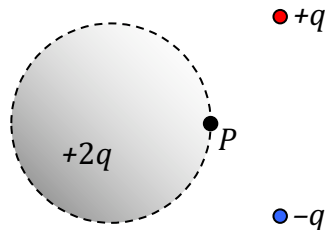


This test covers Coulomb's Law, electric fields, Gauss's Law, electric potential energy, and electric potential, with some problems requiring a knowledge of basic calculus.

Part I. Multiple Choice

- Charges of $+2q$, $+q$, and $-q$ are distributed in an area as shown above. Consider a Gaussian surface located around the $+2q$ charge, with a point P located on the surface as shown. Which of the statements below is true?
 - The electric field at P depends only on the $+2q$ charge.
 - The electric field is the same everywhere *on* the Gaussian surface.
 - The electric field is the same everywhere *inside* the Gaussian surface.
 - The net flux through the Gaussian surface depends only on the $+2q$ charge.
 - The net electric field at point P can be determined using the Gaussian surface shown.
- The electric field in a region of space is given by the function $E = -30x + 2$, where x is in meters and E is in Volts/meter. What is the electric potential at $x = 2$ meters, relative to the origin?
 - +56 V
 - +60 V
 - 30 V
 - 60 V
 - 56 V
- An electric field does 4 J of work on a charged particle, moving it from a potential of 1 V to a potential of 3 V. The particle has a charge of:
 - 8 C
 - +8 C
 - 2 C
 - +0.5 C
 - 0.5 C

4. Two large, parallel conducting plates are separated by a distance d . The two plates are given each given an equal magnitude of charge Q , but with opposite polarities, so a constant electric field E exists between the plates. A particle of mass m and charge $+q$ is released from rest at the surface of the positively-charged plate. Its velocity just before it reaches the surface of the negative plate is:

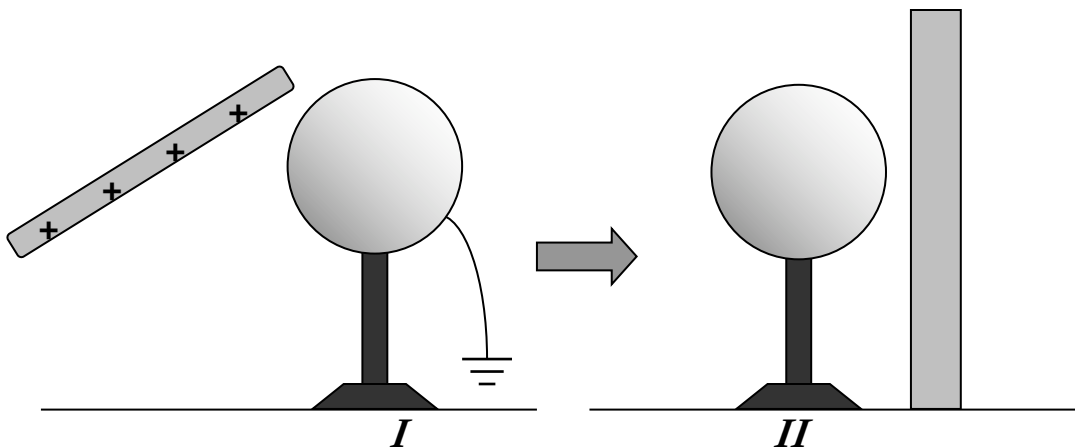
a. $\sqrt{\frac{2qEd}{m}}$

b. $\sqrt{\frac{qEd}{m}}$

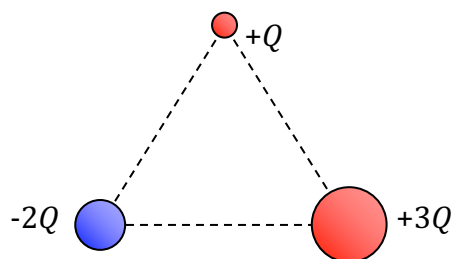
c. $\sqrt{\frac{2QEd}{m}}$

d. $\frac{2qEd}{m}$

e. $\frac{2QEd}{m}$

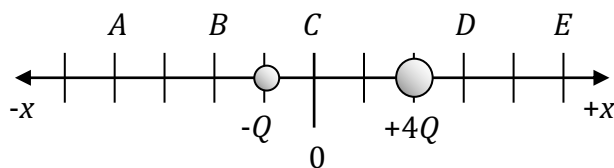


5. A rod-shaped positively-charged insulator is brought near a conducting sphere, and the sphere is momentarily grounded as shown in Diagram *I*. The conducting sphere is then placed near a nonconducting plane as shown in Diagram *II*. Which of the following statements is true for Diagram *II*?
- The conductor has a negative charge and is attracted to the nonconducting plane.
 - The conductor has a negative charge and is not attracted to the nonconducting plane.
 - The conductor has a positive charge and is attracted to the nonconducting plane.
 - The conductor has a positive charge and is not attracted to the nonconducting plane.
 - The conductor has no charge.



