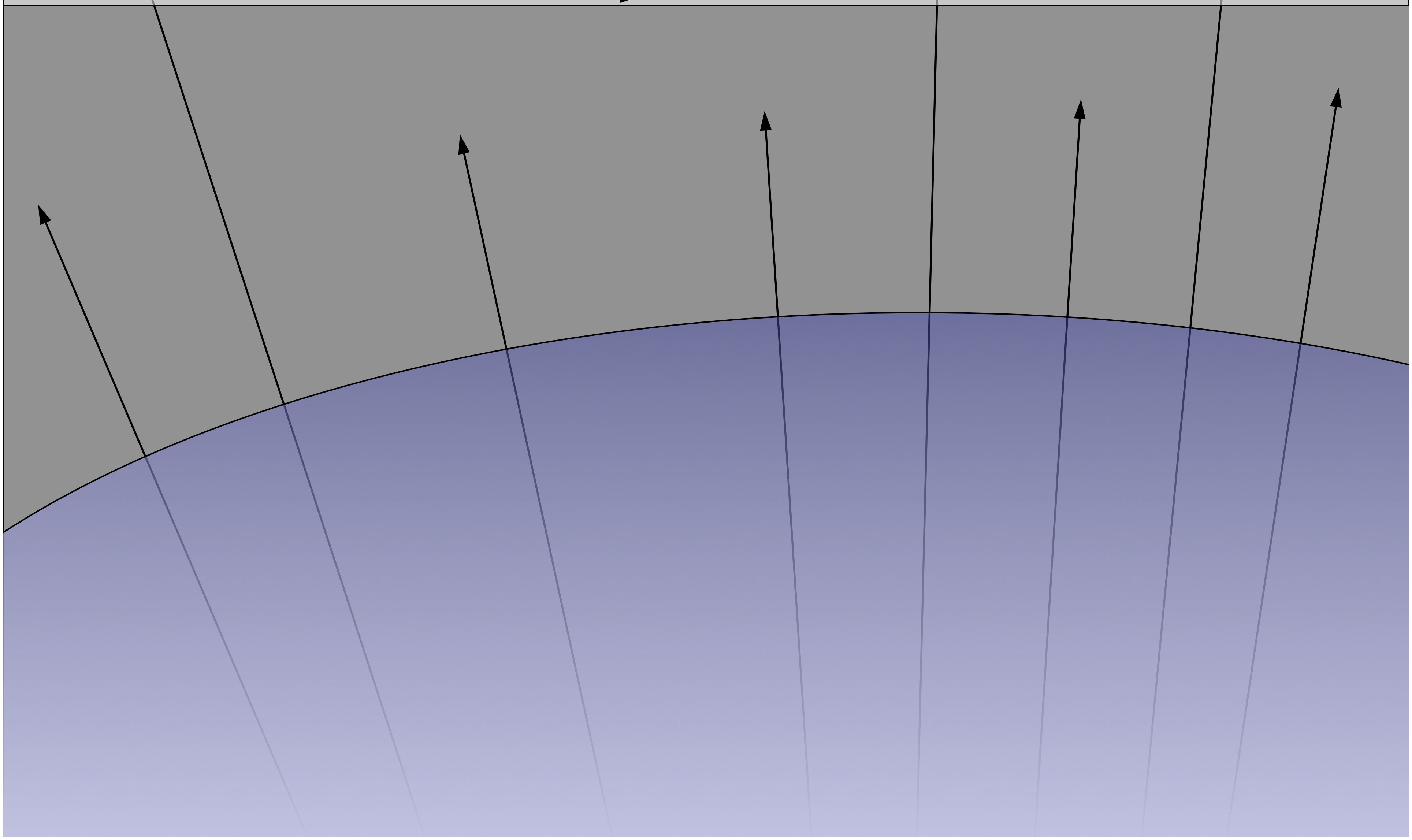
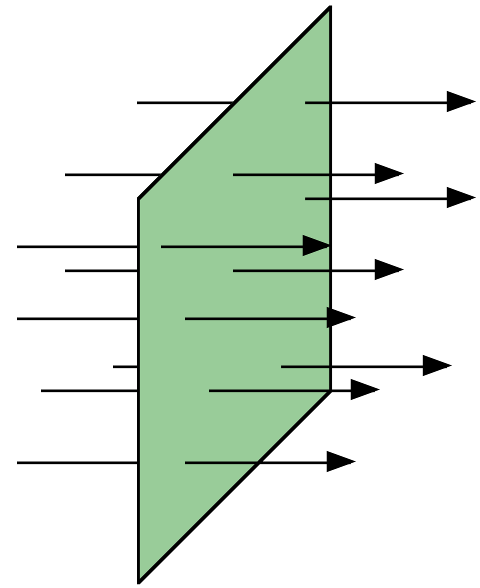
Ch 24
Electric Flux, & Gauss’s Law



Electric Flux

...is related to the number of field lines penetrating a given surface area.

