

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \bullet d\vec{\mathbf{s}}$$

$$\vec{\mathbf{E}} = k\frac{q}{r^2}\hat{\mathbf{r}}$$

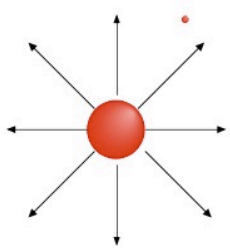
$$V_B - V_A = -\int_A^B k\frac{q}{r^2}\hat{\mathbf{r}} \bullet d\vec{\mathbf{s}}$$

$$V_f - V_i = -\int_{r\ initial}^{r\ final} k\frac{q}{r^2}\ dr$$

$$V_f - V_i = kq\left(\frac{1}{r_{final}} - \frac{1}{r_{initial}}\right)$$

If we set $V_i = 0$ at $r = \infty$,

$$V = k\frac{q}{r}$$



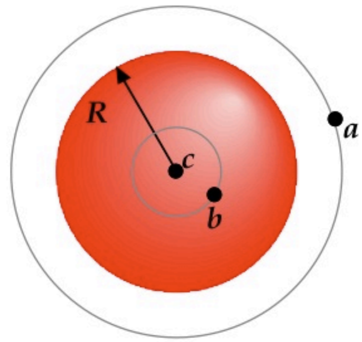
Example 10

An insulating solid sphere of radius *R* has a uniform positive charge density w/ total charge *Q*.

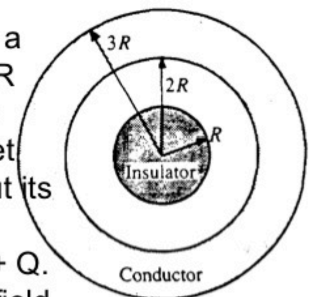
a. Find the electric potential at a point outside the sphere, *r* > *R*. (Take potential to be 0 at *r* = ∞.)

b. Find the potential at a point inside the charged sphere, *r* < *R*.

c. What are the electric field and electric potential at the center of the uniformly charged sphere?



E1. A sphere of radius R is surrounded by a concentric spherical shell of inner radius 2R and outer radius 3R, as shown above. The inner sphere is an insulator containing a net charge + Q distributed uniformly throughout its volume. The spherical shell is a conductor containing a net charge + q different from + Q. Use Gauss's law to determine the electric field for the following values of r, the distance from the center of the insulator.



a. 0 < r < R

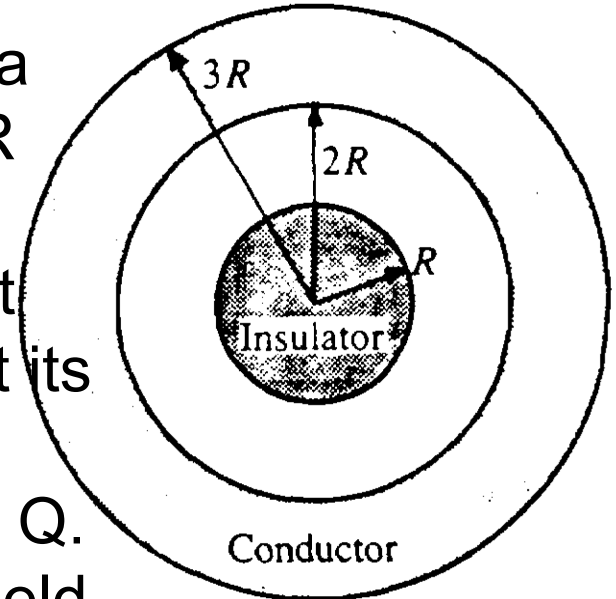
b. R < r < 2R

c. 2R < r < 3R

Determine the surface charge density (charge per unit area) on

d. the inside surface of the conducting shell;

e. the outside surface of the conducting shell.



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a. $0 < r < R$

b. $R < r < 2R$

c. $2R < r < 3R$

Determine the surface charge density (charge per unit area) on

d. the inside surface of the conducting shell;

e. the outside surface of the conducting shell.