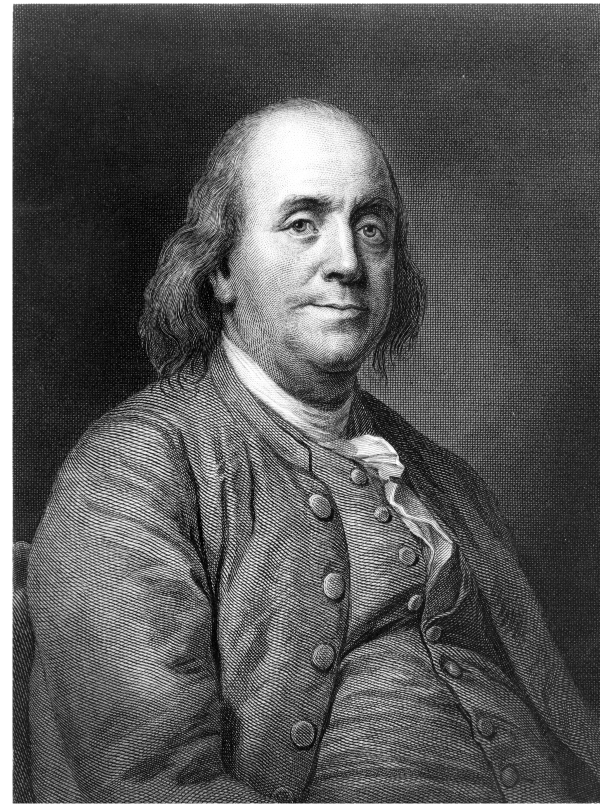
Ch 23 Electric Forces, Fields

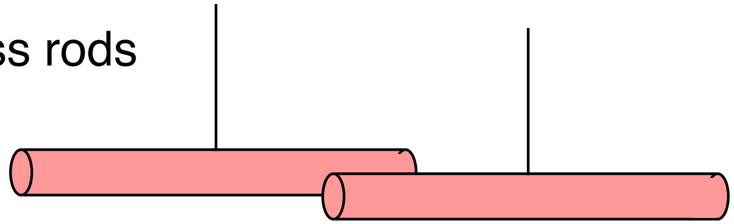
Electrostatic Forces

- Forces between electric charges are responsible for binding atoms and molecules together to create solids and liquids--without electric forces, atoms and molecules would literally fly away from each other.
- Likewise, when one "solid" object comes into contact with another, there is no actual *contact* between the two bodies--the electric force from the molecules in one body push (or pull) on the molecules in the other body.

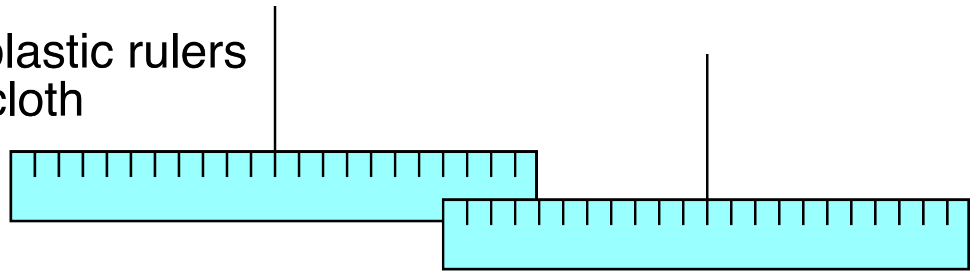
Static Electricity

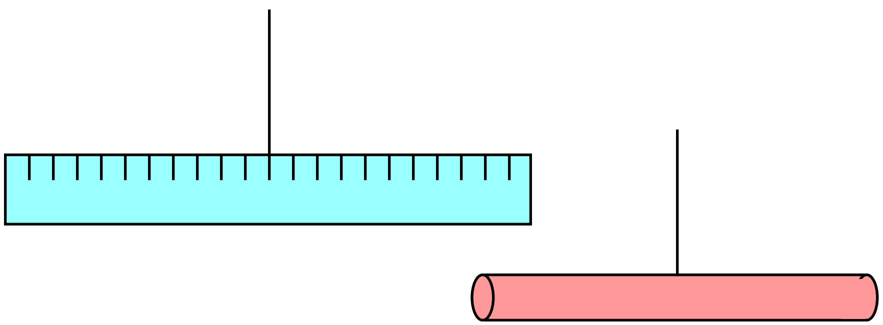


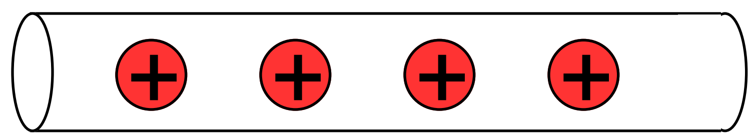
Suspended glass rods
rubbed with silk



Suspended plastic rulers
rubbed with cloth





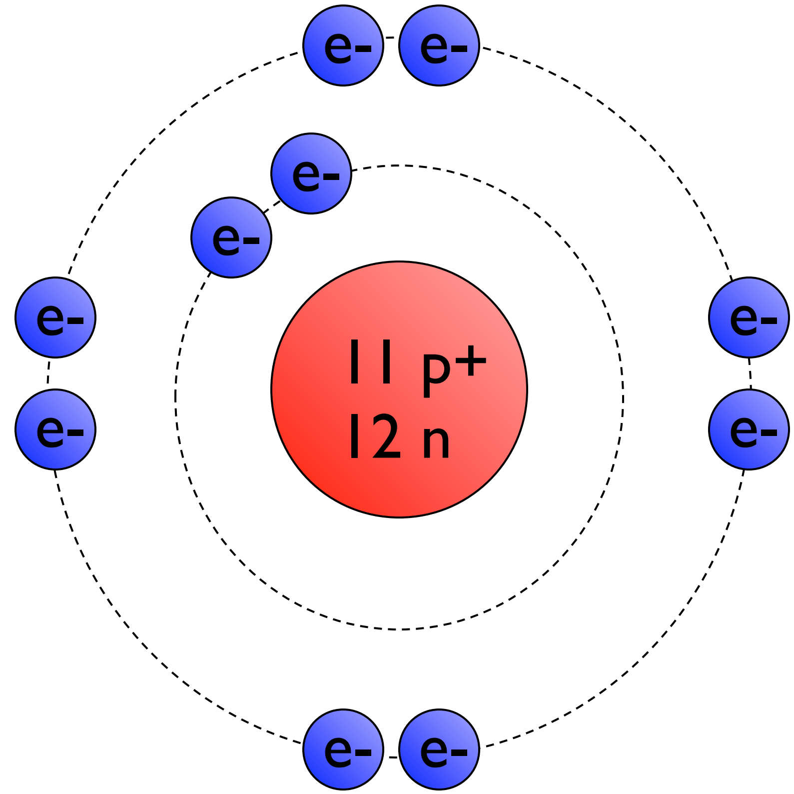


Conservation of Charge

"The net amount of electric charge produced in any process is 0."

Example: When an object (a plastic ruler, say) is rubbed with another object (a paper towel, in this case), if the plastic acquires a negative charge, then the paper towel acquires an equal amount of positive charge.

Sub-atomic particles



The *nucleus* of an atom contains positively charged *protons* and neutrally charged *neutrons*, which each have a mass of 1.67e-27kg.

The nucleus is usually surrounded by negatively charged *electrons*, which each have a mass of 9.11e-31kg.

In its "normal" state, the atom is electrically neutral. If an atom gains or loses electrons, we call it an *ion*, and it has a corresponding negative or positive charge.

Insulators vs. Conductors

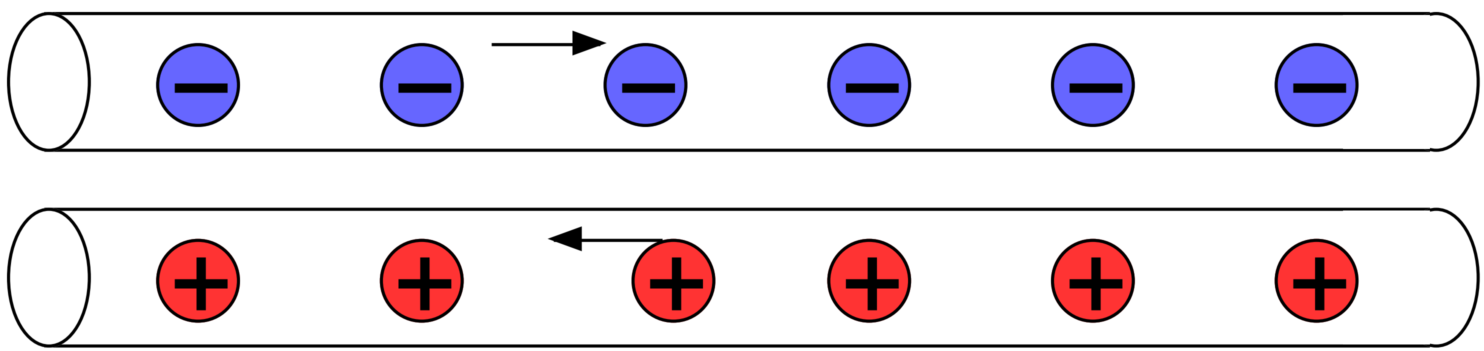
Conductors (copper, gold, aluminum, etc.) conduct charges easily—in these materials, the e- are not very tightly bound to the nucleus of the atoms.

Non-conductors, or *insulators* (rubber, plastic, wood) don't conduct charges very easily—these atoms have e- more tightly bound to the nuclei.

Semiconductors do or don't conduct electric charges, depending on other conditions.

Which charges, + or -, *move*?