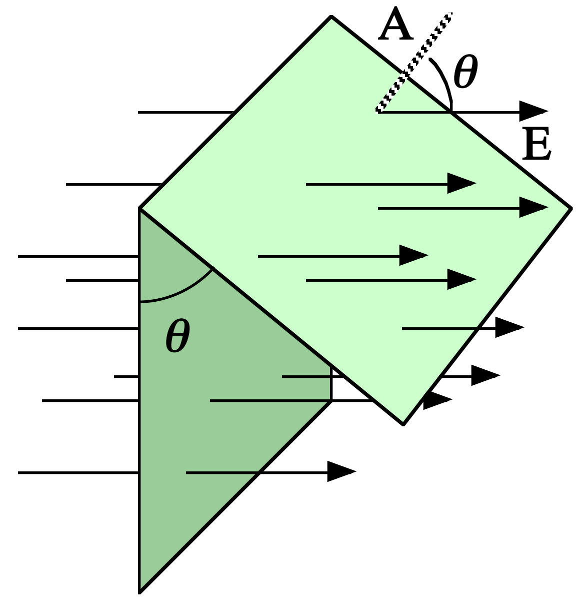


$$\Phi_e = \vec{\mathbf{E}} \bullet \vec{\mathbf{A}}$$

F = “phi” = electric flux

F units are N•m^2/C

Electric Flux



$$\Phi = \vec{\mathbf{E}} \bullet \vec{\mathbf{A}}$$

$$\Phi = EA\cos\theta$$

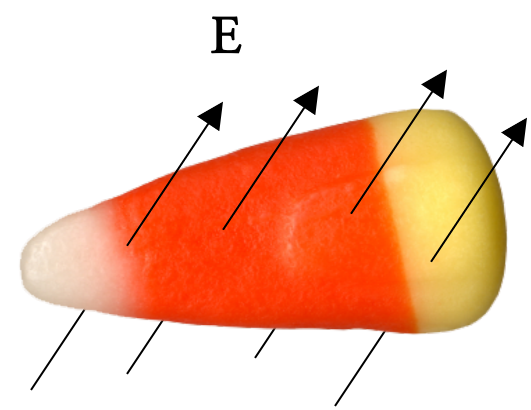
$$\Phi_i = E_i A_i \cos\theta = \vec{\mathbf{E}}_i \bullet \vec{\mathbf{A}}_i$$

$$\Phi = \int_{surface} \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}}$$

Flux for Closed Surfaces

The vector *dA*,by convention points perpendicular to the surface, and outward from the body. For *dA* where θ < 90°, flux is positive. For *dA* where θ > 90°, flux is negative.

Thus, the net flux through a closed surface is proportional to the net number of lines leaving the surface (="lines leaving" - "lines entering").

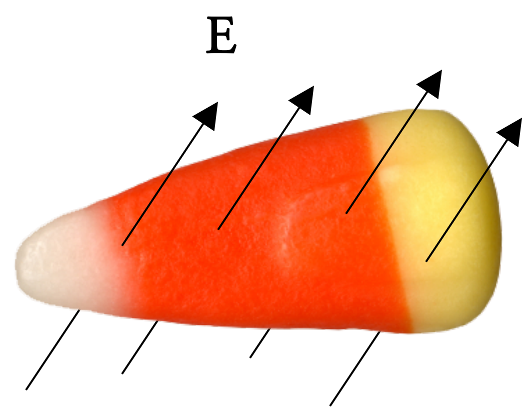


$$\Phi_i = E_i A_i \cos\theta = \vec{\mathbf{E}}_i \bullet \vec{\mathbf{A}}_i$$

$$\Phi = \int_{surface} \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}}$$

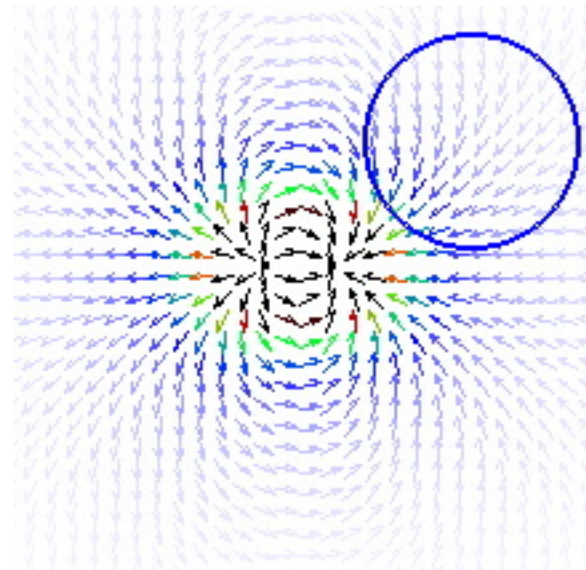
The Surface Integral

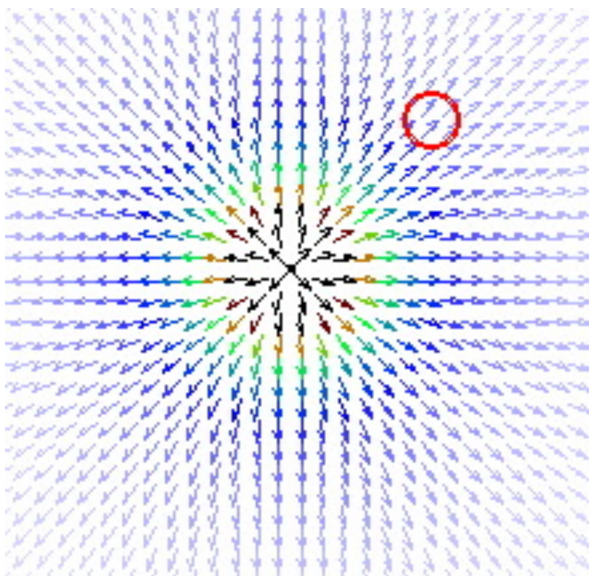
$$\text{Net flux } \Phi_c = \oint \vec{\mathbf{E}} \bullet \mathbf{d}\vec{\mathbf{A}}$$

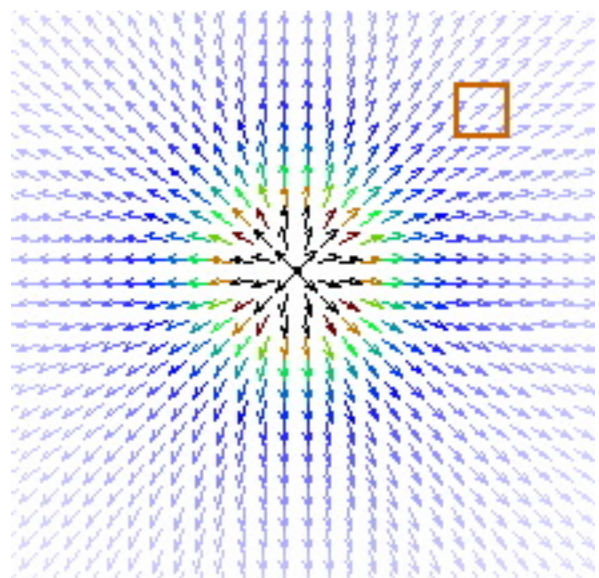


Examples 1

What is the net electric flux through each of the surfaces shown?