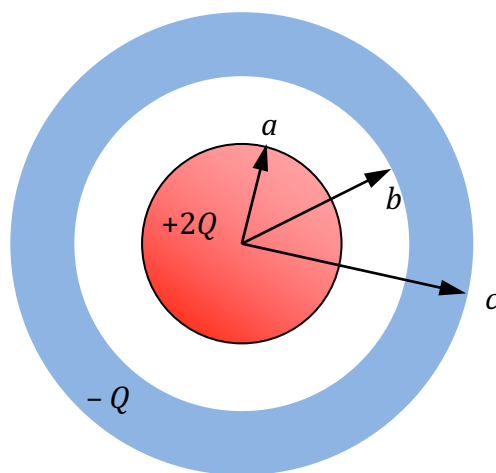
6. Three point charges, of charge $+Q$, $-2Q$, and $+3Q$, are placed equidistant as shown. Which vector best describes the net direction of the electric force acting on the $+Q$ charge?

- a.
- b.
- c.
- d.
- e.



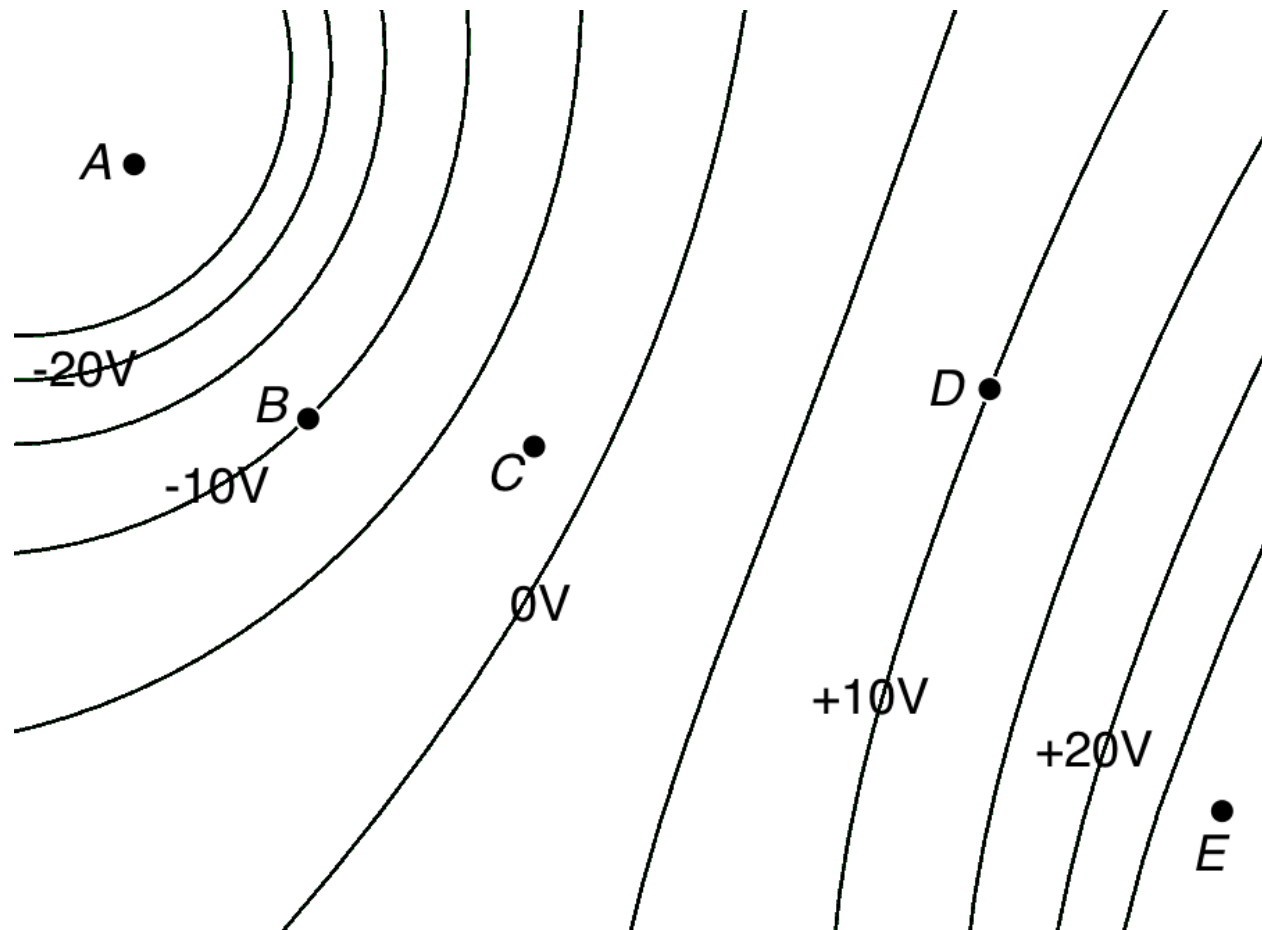
7. Two charges, of magnitude $-Q$ and $+4Q$, are located as indicated in the diagram above. At which position will the electric field due to these two charges be zero?
- a. A
b. B
c. C
d. D
e. E