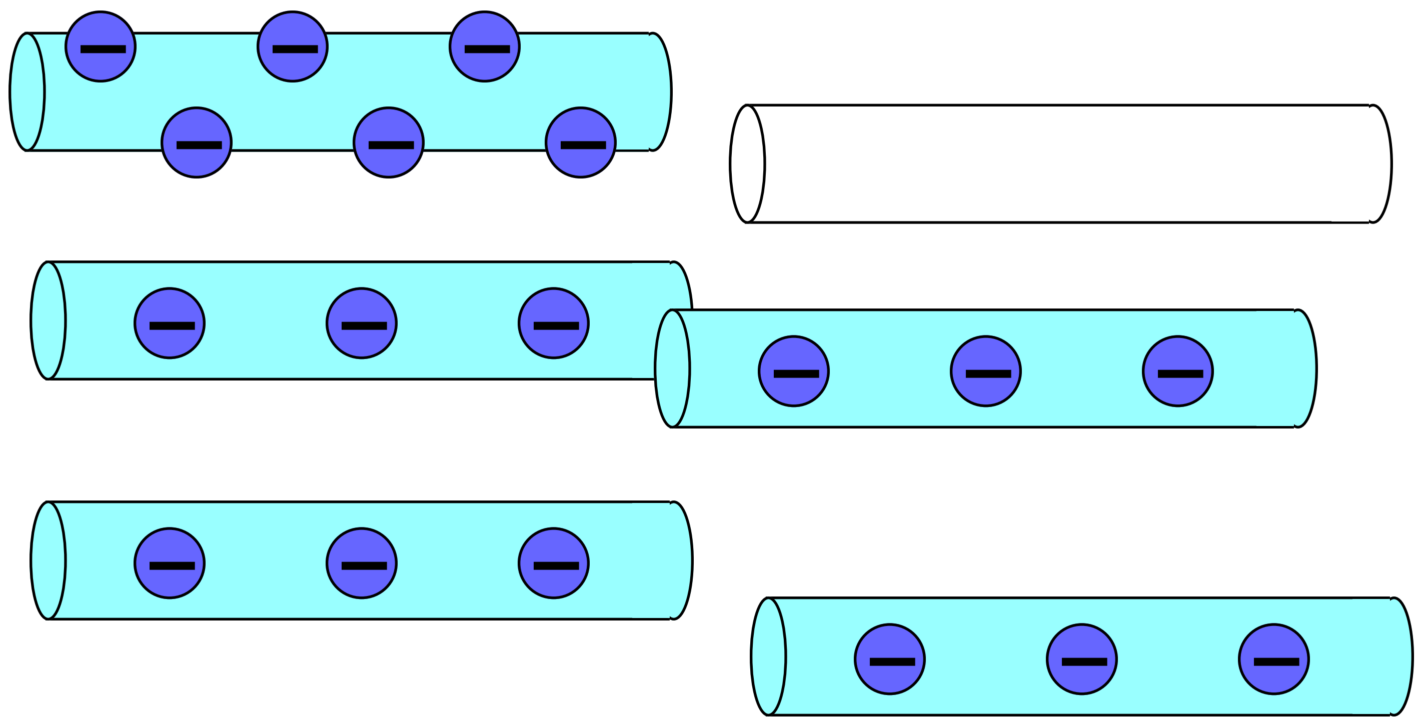
Ben Franklin developed the idea of "positive" and "negative" charges from a conservation perspective: keeping track of gains and losses of charge. At the time, the actual mechanism of charge transfer (the electron), and the fact that electrons even exist, were things of which we were unaware.



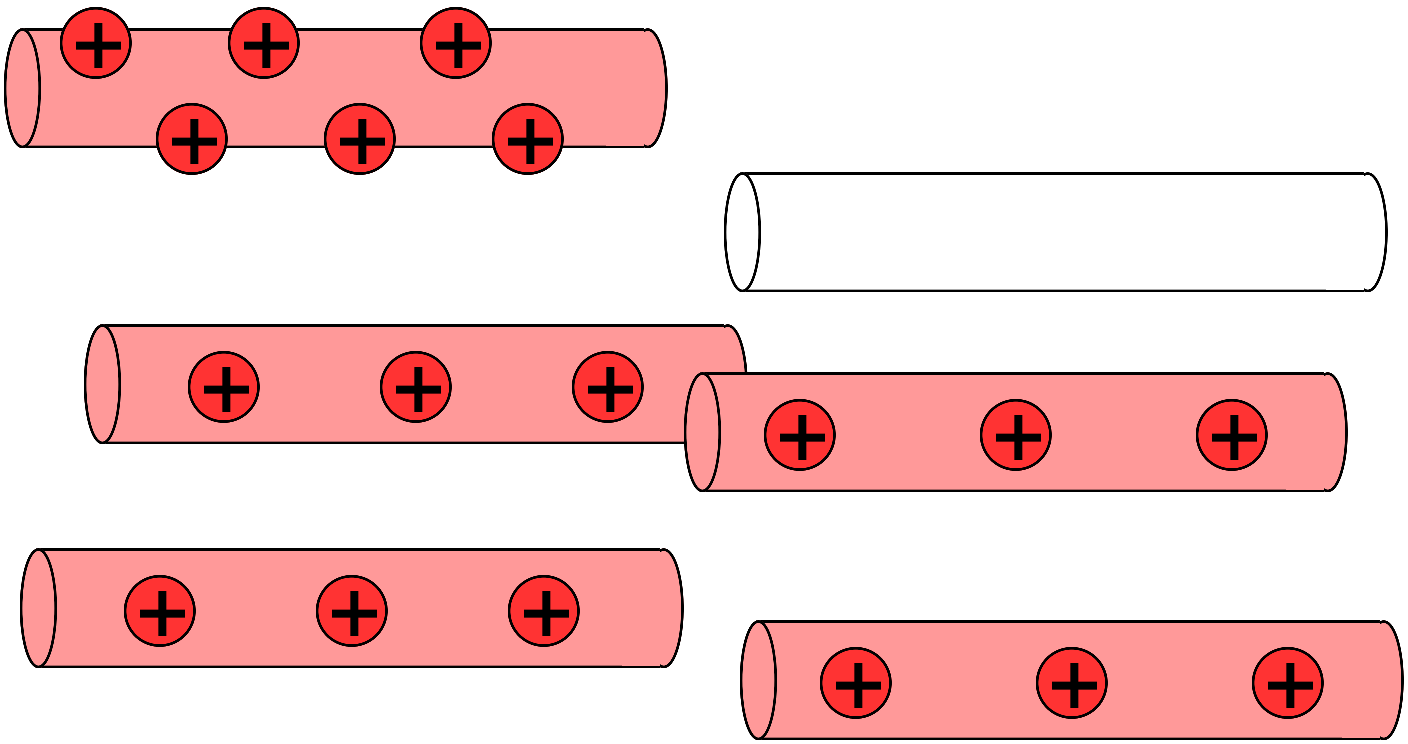
We now know that when charge is transferred, or when charge flows, we're witnessing the flow of electrons. But physicists still talk about "the flow of negative charges" in a certain direction (electrons, which *do* move), and "the flow of positive charges," (which *doesn't actually happen*).

Charging by conduction

"Charging by conduction" occurs when a charged conductor (metal) touches a neutral conductor: some free electrons pass from one object to the other.