"Just in Time"

$$k = \frac{1}{4\pi\varepsilon_o}$$

$$\boxed{F_e = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}}$$

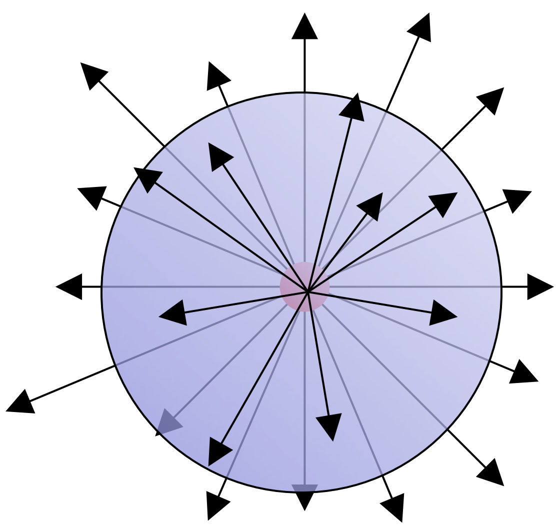
ε_0 refers to the "permittivity of free space." Permittivity refers to the ability of a material--in this case, free space—to transmit (or "permit") an electric field.

You don't really need to know this, but you should be able to recognize the variable and use it to simplify equations when convenient.

What is the value (and units) of ε_0 ?

Example 2 - Internal charge

Find the electric flux passing through the surface of an imaginary sphere of radius *R*, if a charge of +*q* is located at its center.



Because A and E are parallel:

$$\Phi_c = \oint E \bullet dA$$

$$\Phi_c = \oint E \; dA$$

$$\Phi_c = E \oint dA$$

$$\Phi_c = k\frac{q}{r^2} \oint dA$$

$$\Phi_c = k\frac{q}{r^2}\left(4\pi r^2\right)$$

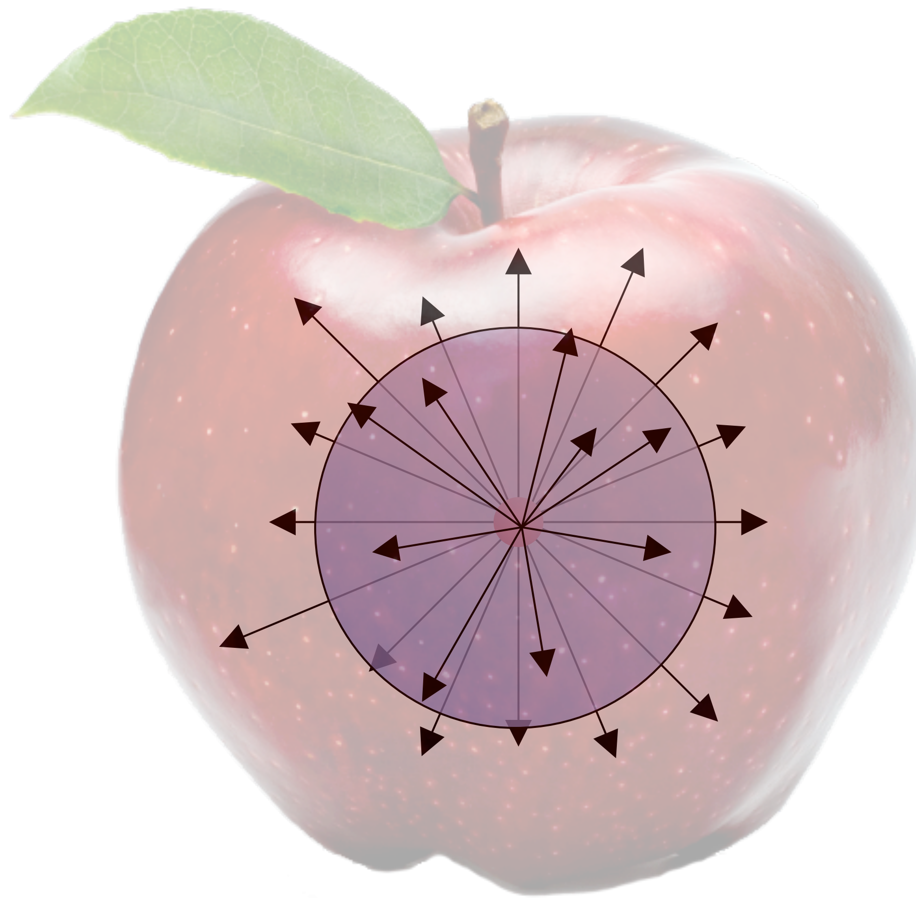
$$\Phi_c = kq4\pi$$

$$\Phi_c = \frac{q}{\varepsilon_o}$$

Example 3

What is the electric flux passing through the surface of the apple shown here?