Part II. Free Response



8. A solid, non-conducting sphere of radius a has a charge of $+2Q$ distributed uniformly throughout its volume. A conducting shell with an inner radius of b and an outer radius of c is located concentrically around the solid sphere, and has a net charge of $-Q$. Express all answers in terms of the given values and fundamental constants.
- a. Use Gauss's Law to determine the magnitude and direction of the electric field E at a point located r away from the center of the spheres, where $r > c$.
- b. Use Gauss's Law to determine the magnitude and direction of the electric field E at a point located r away from the center of the spheres, where $b > r > a$.

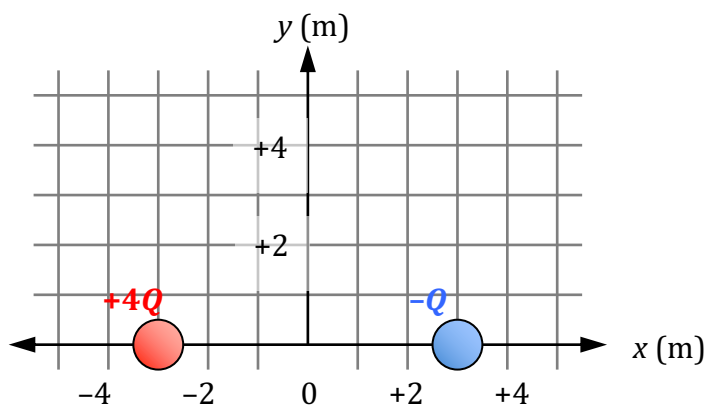
- c. Identify the total amount of charge induced on the inner surface of the conducting shell, at radius b .
- d. Identify the total amount of charge induced on the outer surface of the conducting shell, at radius c .
- e. Use Gauss's Law to determine the magnitude and direction of the electric field E as a function of r , where $r < a$.
- f. Sketch a graph of electric field E as a function of radius r , from $r = 0$ to $2c$, with radii a , b , and c clearly identified.



9. The electric potential diagram above shows equipotentials for a 2-dimensional region of space.
- At which point— A , B , C , D , or E —would an electron have the highest electric potential energy? Briefly explain your answer.

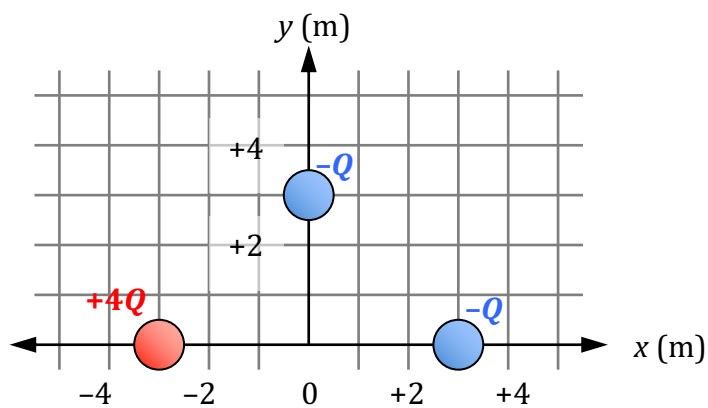
 - In which general direction does the electric field point in this diagram? Briefly explain your answer.

- c. Based on the equipotentials, draw a sketch on the diagram above of the electric field, including at least five field lines.
- d. At which point— A , B , C , D , or E —is the magnitude of the electric field the greatest? Briefly justify your answer.
- e. A proton is released from rest at point C . Qualitatively describe the proton's subsequent:
- direction of motion
 - speed
 - acceleration
- f. For the proton in part e , calculate its velocity after having moved through a potential difference of 10 V.