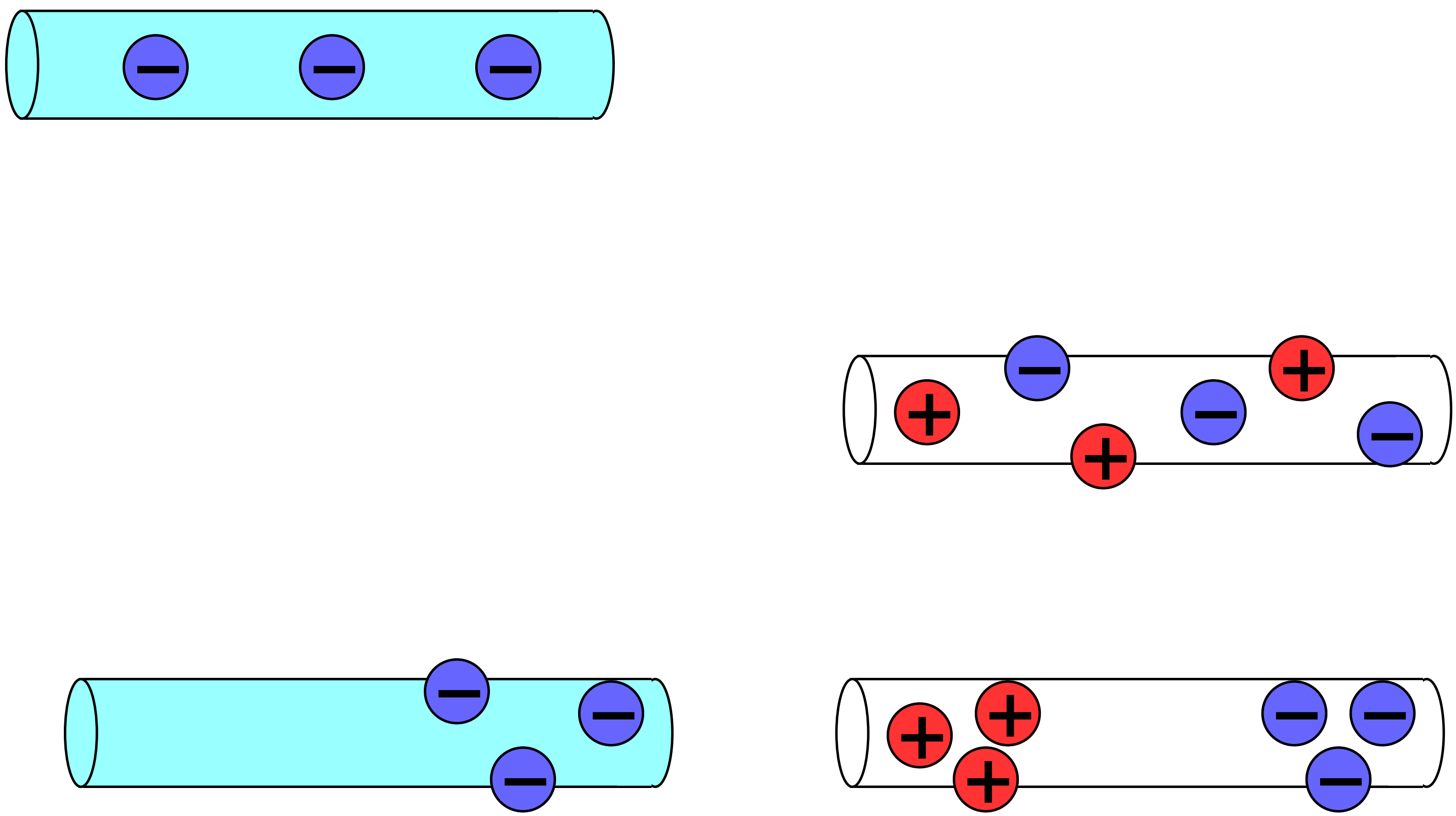


Charging by conduction



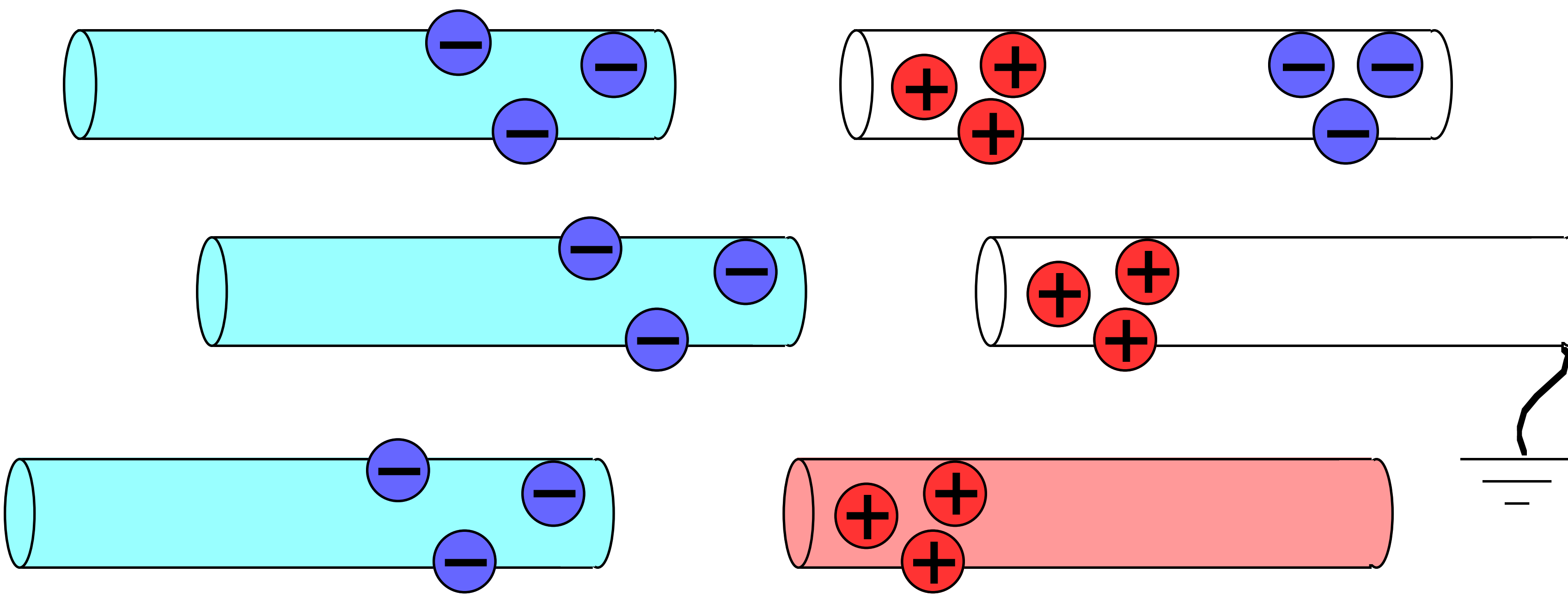
Charging by induction

"Charging by *induction*" occurs when a charged object is brought near a neutral conductor.



Grounding

The term *grounding* refers to connecting a conductor to the literal ground, ie. the earth. The earth readily accepts or gives up electrons—it has plenty to spare—so grounding a conductor allows for the flow of charges. What effect this has depends on the situation.

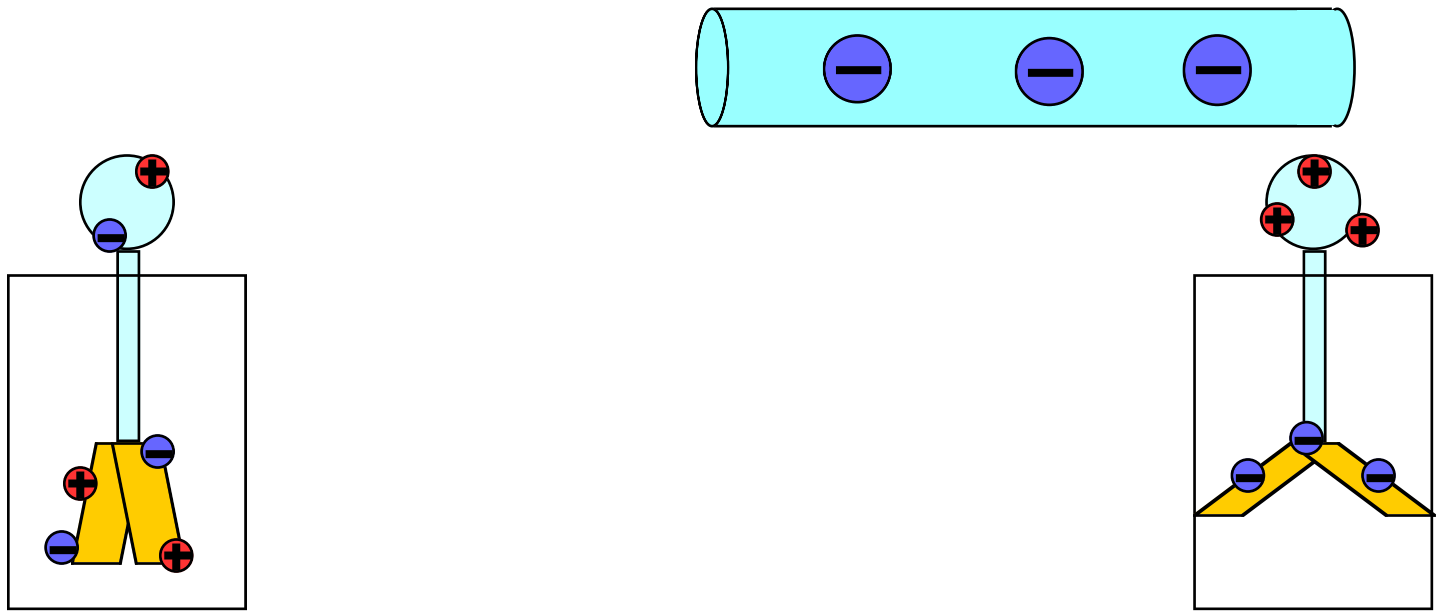


Simulation

file:///Applications/PhET/sims/html/balloons-and-static-electricity/latest/balloons-and-static-electricity_en.html

Electroscope

The *electroscope* is a simple device designed to detect the presence of electric charges. Movable leaves (of gold?!) connected to a metal ball separate when a charged object is brought near, or touched to the ball. But why?



Coulomb's Law

Like charges repel, opposite charges attract, but we need a more quantitative description of electric forces.

$$\mathbf{F}_g = -G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{F}}_{21} = k\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

Coulomb's Law (Fine print)

$$\vec{\mathbf{F}}_{21} = k\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

- Charge of an electron is -1.602e-19C
- Charge of a proton is +1.602e-19C
- k = 9.00e9 N•m^2/C^2
- If the resulting Force is negative then the charges are attracted to each other. If the resulting Force is positive then the charges are repelling each other.
- Coulomb's Law only applies to non-moving charges = *electrostatic situations*.
- Coulomb's Law only applies to two charges. To use it with more than two charges, the net force on any single charge will be equal to the net vector sum of the forces due to the other charges.

More fine print

- The value k can also be expressed in another way:

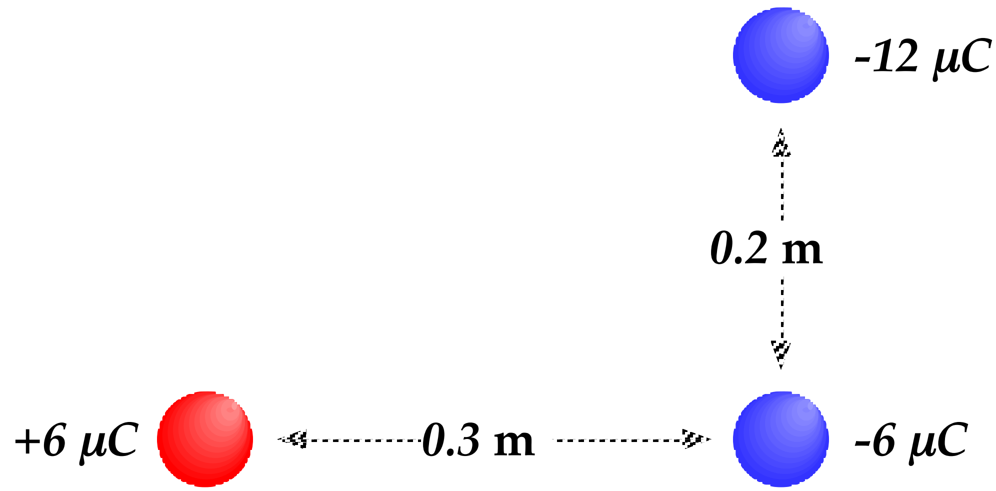
$$k = \frac{1}{4\pi\varepsilon_o}$$

$$F_e = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$$

- ε_0 refers to the "permittivity of free space," which has to do with how quickly electric fields can propagate in a vacuum. For now, just think of it the way you think of the variable π: it seems to show up in a lot of different formulae, and therefore has some value to us.

Example 1

What is the force (magnitude and direction) acting on the -12 μC charge in the situation shown here?



$$F_e = k\frac{q_1 q_2}{r^2}$$

$$F_{12,-6} = (9e9)\frac{(-12e-6C)(-6e-6C)}{(0.2m)^2} = +16.2\mathbf{j}N$$

$$F_{12,+6} = (9e9)\frac{(-12e-6C)(+6e-6C)}{(0.36m)^2} = 5.0N$$

$$\theta = \tan^{-1}\left(\frac{0.2}{0.3}\right) + 180° = 214°$$

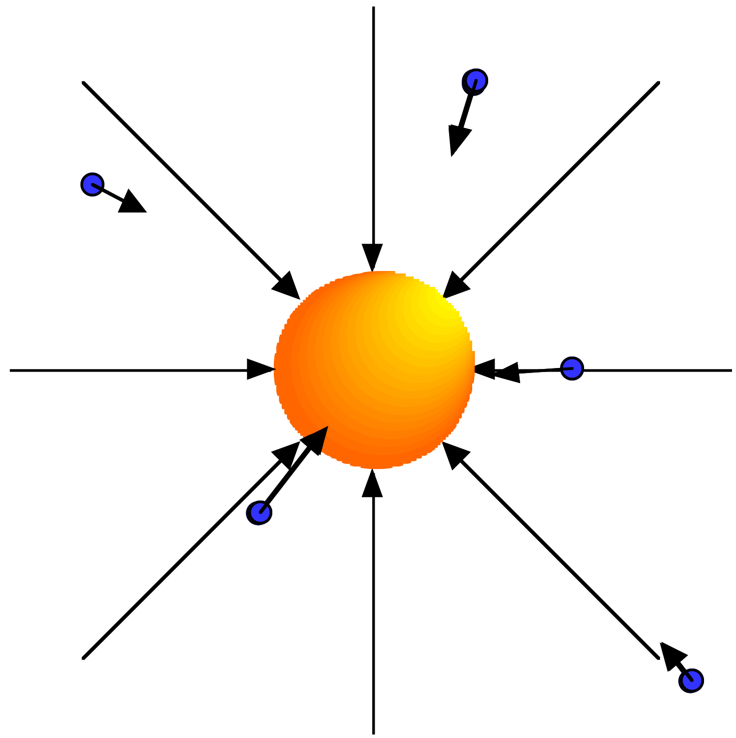
$$F_{12,+6} = (5.0\cos 214°)\mathbf{i} + (5.0\sin 214°)\mathbf{j} = -4.15\mathbf{i} + -2.79\mathbf{j}N$$

$$F_{net} = (-4.15\mathbf{i} + (16.2 - 2.79)\mathbf{j})N = (-4.15\mathbf{i} + 13.4\mathbf{j})N$$

Resolve $(-4.14\mathbf{i} + 13.4\mathbf{j})N$:

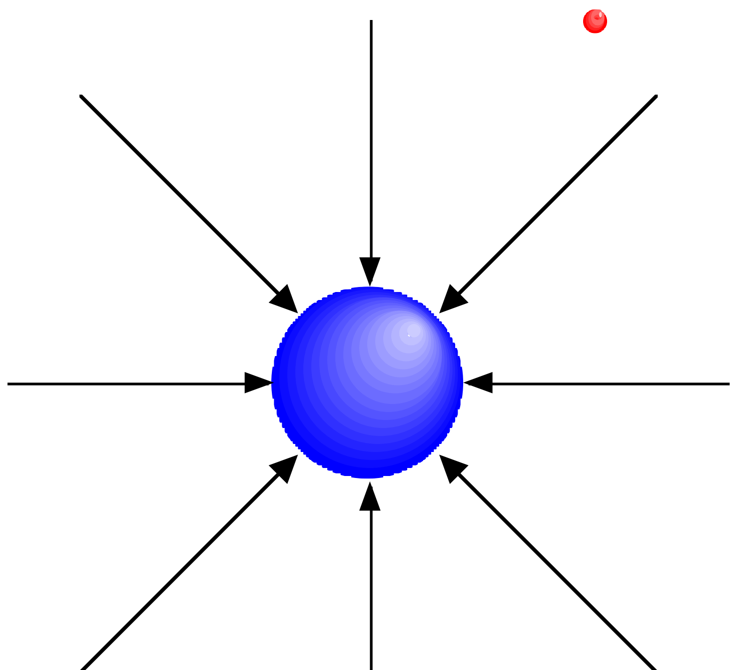
$14.0N @ 107°$

Gravity Field?