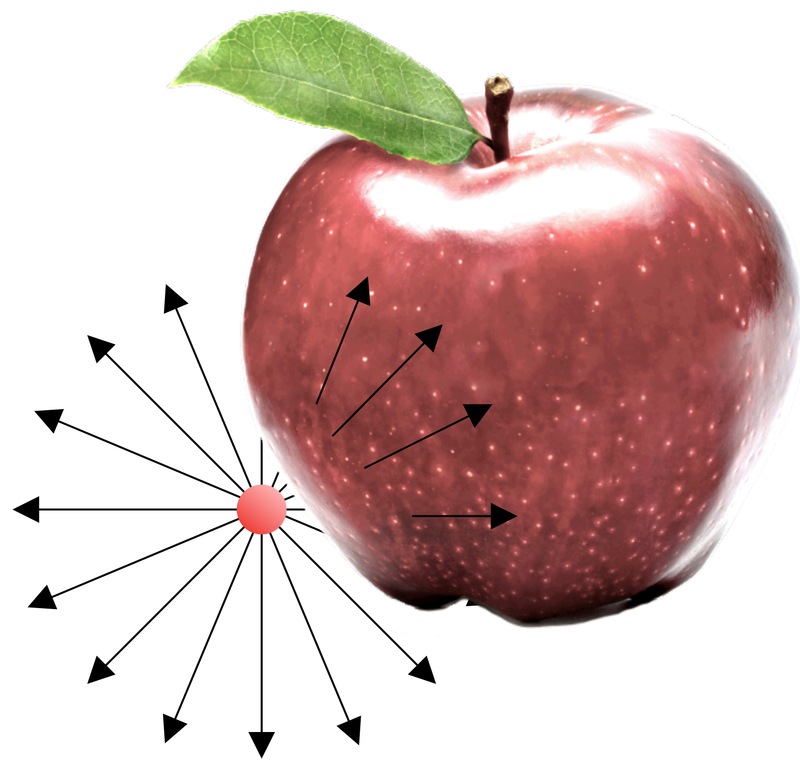
$$\Phi_c = \frac{q}{\varepsilon_o}$$



Example 4

Find the electric flux passing through the surface of an apple if a charge of $+q$ is located outside the surface.

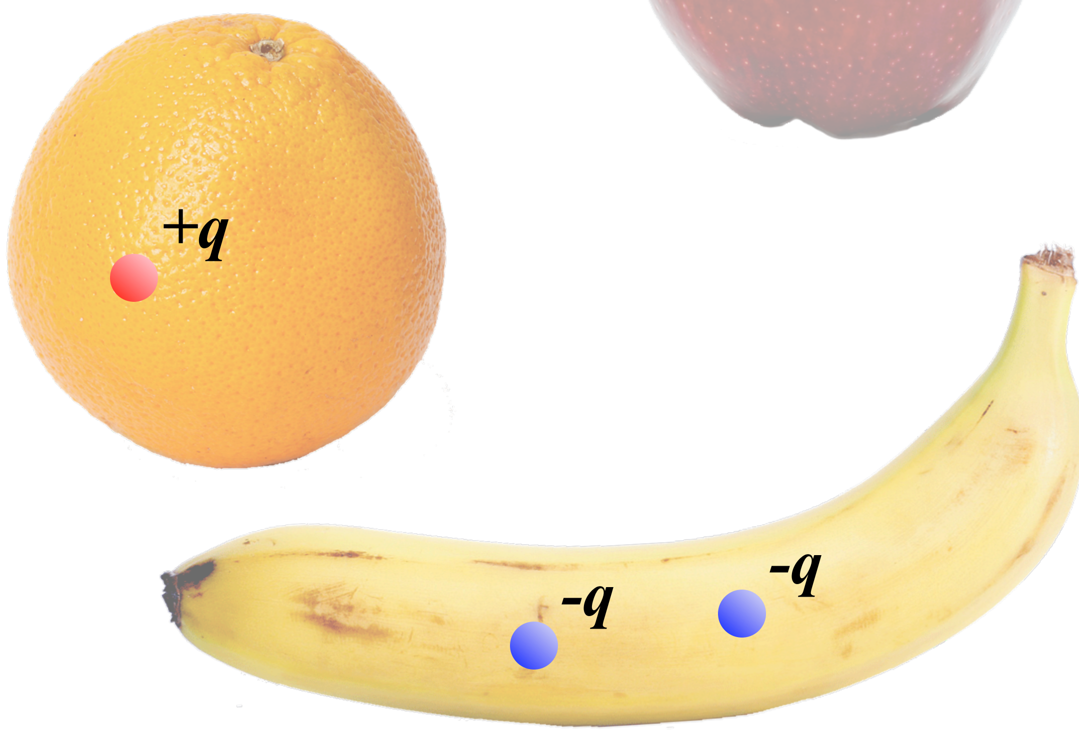
$$\Phi_c = \frac{q}{\varepsilon_o}$$



Electric Flux ≠ Electric Field!

$$\Phi_c = \oint \vec{\mathbf{E}}_{net} \bullet d\vec{\mathbf{A}}$$

$$\Phi_c = \oint (\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3) \bullet d\vec{\mathbf{A}}$$



Electric flux is determined for surfaces, open or closed. Electric field at any point in space is based on distribution of charges.

Gauss's Law

"The net electric flux through any closed surface is equal to the net charge inside the surface divided by ε_0."

$$\Phi_c = \oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_o}$$

Net charge *inside* the surface

Electric field at any point *on* the surface

Example 5

A spherical Gaussian surface surrounds a charge. Describe what happens to the electric flux if...

a. the charge is tripled

   The flux is tripled

b. the volume of the sphere is doubled

   Flux remains the same

c. the surface is changed to a cube

   Flux remains the same

d. the charge is moved to a different location inside the surface

   Flux remains the same

e. the charge is moved outside the surface

   Flux drops to zero

Example 6

The net flux through a Gaussian surface is 0. Which of the following statements *must* be true?

a. There are no charges inside the surface

Might be true, but could be a dipole.

b. The net charge inside the surface is 0.

This *must* be true.

c. The electric field everywhere on the surface is 0.

No, there may be field from external charge, but no net flux

d. The number of electric field lines entering the surface equals the number leaving the surface.

This *must* be true.

Example 7 - E Field of a Point Charge

Use Gauss's Law to calculate the electric field at a distance *r* from a point charge *q*. Then derive Coulomb's Law from your result.