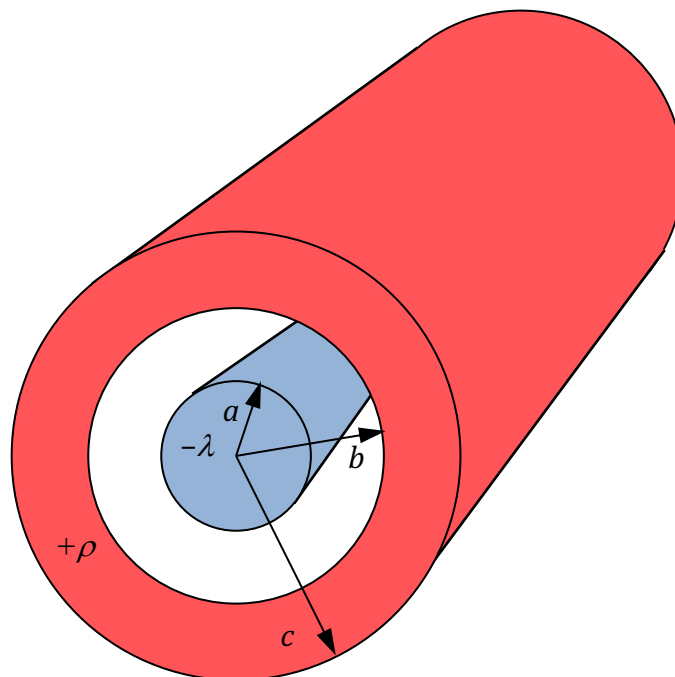


10. A charge of $+4Q$ is placed at $(-3, 0)$ m and a charge of $-Q$ is placed at $(+3, 0)$ m as shown in the diagram above.
- Determine the x coordinate of the location where the electric field due to the two charges is 0.
 - Determine the x coordinate of the location where the electric potential due to the two charges is 0.
 - If $Q = 1.00\mu\text{C}$, calculate how much Work must be done to bring an electron from infinitely far away to the origin.

- d. As the electron moves toward the origin, is the Work done by the field *positive*, *negative*, or *zero*?
- e. If $Q = 1.00\mu\text{C}$, calculate the acceleration—magnitude and direction—of the electron released from rest at the origin.



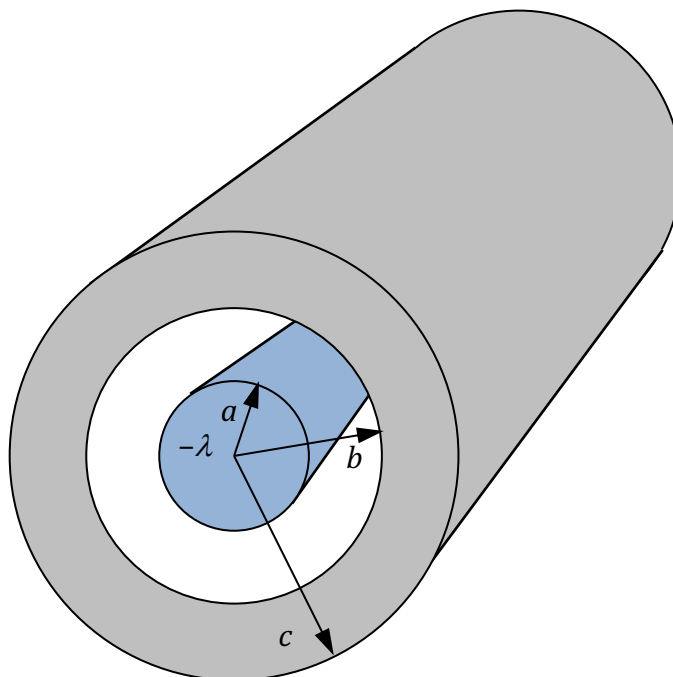
- f. Now another $-1.00\mu\text{C}$ charge is placed at $(0, 3)$ m as shown.
- Determine the electric potential at the origin with this new charge configuration.
 - Determine the electric field at the origin—magnitude and direction—with this new charge configuration.



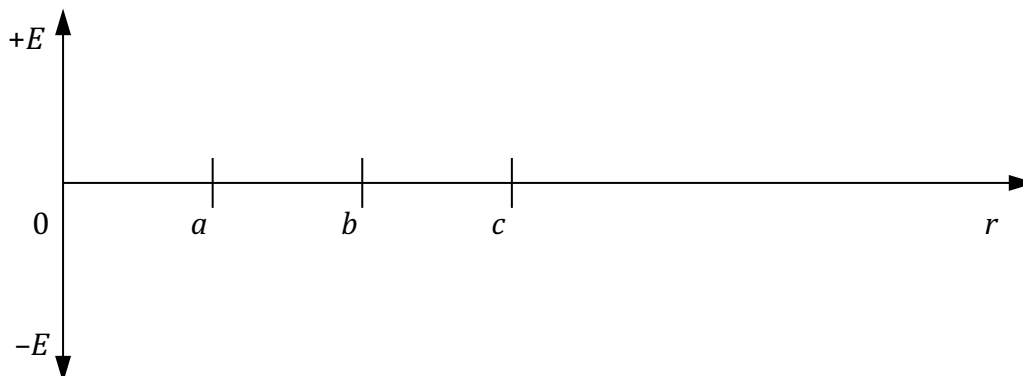
11. An infinitely long cylindrical conductor has a radius a and a linear charge density of $-\lambda$ as shown above. The conductor is surrounded by a cylindrical shell made of a nonconducting material of inner radius b , outer radius c , and with a constant volume charge density of $+\rho$. The conductor and nonconductor are located concentrically about a common axis.
- Determine the net electric flux per unit length passing through a cylindrically symmetric Gaussian surface located just outside the surface of the conductor.
 - Use Gauss's Law to determine the magnitude of the electric field E as a function of radius r , where:
 - $r < a$