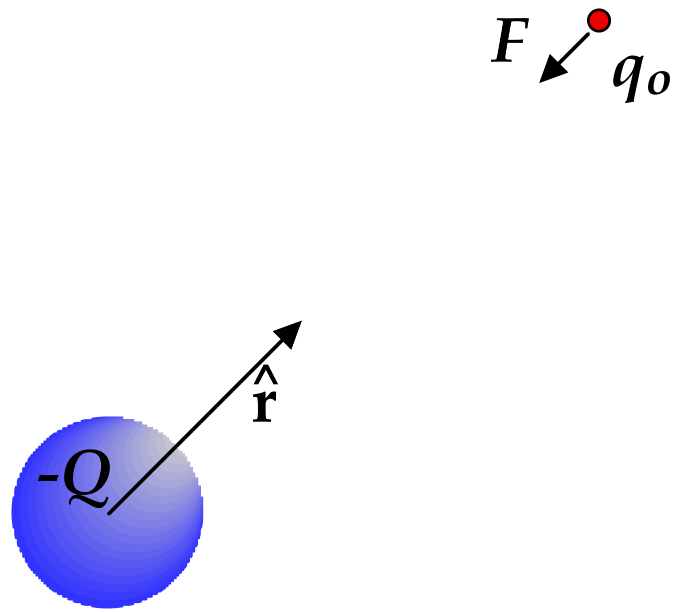
$$g = \frac{F_g}{m_o} = \frac{G\frac{m_1 m_o}{r^2}}{m_o} = G\frac{m_1}{r^2}$$

Electric Field!



$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_o} = \frac{k\dfrac{q_1 q_o}{r^2}}{q_o} = k\frac{q_1}{r^2}$$

Electric Field w/Unit Vectors



$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_o}\hat{\mathbf{r}} = k\frac{q_1}{r^2}\hat{\mathbf{r}}$$

Field Visualization

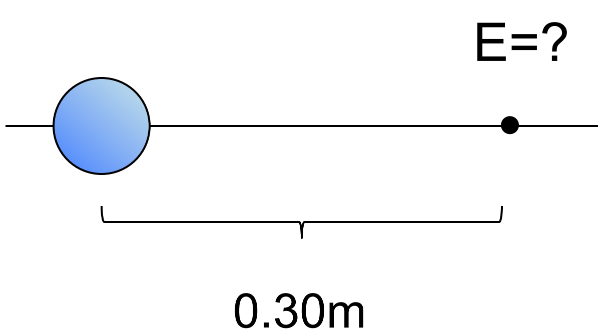
http://falstad.com/emstatic

Example 2

Computer models of Coulomb's Law (Ch 22, 23 Explorations)

Example 3

A point charge has a charge of -3.0e-6C.



a) What is the magnitude & direction of the E field 30 cm to the right of the charge?

$E=kq/r^2$=3.0e5 N/C to the left

b) What is the Force (mag & direction) on an electron placed at this position?

F=qE=4.8e-14 N to the right

c) How many electrons were deposited on the point charge to give it this magnitude charge?

$n=q/e^-$ = 1.9e13 electrons

If there are more point charges?

$$\vec{\mathbf{E}}_{net} = \sum_i k \frac{q_i}{r_i^2}\hat{\mathbf{r}} = k \sum_i \frac{q_i}{r_i^2}\hat{\mathbf{r}}$$

Example 4

A charge, q_1=7μC is located at the origin, and a second charge, q_2=-5.0μC, is located along the x-axis at 0.30 m from the origin. Find the electric field (magnitude and direction) at the point (0, 0.40) m.

1. The *x*-component of this Electric field is approximately:

a. 1.08e5 N/C
b. -1.08e5 N/C
c. 2.5e5 N/C
d. none of these

2. The *direction* of this Electric field is approximately:

a. 74°
b. 70°
c. 66°
d. 60°

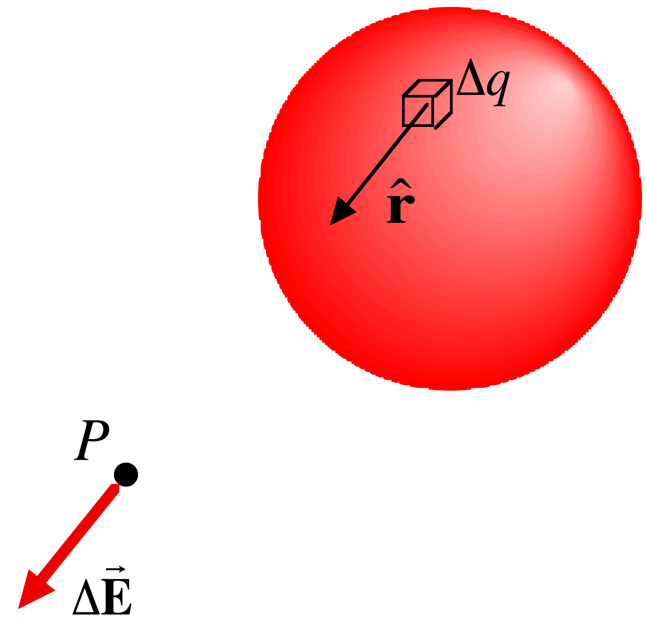
Continuous Distribution??!

To find the electric field **E** at a point *P* due to continuous charge +*Q*:

a. divide the charge distribution into small elements, each with charge *dq*;

b. use Coulomb's Law to calculate the electric field *dE* due to each of these point charges;

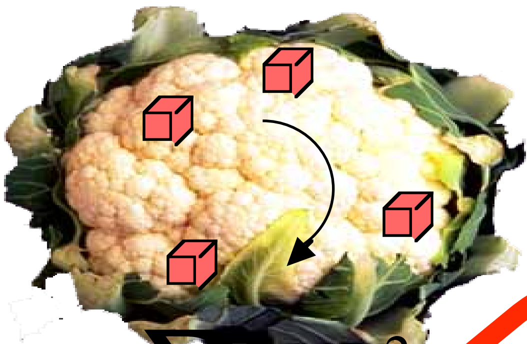
c. evaluate the total **E** field at *P* by summing the contributions of all the individual charge elements.



$$\vec{\mathbf{E}}_{net} = k \int \frac{dq}{r^2} \hat{\mathbf{r}}$$

I for continuous distributions

Guess what? If you want to find the moment of inertia for a larger object, say--oh, I don't know... a *cauliflower*, for instance--you can probably figure out what we're going to do...



$$I = \sum m_i r_i^2$$

$$I = \lim_{\Delta m_i \to 0} \sum \Delta m_i r_i^2$$

$$I = \int r^2 dm$$

- As before, the key to using this equation is expressing *dm* in terms of other quantities. We may use:

$$dm = \lambda \; dr$$

$$dm = \sigma \; dA$$

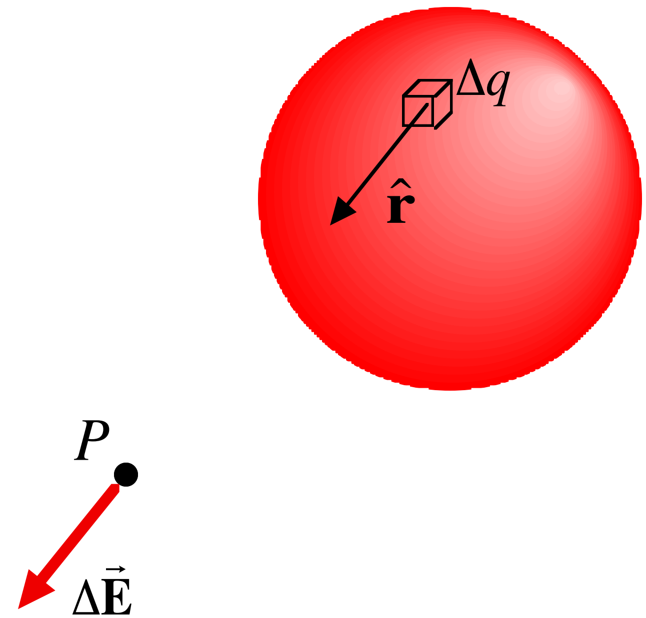
$$dm = \rho \; dV$$

Substitution Strategy

$$\lambda = \frac{Q}{L}, \text{ or } dq = \lambda \; dl$$

$$\sigma = \frac{Q}{A}, \text{ or } dq = \sigma \; dA$$

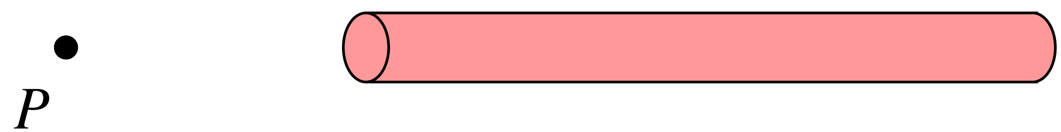
$$\rho = \frac{Q}{V}, \text{ or } dq = \rho \; dV$$



$$\vec{\mathbf{E}}_{net} = k \int \frac{dq}{r^2} \hat{\mathbf{r}}$$

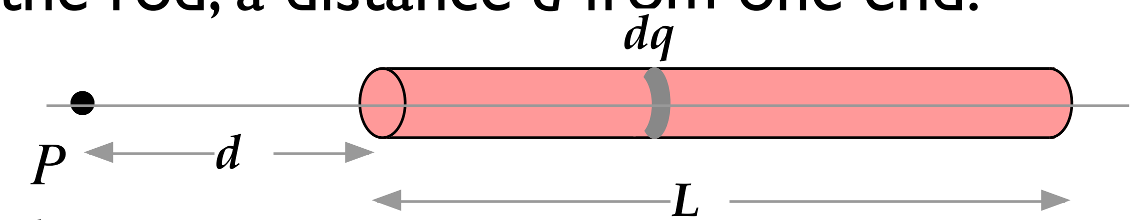
Example 5
E Field of a Charged Rod

A rod of length *L* has a uniform positive charge per unit length λ and a total charge of *Q*. Calculate the electric field at a point *P* along the axis of the rod, a distance *d* from one end.