Strategy: Choose a Gaussian sphere.

$$\Phi_c = \oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_o}$$

$$\vec{\mathbf{E}} \oint d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_o}$$

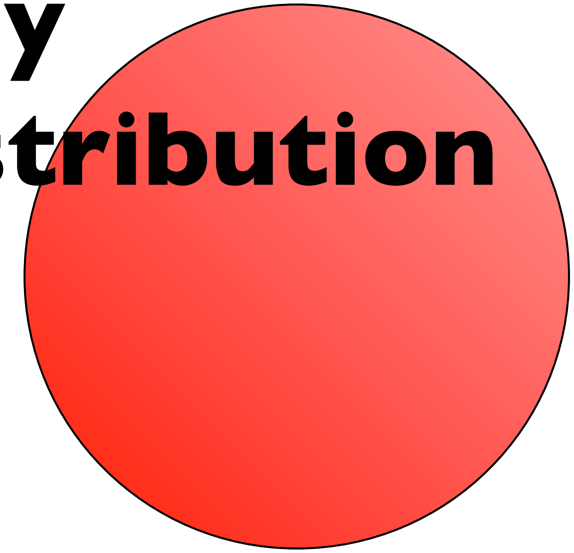
$$\vec{\mathbf{E}}(4\pi r^2) = \frac{q_{in}}{\varepsilon_o} = \frac{q_{in}}{1/4\pi k} = q_{in} 4\pi k$$

$$\vec{\mathbf{E}} = \frac{kq}{r^2}$$

$$F = Eq_o, \ so$$

$$F = \frac{kqq_o}{r^2}$$

Example 8 - Spherically Symmetric Charge Distribution



An insulating sphere of radius a has a uniform charge density *r* and a total charge of +Q.

a. Calculate the magnitude of the electric field at a point *outside* the sphere.

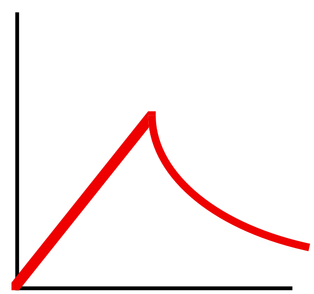
b. Find the magnitude of the field at a point *inside* the sphere.

c. Sketch the graph of *E* as a function of *r*.

a. Strategy: choose Gaussian sphere of radius r>a. $E=kQ/r^2$.

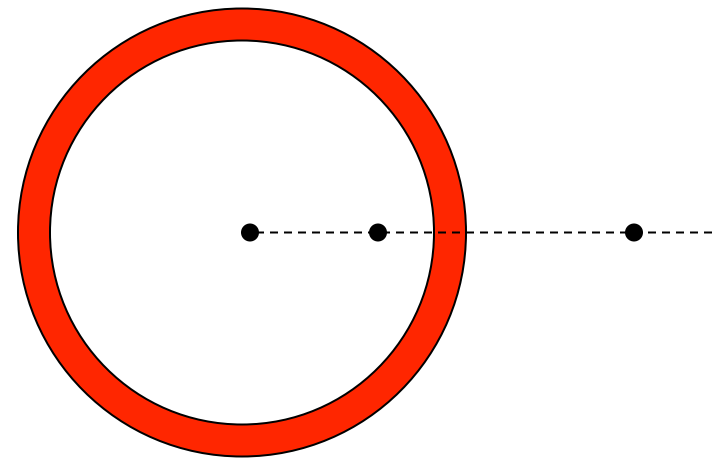
b. Choose Gaussian sphere of radius r<a. Let q_{in}= V $E=kQr/a^3$.

c.



Example 9 - E of Thin Spherical Shell

A thin spherical shell of radius *a* has a total charge of *Q* distributed uniformly over its surface. Find the electric field at points inside and outside the shell.

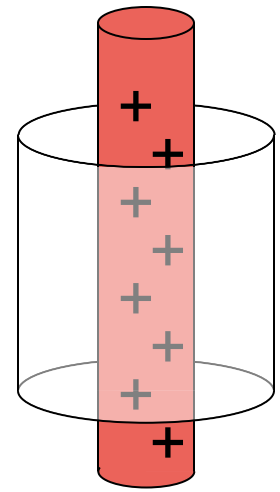


Choose large Gaussian sphere to get same result as for point charge: $E=kq/r^2$ for $r>a$.

Inside the shell, there is no internal charge. Gauss's law shows that E is 0 for $r<a$.

Example 10 - E of Linear Charge Distribution

Find the electric field a distance *r* from a uniform positive line charge of infinite length whose charge per unit length λ is constant.



Strategy: Choose a Gaussian cylinder coaxial with the wire. *E* is perpendicular to the curved surface everywhere, and 0 at the flat ends.

$$Q = \lambda L$$

$$\Phi = \oint E \bullet dA = E \oint dA = \frac{q_{in}}{\varepsilon_o} = \frac{\lambda L}{\varepsilon_o}$$

$$E(2\pi r L) = \frac{\lambda L}{\varepsilon_o}$$

$$E = 2k\frac{\lambda}{r}$$

Example 11 - A Dipole

What type of Gaussian surface would one use to analyze an electric dipole?

You *can't.* The dipole doesn't have sufficient symmetry for us to be able to analyze it with a Gaussian surface.