ii. $a < r < b$

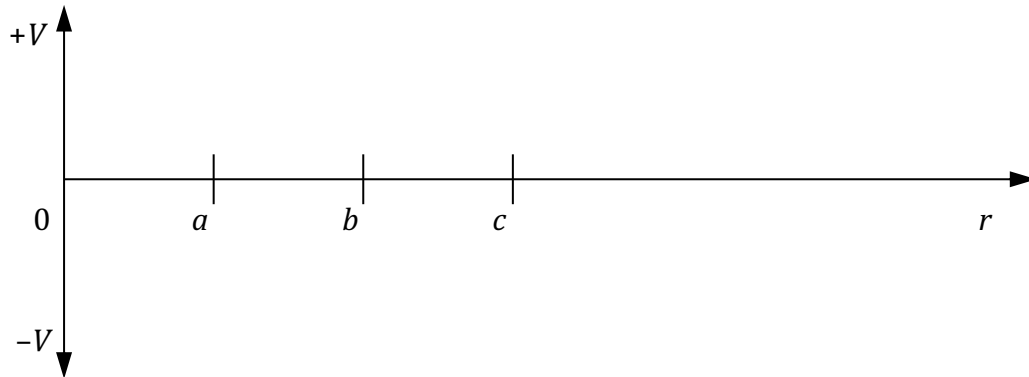
iii. $r > c$



- c. The outer non-conducting cylinder is now replaced with a conducting cylinder with no net charge, while the inner conducting cylinder still has a linear charge density of $-\lambda$.
- i. In the space below, sketch a graph of the electric field as a function of radius, where a field pointing away from the cylinder is positive.



ii. In the space below, sketch a graph of the electric potential as a function of radius.



1. The correct answer is *d*. The electric field varies with position in the vicinity of these three charges, and can be calculated by applying $E = \sum k \frac{q}{r^2}$. While there is a net flux through the Gaussian sphere shown due to the internal charge $+2q$, the other charges also contribute to the electric field at *P*, so an analysis using Gauss's Law doesn't tell us the net electric field there.
2. The correct answer is *a*. The relationship between electric field and electric potential is given by the integral $V = -\int E \cdot dr$, and determined for this problem as follows:

$$\Delta V = -\int_{x_i}^{x_f} E \cdot dx$$

$$\Delta V = -\int_0^{2m} -30x + 2 \cdot dx$$

$$\Delta V = -(-15x^2 + 2x)\Big|_0^{60} = (60 - 4) - (0 - 0) = +56V$$

3. The correct answer is *c*. The electric field does work on the particle, moving it from a position of high electric potential energy to low electric potential energy according to

$$\Delta V = \frac{\Delta U}{q}$$

$$q = \frac{\Delta U}{\Delta V} = \frac{U_f - U_i}{V_f - V_i} = \frac{-4J}{2V} = -2C$$

The negative sign on the charge can also be deduced by the fact that the field is doing Work on the particle in moving it from a lower potential to a higher potential. If it were a positive charge, the field would be doing Work in going from higher to lower potential.

4. The correct answer is *a*. The particle is accelerated by the electric field, and so its velocity can be determined either by using the Work done on the particle by the field, or by calculating the electric force's acceleration and solving using kinematics. Here, we'll use the energy analysis:

$$U_{initial} = K_{final}$$

$$U = qV, V = Ed, \text{ and } K = \frac{1}{2}mv^2$$

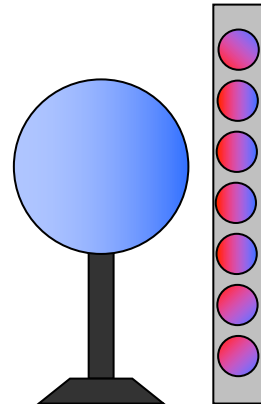
$$qEd = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2qEd}{m}}$$