Example 5
E Field of a Charged Rod

A rod of length *L* has a uniform positive charge per unit length λ and a total charge of *Q*. Calculate the electric field at a point *P* along the axis of the rod, a distance *d* from one end.



$$dq = \lambda \; dx$$

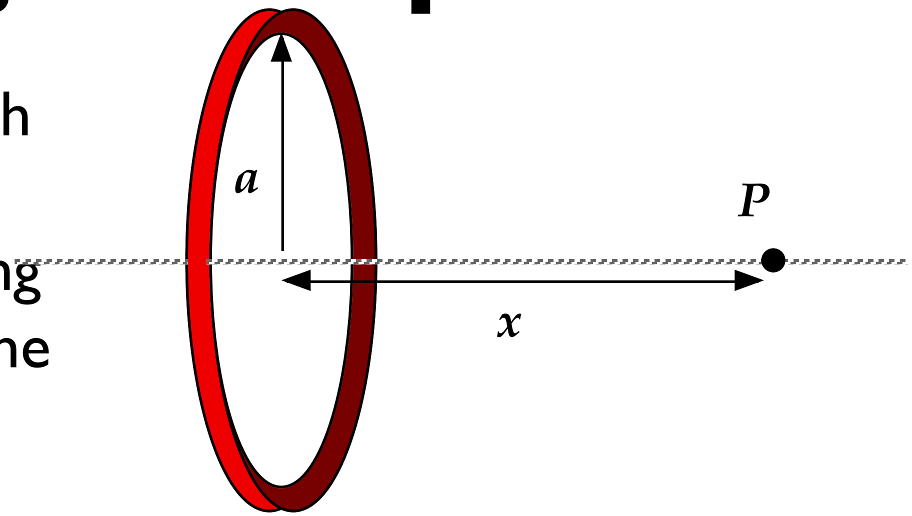
$$\vec{\mathbf{E}} = k\int \frac{dq}{r^2}\hat{\mathbf{r}} = k\int \frac{dq}{x^2}$$

$$\vec{\mathbf{E}} = k\int_d^{d+L} \frac{dq}{x^2} = k\int_d^{d+L} \frac{\lambda}{x^2}dx$$

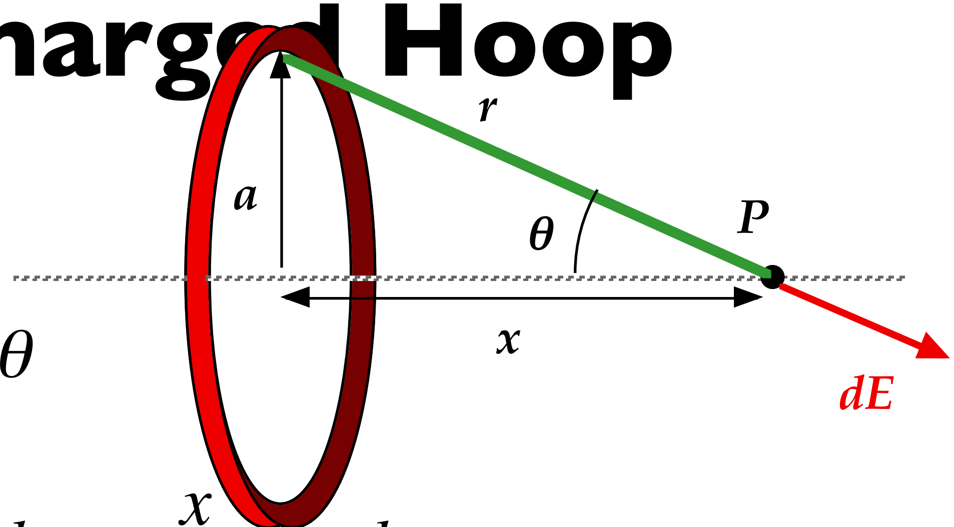
$$\vec{\mathbf{E}} = k\lambda\int_d^{d+L} \frac{1}{x^2}dx = k\lambda\left[\frac{-1}{x}\right]_d^{d+L} = \frac{kQ}{d(L+d)}$$

Example 6
E Field of a Charged Hoop

A ring of radius *a* has a uniform positive charge per unit length *L*, with a total charge of *Q*. Calculate the electric field along the axis of the ring at a point *P* lying a distance *x* from the center of the ring.



Example 6
E Field of a Charged Hoop



$$dE = k\frac{dq}{r^2}$$

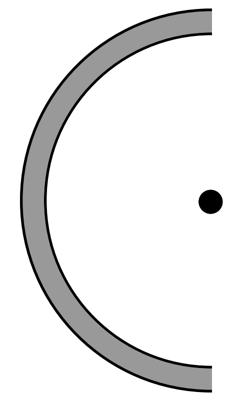
$$dE_x = dE\cos\theta = k\frac{dq}{r^2}\cos\theta$$

$$dE_x = k\frac{dq}{r^2}\left(\frac{x}{r}\right) = k\frac{dqx}{r^3} = k\frac{x}{\left(x^2 + a^2\right)^{\frac{3}{2}}}dq$$

$$\vec{\mathbf{E}} = \int dE_x = \int k\frac{x}{\left(x^2 + a^2\right)^{\frac{3}{2}}}dq$$

$$\vec{\mathbf{E}} = k\frac{x}{\left(x^2 + a^2\right)^{\frac{3}{2}}}Q$$

Example 8
E Field of a Curved Rod



Assume a rod of length *L* and continuous charge distribution -*Q*, curved into a semi-circle of radius *R*. Find the magnitude and direction of the electric field at the center of the semi-circle.

$$E = \int dE\cos\theta = \int \frac{k \bullet dq}{r^2}\cos\theta$$

$$dq = \lambda \bullet ds = \lambda r \bullet d\theta$$

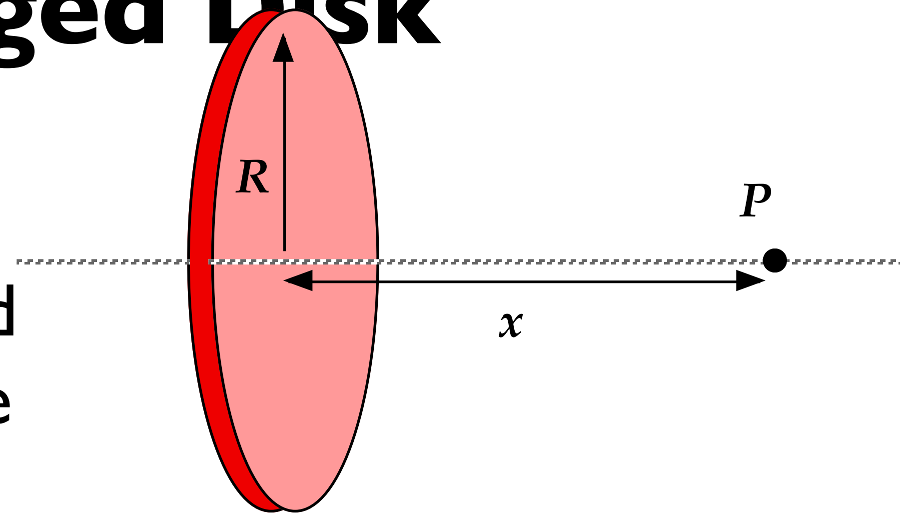
$$E = \int_{\pi/2}^{3/2\pi} \frac{k\lambda r\cos\theta}{r^2}d\theta$$

$$E = \frac{k\lambda}{r}\int_{\pi/2}^{3/2\pi} \cos\theta \bullet d\theta$$

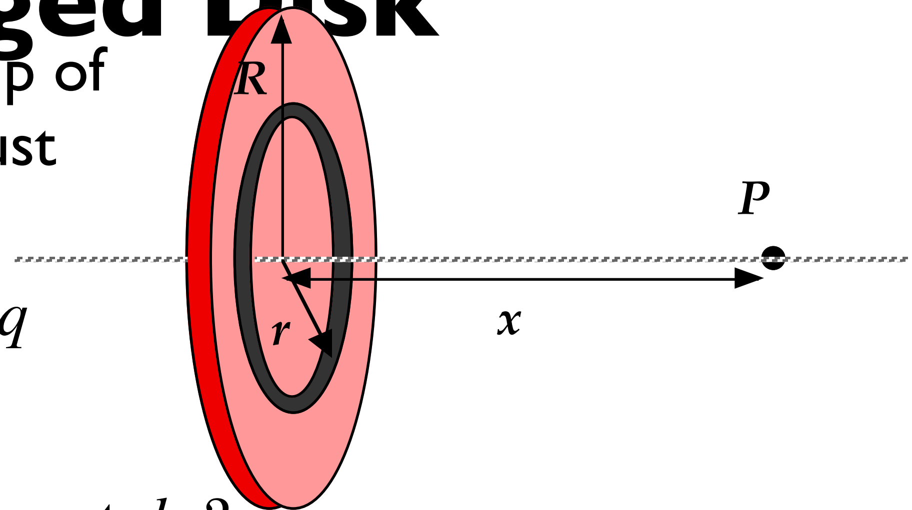
$$E = \frac{kQ}{rL}\left[\sin\theta\right]_{\pi/2}^{3/2\pi} = \frac{2kQ}{rL}\ \textit{to the left}$$

Example 7
E Field of a Charged Disk

A disk of radius *R* has a uniform charge per unit area [illegible]. Calculate the electric field at a point P that lies along the central axis of the disk, at a distance *x* from its center.