Example 12 - Nonconducting Plane sheet of charge

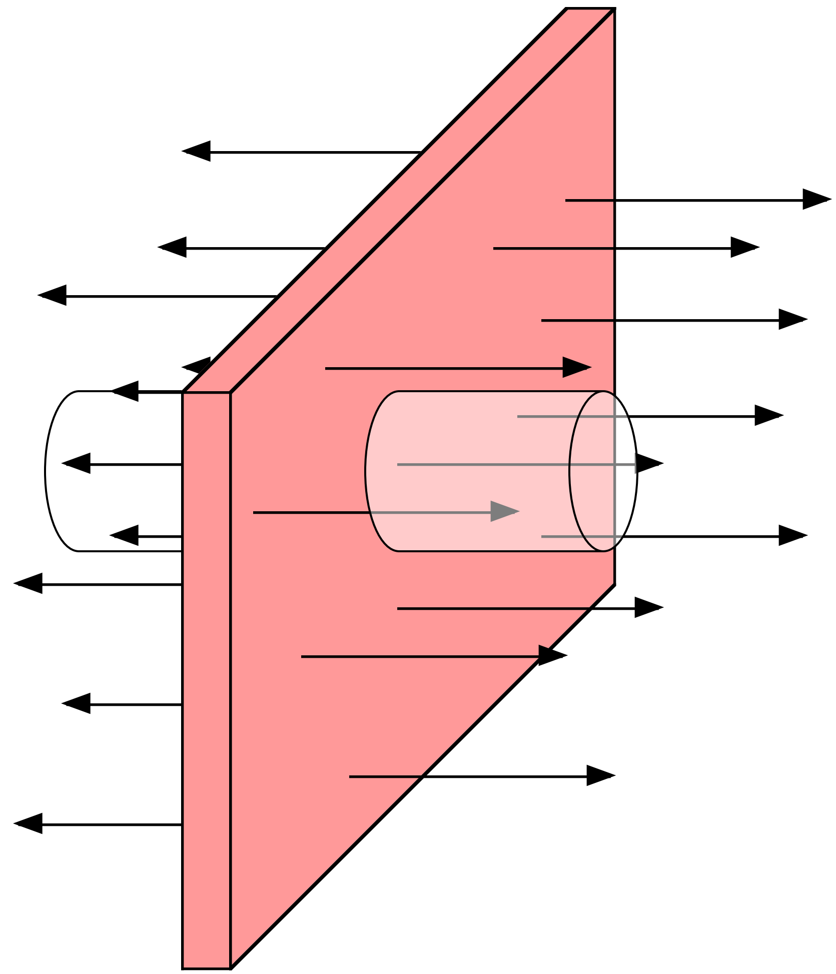
Find the electric field due to a non-conducting infinite plane with uniform charge per unit area .

$$\oint E \bullet dA = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint dA = \frac{\sigma A}{\varepsilon_o}$$

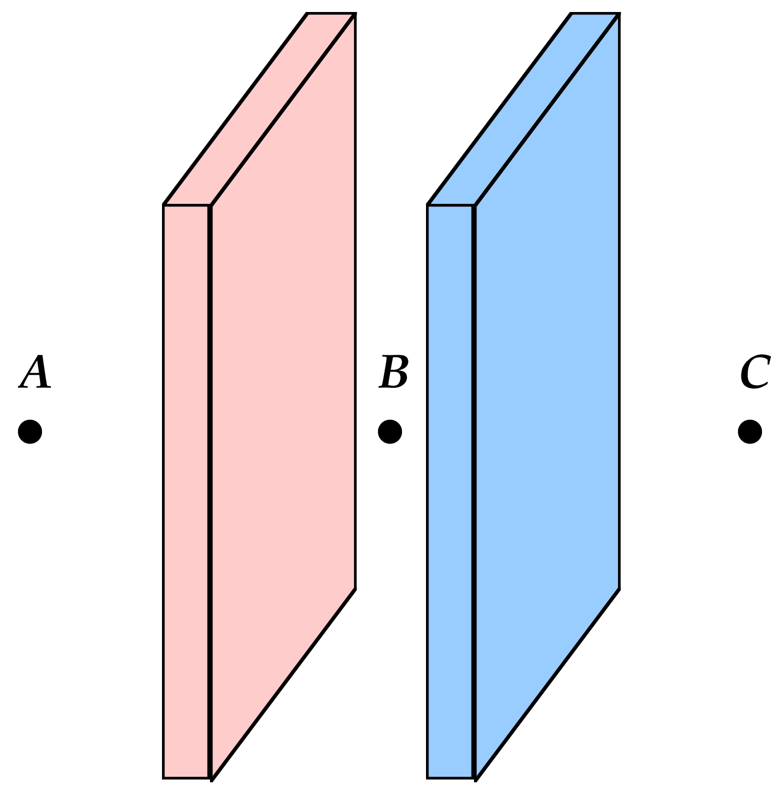
$$E2A = \frac{\sigma A}{\varepsilon_o}$$

$$E = \frac{\sigma}{2\varepsilon_o}$$



Example 13 - E near a pair of charged sheets

Find the electric field at points A, B, and C for these infinite sheets of charge.



Example 14 - E near a charged disk

What type of Gaussian surface would you use to analyze the E field near a charged disk?

You *can't*. The disk doesn't have sufficient symmetry for us to be able to analyze it with a Gaussian surface.

Conductors

For now: charges are free to move in a conductor, but currently under electrostatic conditions.

Conductors

1. The electric field is 0 everywhere inside a conductor
2. Any charge on an isolated conductor resides on its surface
3. The electric field just outside a charged conductor is perpendicular to the surface, and has a magnitude E=[illegible]/ [illegible]$_0$.
4. On an irregularly-shaped conductor, charge tends to accumulate at locations where the radius of curvature of the surface is the smallest, i.e. at sharp points

1. If $E_{external}$ exists, $E_{internal}$ opposes it.
2. E = 0 inside, so must not be any $q_{internal}$.
3. Magnitude E=[illegible]/ [illegible]$_0$ is derived from Gauss's Law. If E field wasn't perp., horz component would cause F.
4. Discussed later