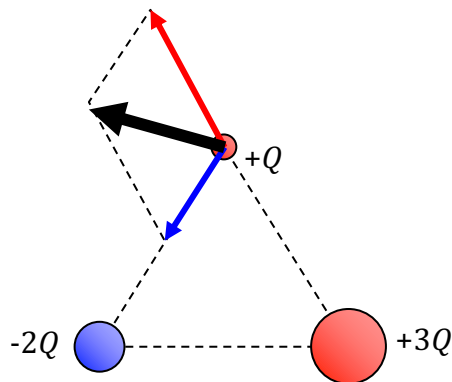
The kinematics-based analysis arrives at the same result by using $F_{net} = ma$, $F_e = qE$, and

$$v_f^2 = v_i^2 + 2ax.$$

5. The correct answer is *a*. When the conducting sphere is grounded in the presence of the positively-charged rod, electrons travel to the conductor, giving it a negative charge. When the negatively-charged conductor is brought near the plane, although the plane has no net charge, electrons will be repelled to one side of the atoms in the plane. This leaves the positive charges in those atoms relatively closer, which results in a net attractive force.



6. The correct answer is *b*. The direction of the electric force due to $-2Q$ and $+3Q$ can be visualized with a quick free-body sketch of the electric forces:



7. The correct answer is *a*. The electric field due to each charge is determined according to $E = k \frac{q}{r^2}$. With the positive charge having 4 times the magnitude of $-Q$, it's going to need to be located at a position such that

$$E_{-Q} = E_{+4Q}$$

$$k \frac{Q}{r_{-Q}^2} = k \frac{4Q}{r_{+4Q}^2}$$

$$\frac{1}{r_{-Q}^2} = \frac{4}{r_{+4Q}^2}$$

$$r_{+4Q}^2 = 4r_{-Q}^2$$

$$r_{+4Q} = 2r_{-Q}$$

Location \mathcal{A} is the position that satisfies this equation.

8.

- a. At the exterior of the spheres, create a concentric Gaussian surface and analyze it using Gauss's Law:

$$\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$E \oint dA = \frac{q_{in}}{\epsilon_0}$$

$$E(4\pi r^2) = (+2Q - Q)4\pi k$$

$$E = \frac{kQ}{r^2}$$

- b. Use the same approach as above, but now the Gaussian surface is smaller, with a radius r between a and b . Here, the charge internal to the Gaussian surface is $+2Q$, so our analysis yields

$$\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$E(4\pi r^2) = (+2Q)4\pi k$$

$$E = \frac{2kQ}{r^2}$$

- c. On the inner surface of the conducting shell, $-2Q$ worth of charge is attracted to the $+2Q$ charge on the inner sphere. This can be determined analytically by creating a Gaussian sphere with a radius between b and c . Because we know that the electric field E in this conductor is zero, we can conclude that the net internal charge for our Gaussian surface must be zero, and that can only be the case if there is $-2Q$ worth of charge located on the interior of the conducting shell.

- d. If there is a net charge of $-1Q$ on the conducting shell, and $-2Q$ at the interior, there must be $+1Q$ on the outer shell.

$$Q_{net} = Q_{inner} + Q_{outer}$$

$$-1 = -2 + Q_{outer}$$

$$Q_{outer} = +1$$

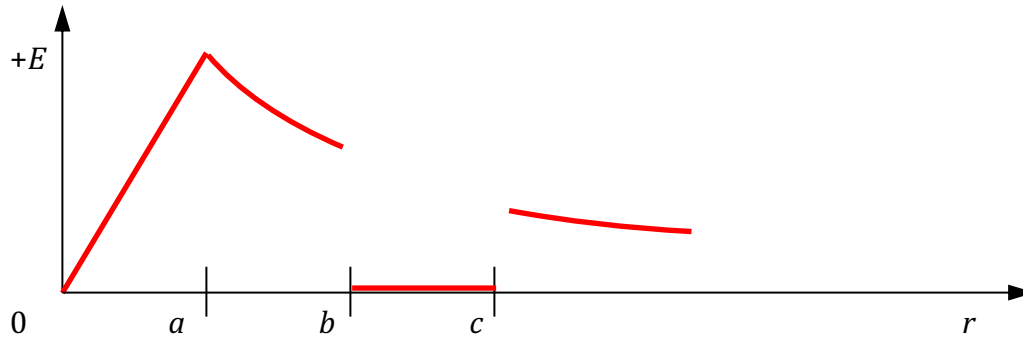
- e. We can use Gauss's Law again, but we'll need to take into account that the $q_{internal}$ is a fraction of what it was before.

$$\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$\rho = \frac{Q_{total}}{V_{total}} = \frac{q_{in}}{V_{in}} \Rightarrow q_{in} = \frac{V_{in}}{V_{total}} Q_{total} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} (2Q)$$