Example 7
E Field of a Charged Disk



Strategy: consider the disk as a group of concentric rings, for which we've just learned how to calculate **E**.

$$dE \text{ for one ring: } dE = \frac{kx}{(x^2+r^2)^{3/2}}dq$$

$$E = \int_0^R \frac{kx}{(x^2+r^2)^{3/2}}dq, \text{ but how do we get } dq?$$

$$dq = \sigma dA = \sigma 2\pi r dr$$

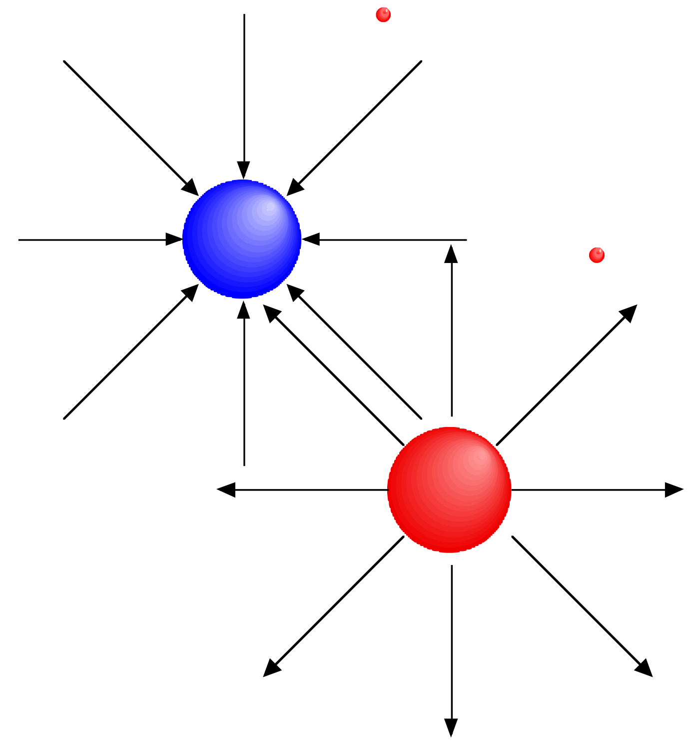
$$E = \int_0^R \frac{kx\sigma 2\pi r}{(x^2+r^2)^{3/2}}dr = kx\sigma\pi \int_0^R \frac{2r}{(x^2+r^2)^{3/2}}dr$$

Integrate by substitution to get:

$$E = 2k\sigma\pi\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

Field Lines & Field Diagrams

By looking at the charged bodies in an area, and determining **what Force would a small, positive, test charge feel at each position**, we can draw *lines of force*--also called *field lines*--to indicate what the field "looks" like.