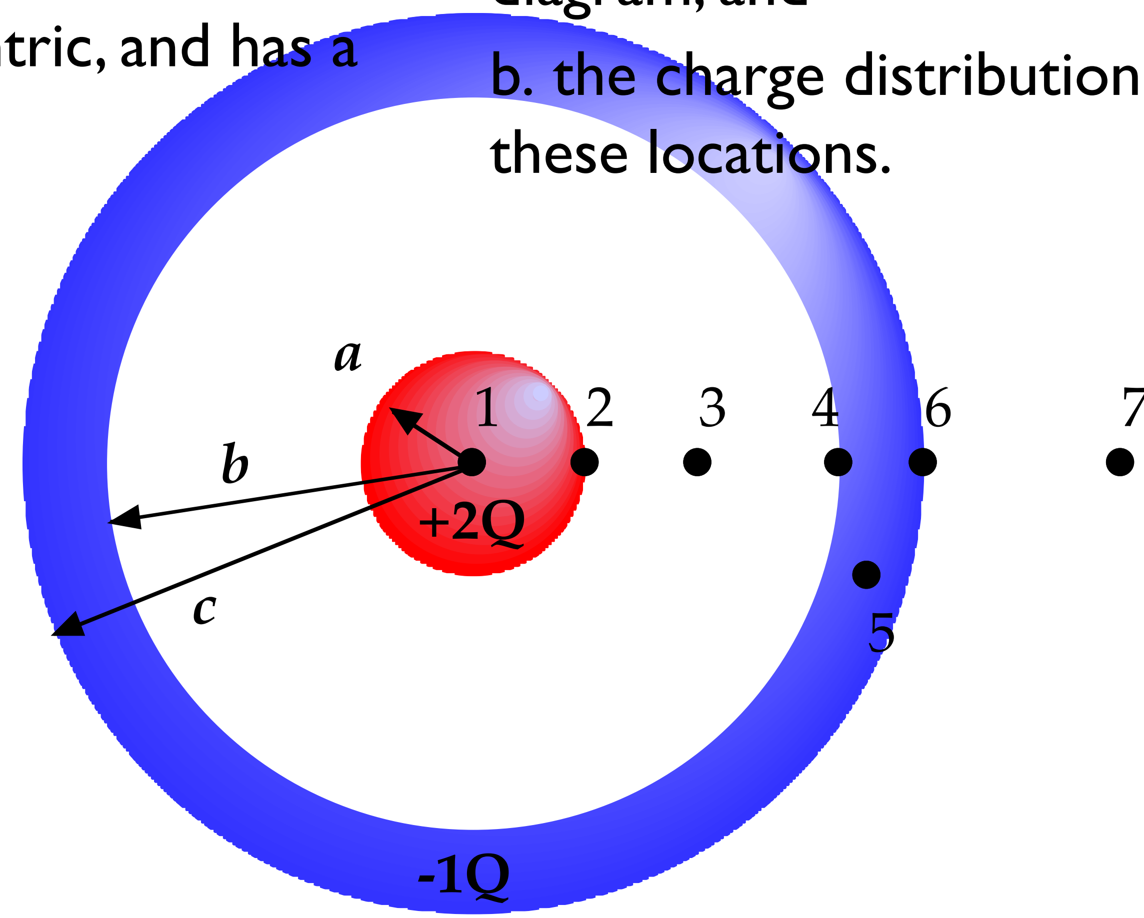
Example 15 - E for conductors

A solid conducting sphere of radius *a* has a net positive charge +2Q. A conducting spherical shell of inner radius *b* and outer radius *c* is concentric, and has a net charge -Q.

Use Gauss' s Law to find

a. the electric field at the 7 regions (1-7) shown in the diagram, and

b. the charge distributions at these locations.



Example 15 - Answers!

1.E=0, the electric field inside a conductor is 0. No net charge in body of a conductor.

2.On surface is a charge of+2Q, but E=0 (still in conductor). Just above surface, no charge, and E=s/e_0

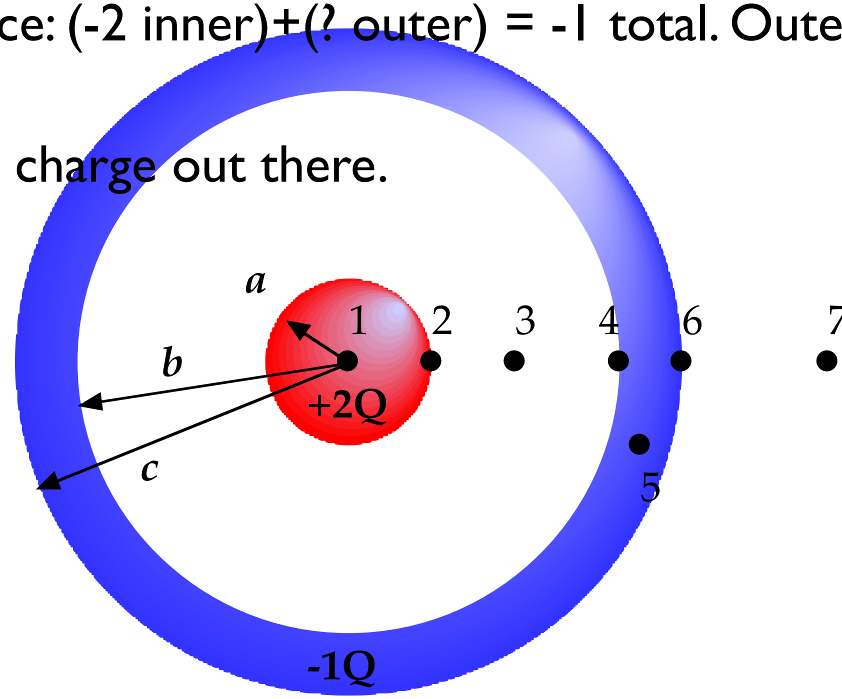
3.No charge located in this space, but use Gauss' s Law to get E=k(2Q)/r^2.

4.At inner surface, charge is -2Q, but E=0 on the inside of a conducting shell.

5.E=0, and there' s no net charge there.

6.E=0 on the surface, and q at surface: (-2 inner)+(? outer) = -1 total. Outer charge is +1Q.