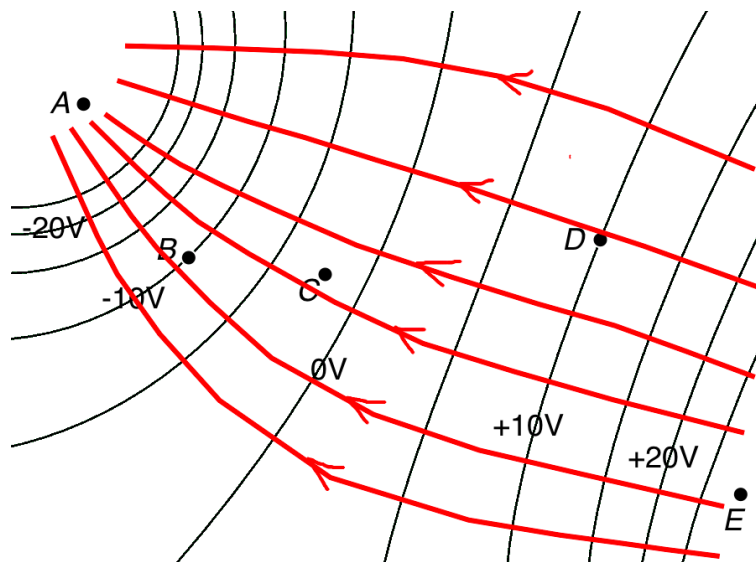
$$E(4\pi r^2) = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} (2Q)(4\pi k) \Rightarrow E = \frac{2kQ}{a^3} r$$

- f. Using the information from the previous parts of this problem:



9.

- At **A**. Electron has highest potential energy where electric potential is lowest, the opposite of what a positive test charge experiences.
- The electric field points from lower-right to upper-left. Electric field lines point in the direction of high-to-low electric potential.
- Field lines should be perpendicular to equipotentials.



- d. At **A**. Field lines become more closely spaced there.
- e.
- i. The proton accelerates in the same direction of field, ie, towards upper left of diagram.
 - ii. The proton has a speed that *increases* over time; the electric field applies a force to it, causing it to accelerate.
 - iii. The proton's acceleration *increases* as it moves into an area with an increasingly strong electric field.

- f. This is a conservation of energy problem, with the field doing Work on the charge to increase its kinetic energy. Here, we can look at the change in electric potential energy and see how that converts to kinetic energy.

$$\Delta U = \Delta K$$

$$q\Delta V = K_f - K_i = \frac{1}{2}mv^2 - 0$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2(1.60e-19C)(10V)}{(1.67e-27kg)}} = 4.38e4m/s$$

10.

- a. The $+4Q$ charge produces a field away from the $-3m$ position, and the $-Q$ charge produces a field towards its $+3m$ position. Because the $+4Q$ charge has a greater magnitude, we know that the position where electric field $E = 0$ is farther away from it. The $E = 0$ position will also have to be some place that is *not* between the two charges—in that region, both charges produce fields pointing to the right. So the $E = 0$ position must be to the right of $x = +3$.

Conceptually, if the positive charge has 4 times the negative charge, then E must be located $\sqrt{4}$ times away, based on the inverse-square relationship to distance. This can be counted out in the diagram to determine that $x = 9m$ is the location.

Analytically:

$$\sum E = \sum \frac{kq}{r^2} = 0$$

$$\frac{4kQ}{(x+3)^2} = \frac{kQ}{(x-3)^2}$$

$$4(x^2 - 6x + 9) = (x^2 + 6x + 9)$$

$$x = \{1, 9\}$$

Because $x = 9$ is in the correct region, we choose this as our answer.

- b. Electric potential is not a vector so we're not required to keep track of the directions. We do need to account for the positive and negative polarity on the charges, however.

$$\sum V = \sum \frac{kq}{r} = 0$$

$$\frac{k(+4Q)}{(x+3)} + \frac{k(-Q)}{(x-3)} = 0$$

$$4(x-3) = x+3$$

$$x = 5m$$

- c. The Work done in bringing an electron from infinitely far away is just the change in potential energy U.

$$Work = \Delta U = q \Delta V$$

$$Work = (1.602e-19C)(V_{origin} - V_{\infty})$$

$$V_{origin} = \sum \frac{kq}{r} = \frac{k(+4Q)}{3} + \frac{k(-Q)}{3} = kQ = (8.99e9)(1.00e-6) = 8.99e3J/C$$

$$Work = (1.602e-19C)(8.99e3J/C - 0) = 1.44e-15J$$

- d. Work done by the field is *positive*. The electron moves along E field lines spontaneously, with U decreasing, so E field is doing Work, which is being converted to K.