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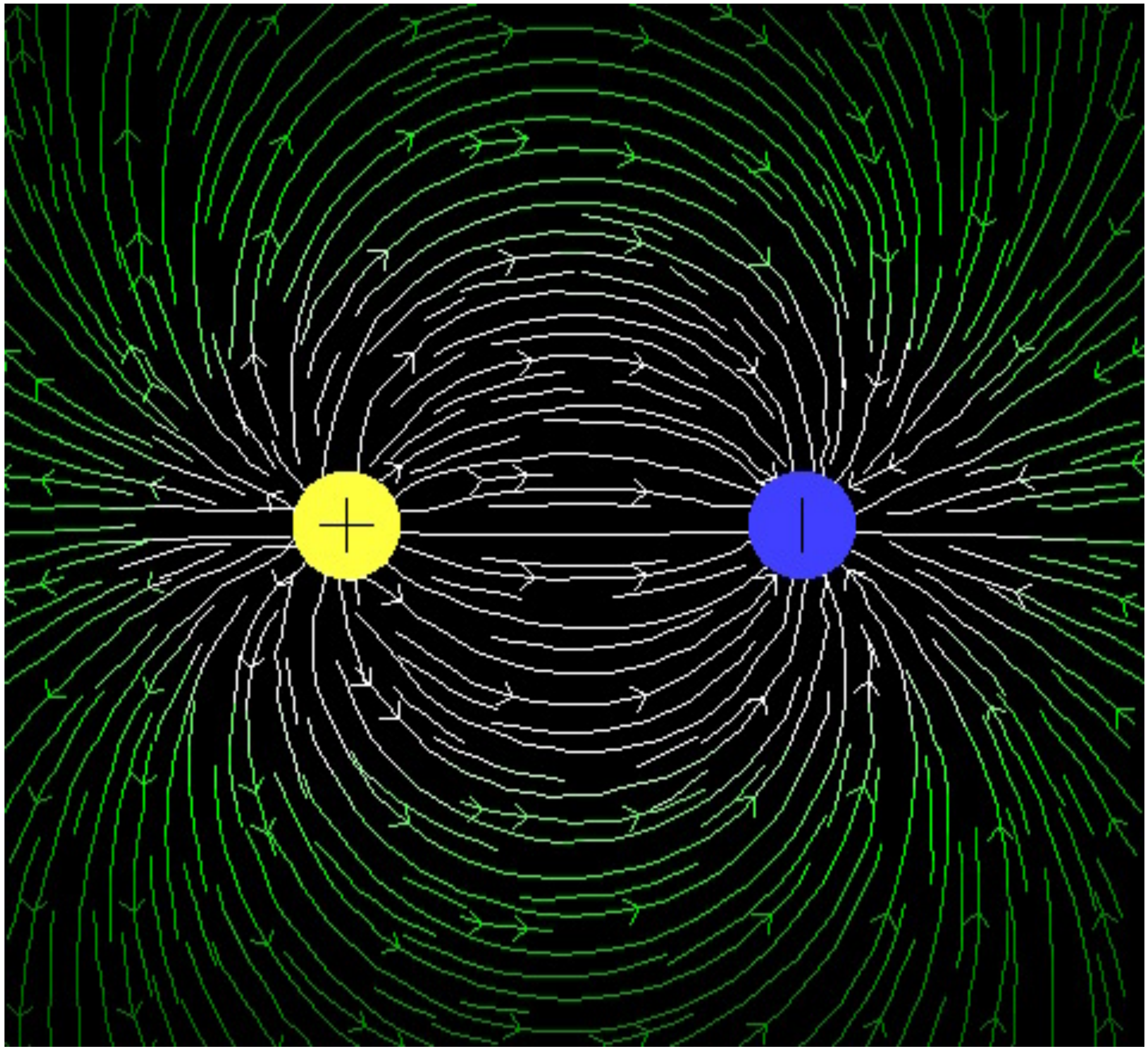
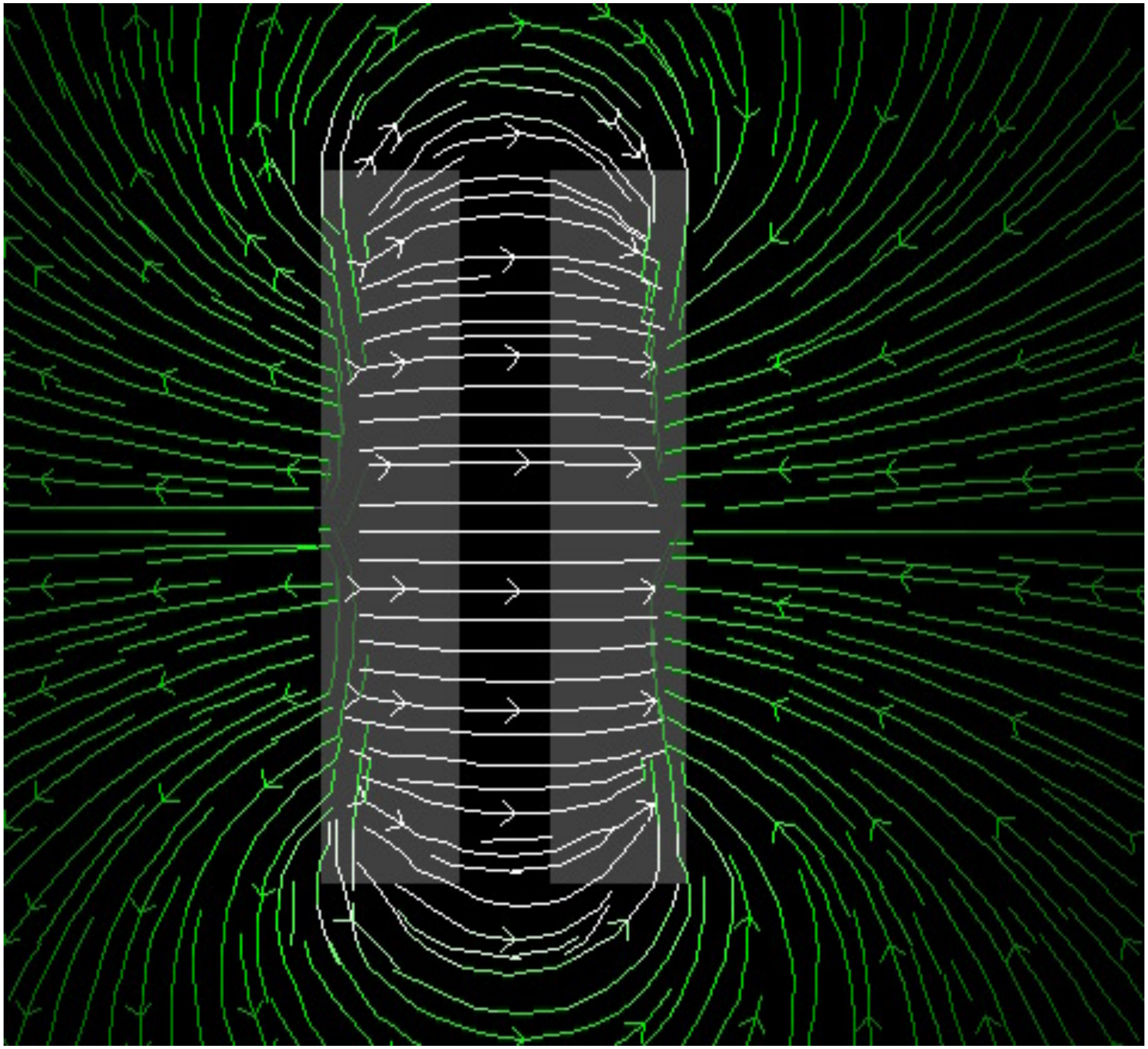
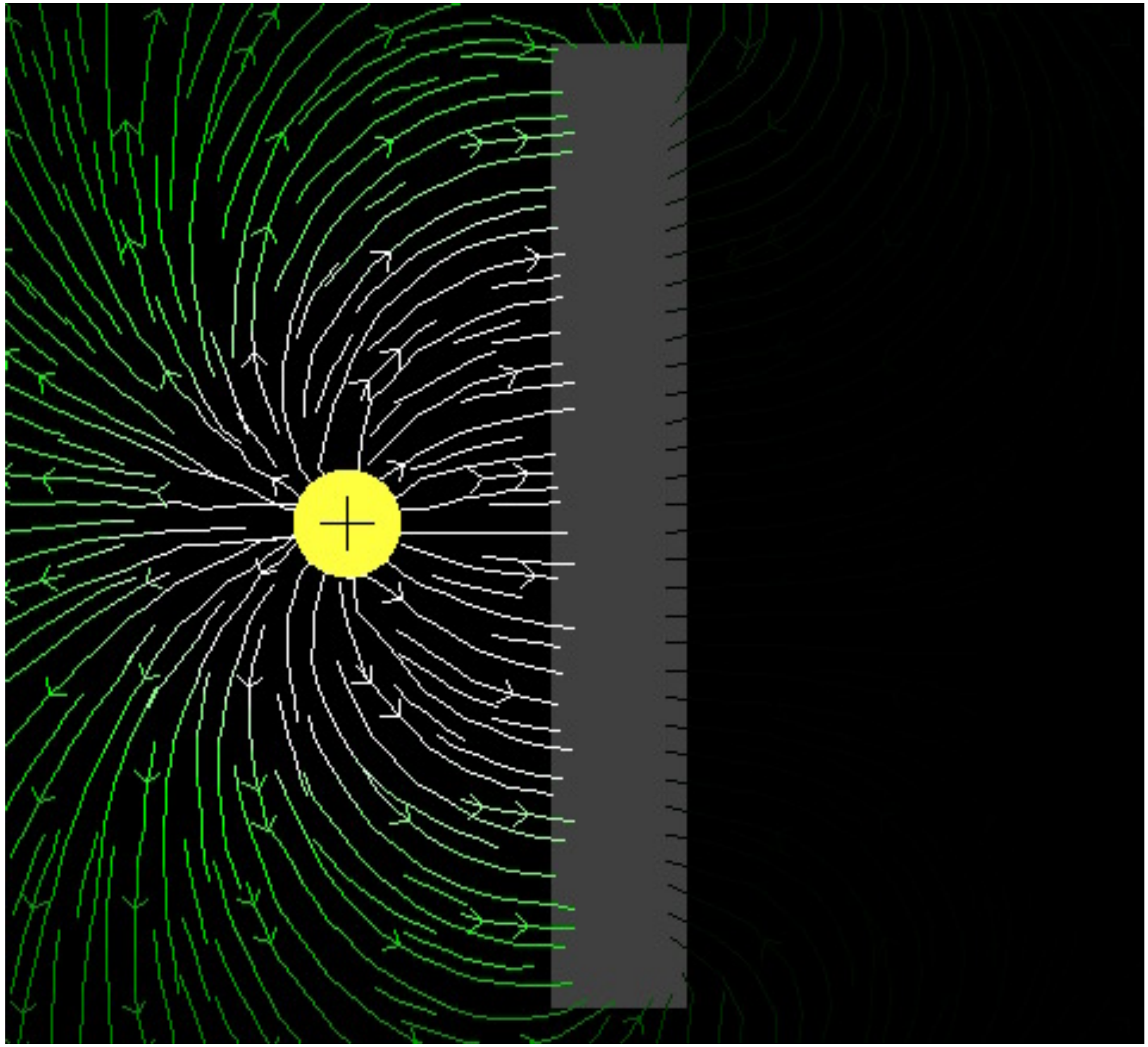
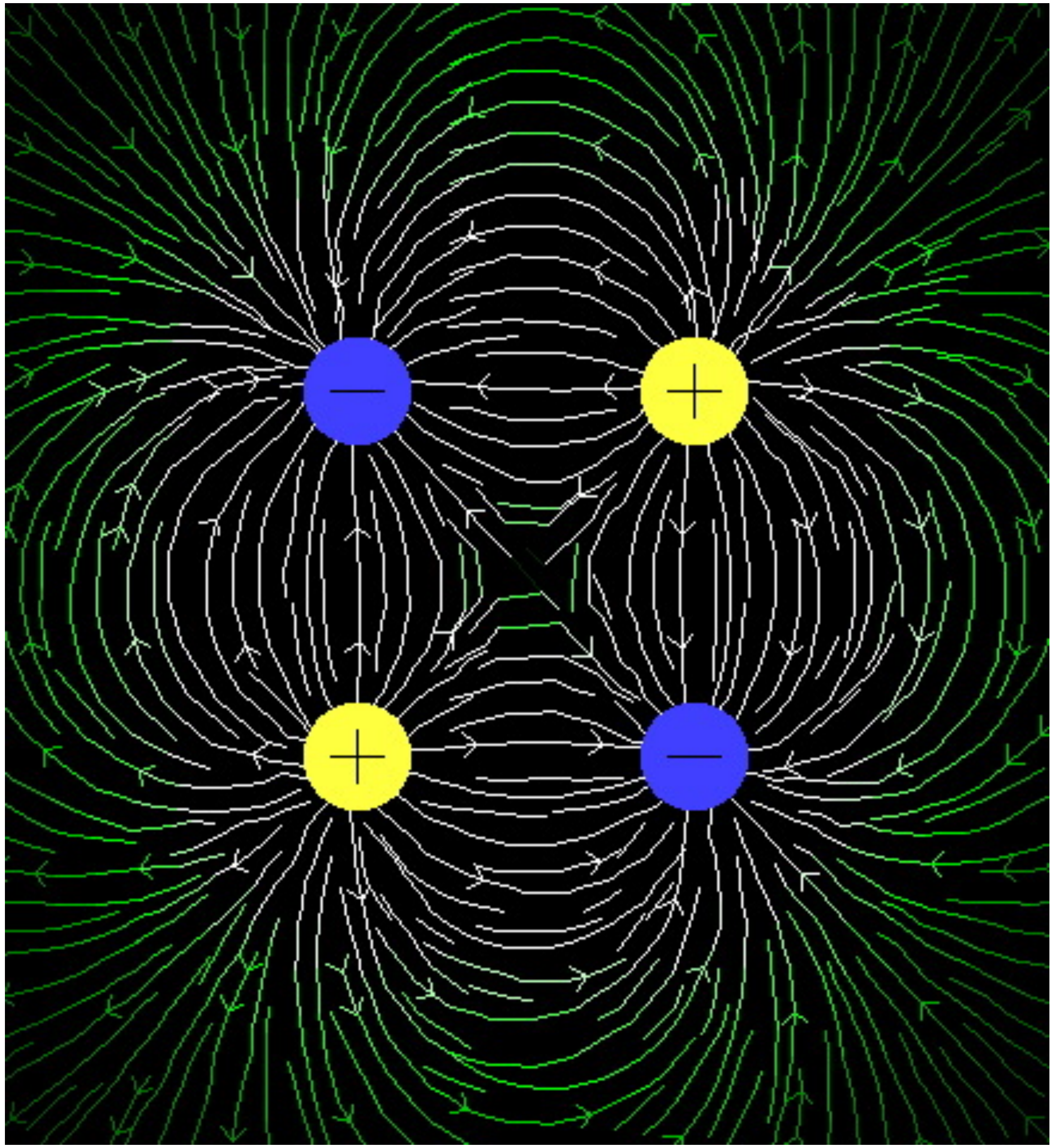
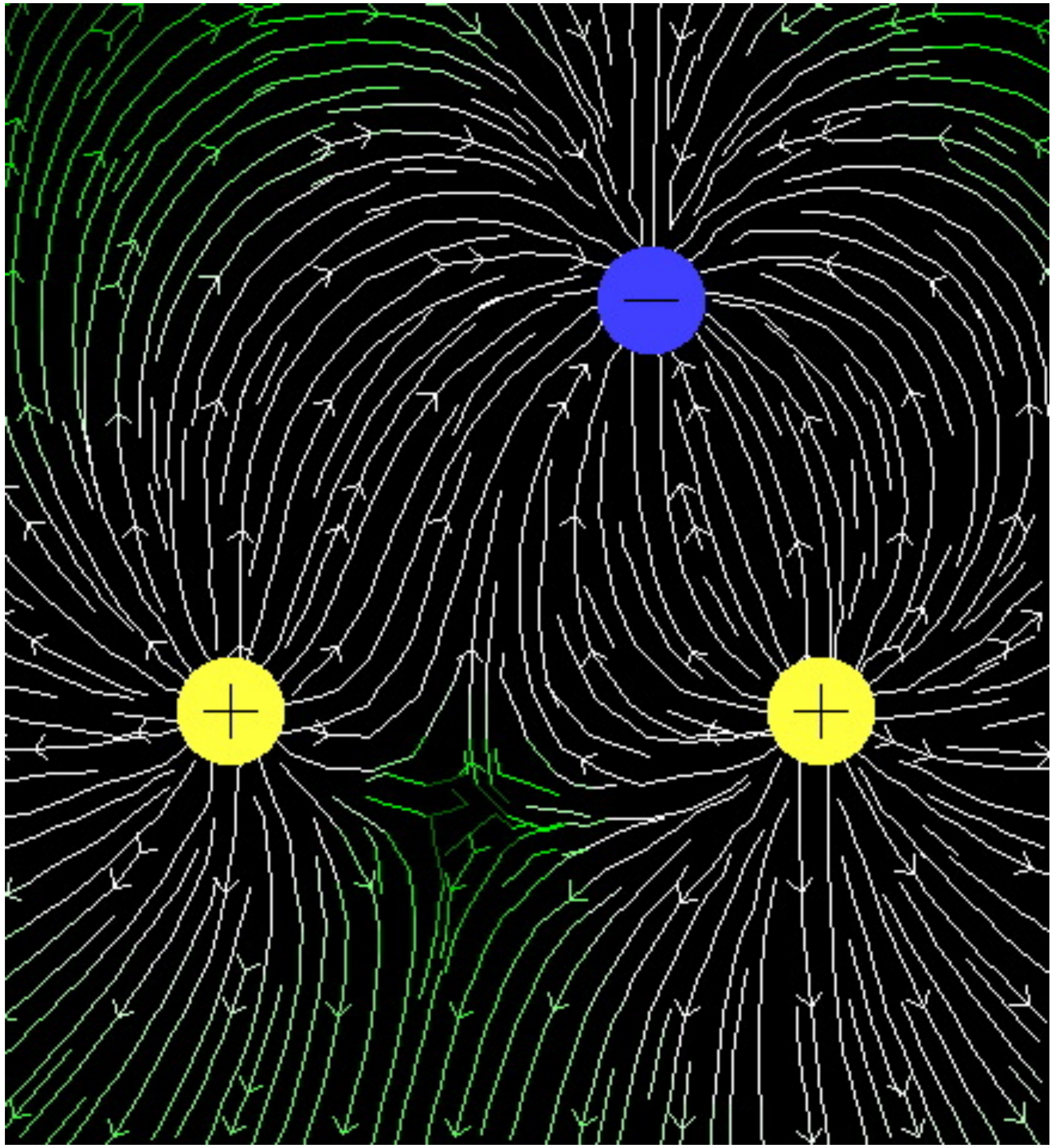








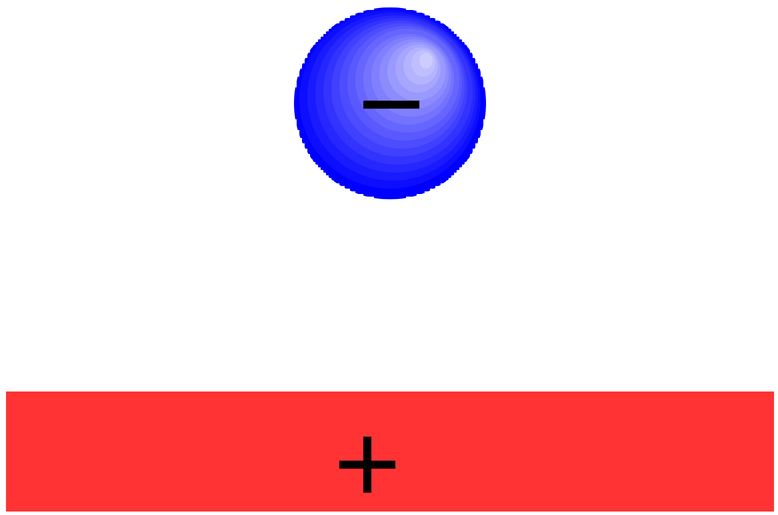


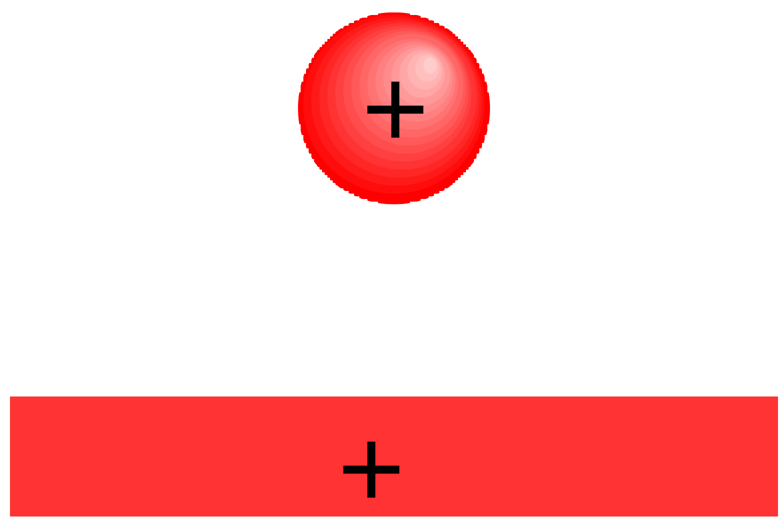
Field Lines...