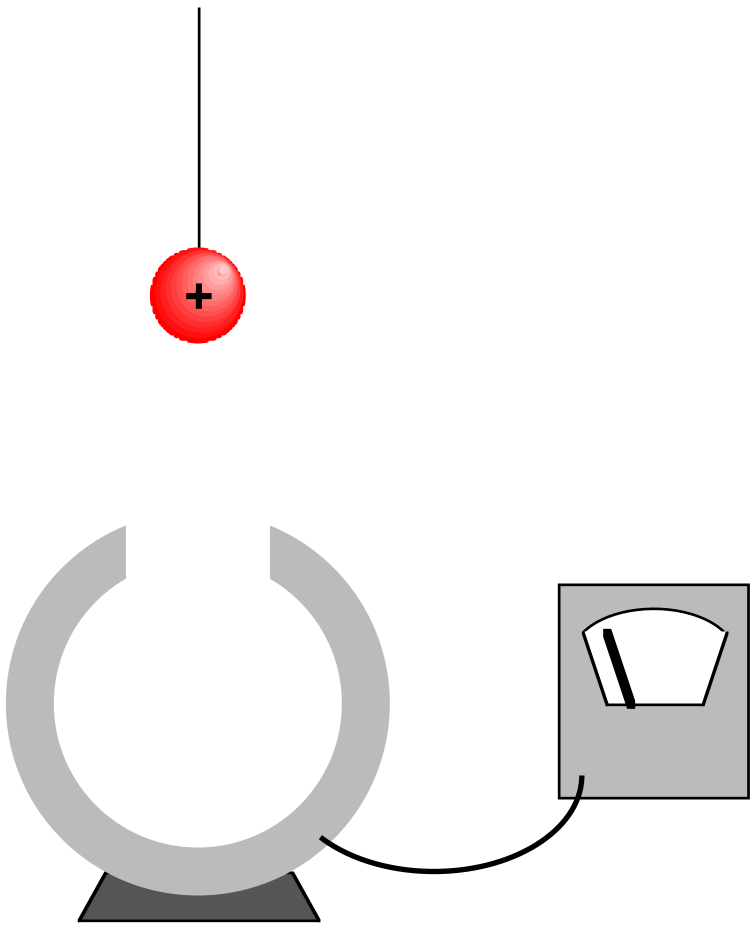
7.Gaussian sphere, get E=kQ/r^2. No charge out there.



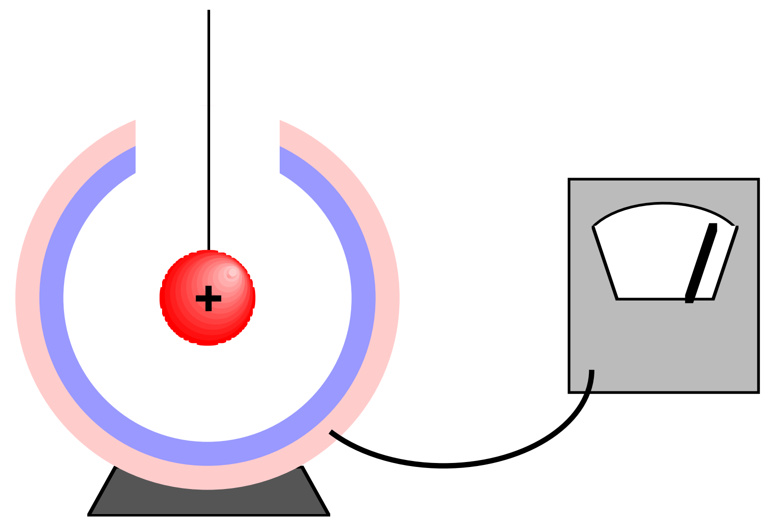
Example 16

Physlet E.24.3

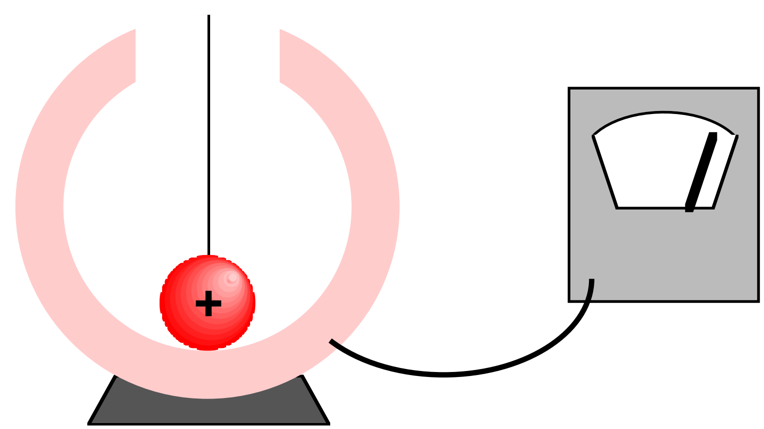
Example 17 - Faraday's Ice Pail Experiment



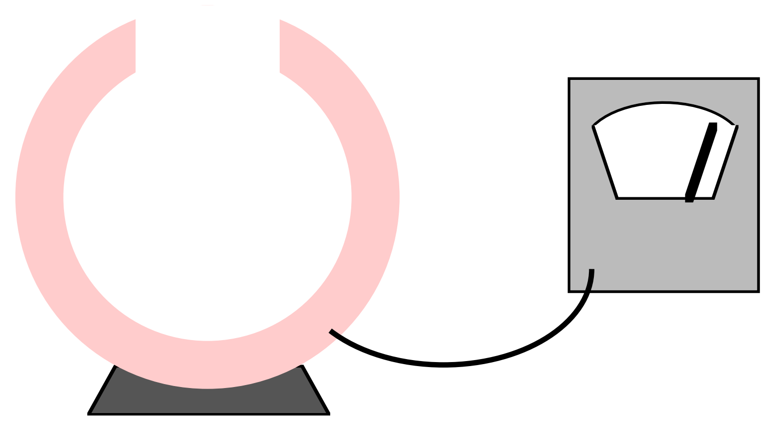
Example 17 - Faraday's Ice Pail Experiment



Example 17 - Faraday's Ice Pail Experiment



Example 17 - Faraday's Ice Pail Experiment



Example 17 - Faraday's Ice Pail Experiment