- e. The force of the electric field on the charged particle causes it to accelerate according to $F_{net} = ma$:

$$F_{net} = ma$$

$$qE = ma$$

$$a = \frac{qE}{m}$$

$$E \text{ at origin} = \sum \frac{kq}{r^2} \hat{r} = \frac{k(4Q)}{3^2} (\text{to the right}) + \frac{k(Q)}{3^2} (\text{to the right}) = 5.00e3N/C \text{ to the right}$$

$$a = \frac{(1.602e-19C)(5.00e3N/C)}{9.11e-31kg} = 8.79e14m/s^2 \text{ to the left}$$

- f.

- i. Again, no vectors are needed. Just add the potentials from the three charges now.

$$V = \sum \frac{kq}{r} = \frac{k(+4Q)}{r} + \frac{k(-Q)}{r} + \frac{k(-Q)}{r}$$

$$V = \frac{k(2Q)}{r} = \frac{(8.99e9)(2 \cdot 1e-6)}{3} = 5.99e3V$$

- ii. We've already determined E_x in part (e) above. We can easily determine the y -component of the electric field at the origin by considering this new charge:

$$E = \frac{kq}{r^2} = \frac{(8.99e9)(1e-6)}{3^2} = 999N/C, \text{ up}$$

Combine this with E_x :

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{5000^2 + 999^2} = 5.10e3N/C$$

$$\theta = \tan^{-1}\left(\frac{999}{5000}\right) = 11.3^\circ$$

11.

- a. The flux per unit length might be calculated using E if we had that function available. It's easier, though, to consider flux according to the definition of Gauss's Law:

$$\Phi = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$\Phi = \frac{q_{in}}{\epsilon_0}$$

The $q_{internal}$ is just the charge Q on the cylinder, which we can calculate in terms of linear charge density.

$$\lambda = \frac{Q}{L} \Rightarrow Q = \lambda L$$

Combining this with the first equation:

$$\Phi = \frac{q_{in}}{\epsilon_0} = \frac{-\lambda L}{\epsilon_0}$$

$$\frac{\Phi}{L} = \frac{-\lambda}{\epsilon_0}$$

The negative sign here indicates that the net flux is *into* the cylindrical Gaussian surface just outside the conductor.

b.

- i. Inside the conductor, there is no charge—charges in an electrical conductor all reside at the surface. Therefore, $E = 0$.
- ii. For the open area between the two cylinders:

$$\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$E(2\pi rL) = -\lambda L(4\pi k)$$

$$E = \frac{-2k\lambda}{r}$$

- iii. At the exterior of the cylindrical shells, we need to know the $q_{internal}$ for the cylinders, which we can calculate in terms of their charge densities, a linear density for the conductor and a volume density for the non-conducting shell.

$$q_{in} = q_{conductor} + q_{nonconductor}$$

$$q_{in} = -\lambda L + \rho V$$

$$V_{nonconductor} = V_{outer} - V_{inner} = \pi r_{outer}^2 L - \pi r_{inner}^2 L = \pi L(c^2 - b^2)$$

$$q_{in} = -\lambda L + \rho(\pi L(c^2 - b^2))$$

Now, using Gauss's Law:

$$\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

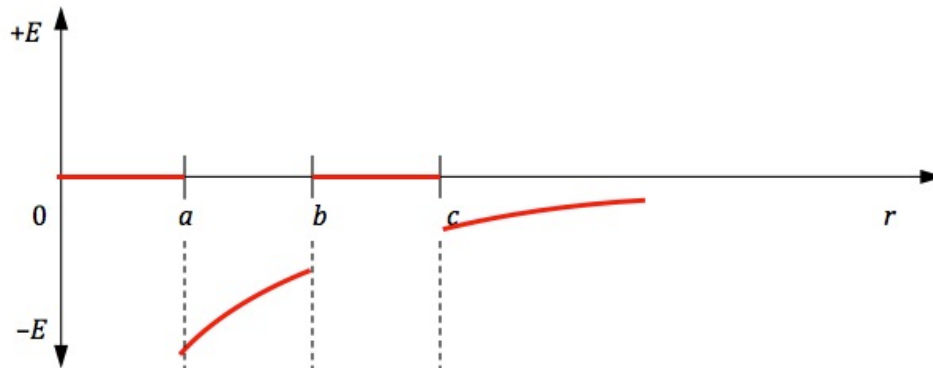
$$E(2\pi rL) = (-\lambda L + \rho\pi L(c^2 - b^2))(4\pi k)$$

$$E = \frac{2k(\rho\pi(c^2 - b^2) - \lambda)}{r}$$

- c. Putting an uncharged conducting cylinder around the linear charge density causes the uncharged cylinder to become polarized, with positive charges at the inner surface and negative charges at the outer surface. There is no electric field inside either conductor, and thus no change in electric potential inside the conductors.

The following graphs require a general qualitative understanding of how electric field and electric potential changes work, without having to solve for specific functions.

i.



ii.