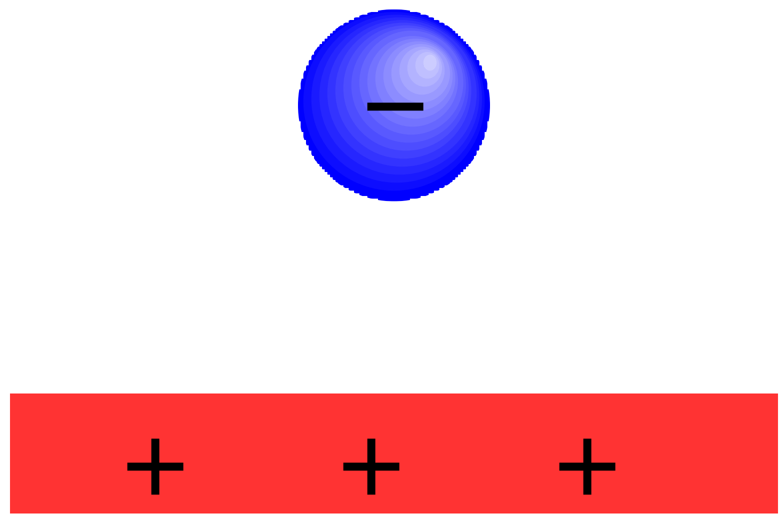
1. The number of lines starting on a positive charge or ending on a negative charge is ***proportional to the magnitude*** of the charge.

2. ***The closer*** the lines are together in a region, ***the stronger*** the electric field is in that region.

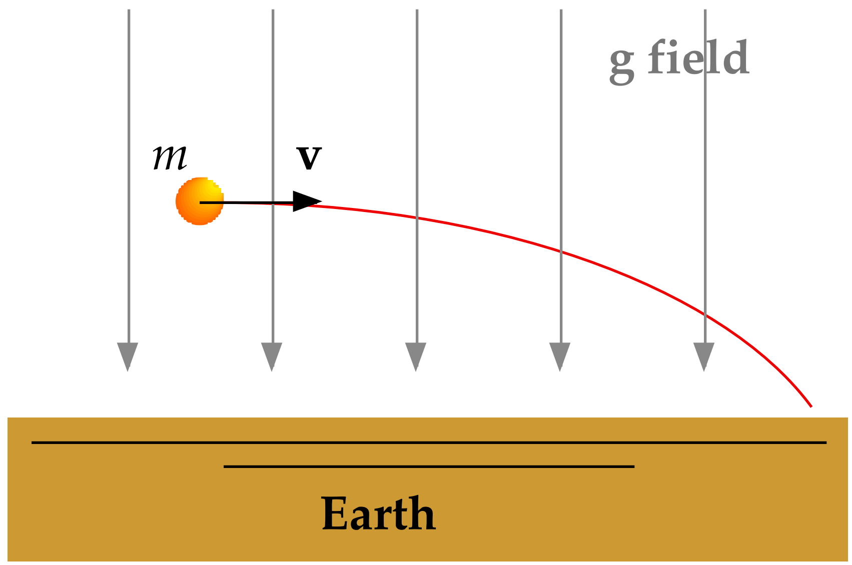
3. Field lines indicate the direction of the electric field, because the ***E field*** at any position ***points in a direction tangent to the field line*** at that point.



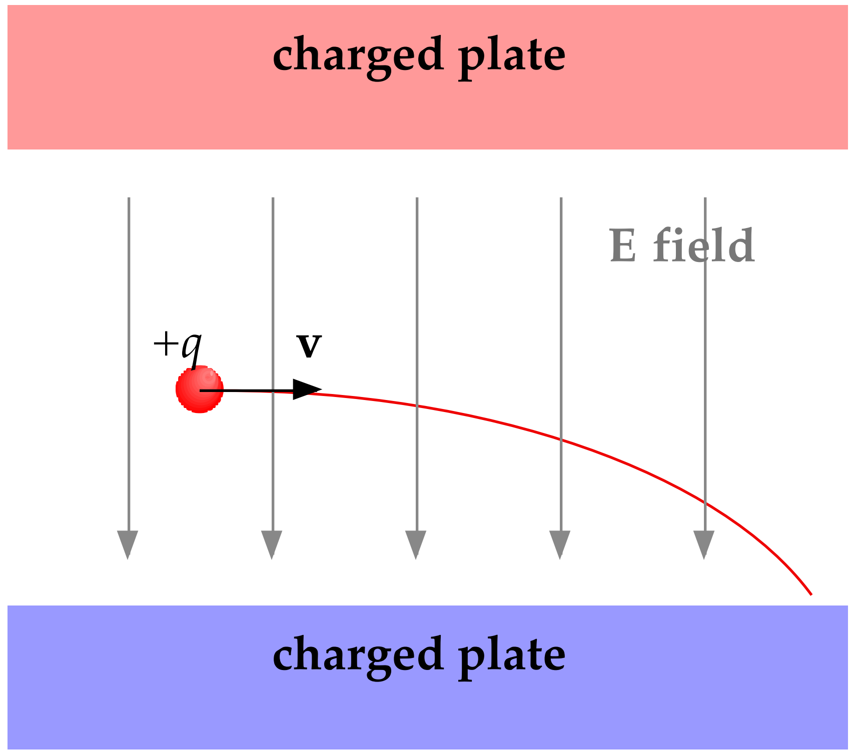




Motion of Mass in g Field



Motion of Charge in E Field

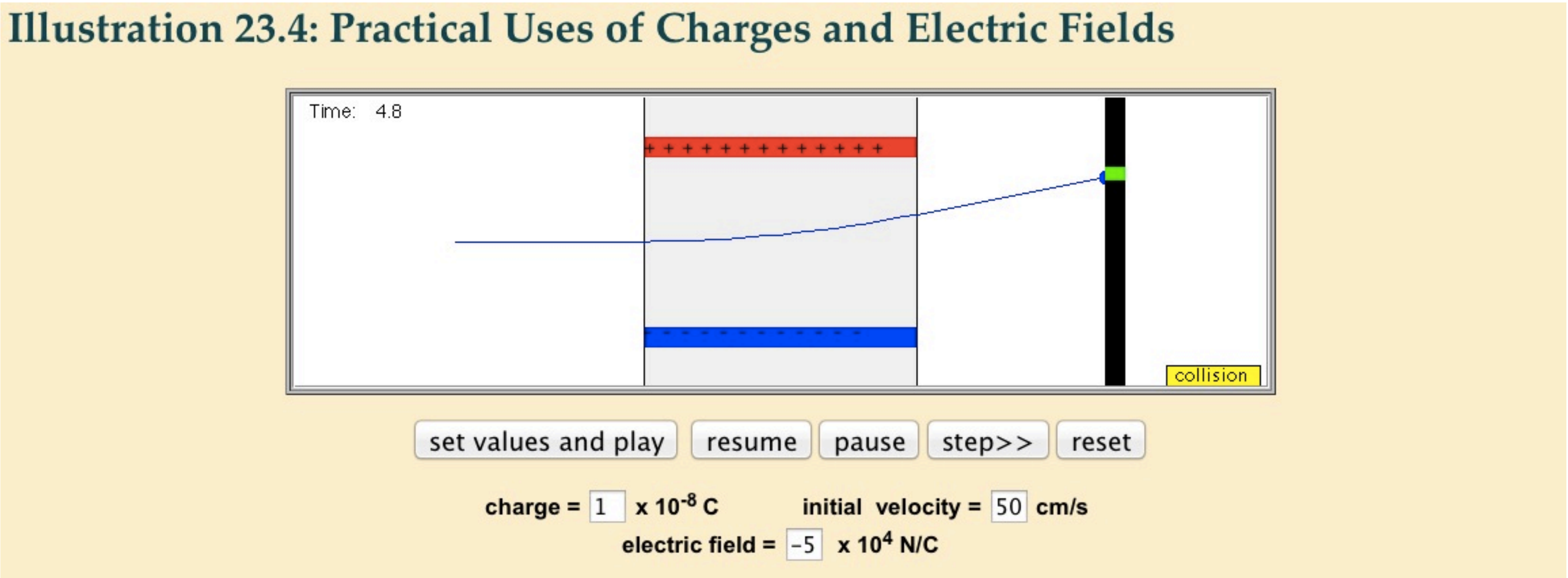


$$\vec{\mathbf{F}}_e = \vec{\mathbf{E}}q$$

$$m\vec{\mathbf{a}} = \vec{\mathbf{E}}q$$

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{E}}q}{m}$$

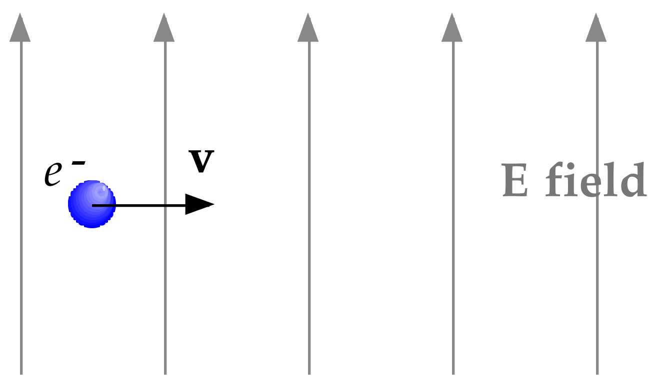
Motion of Charge in E Field



Physlet I.23.4

Example 9

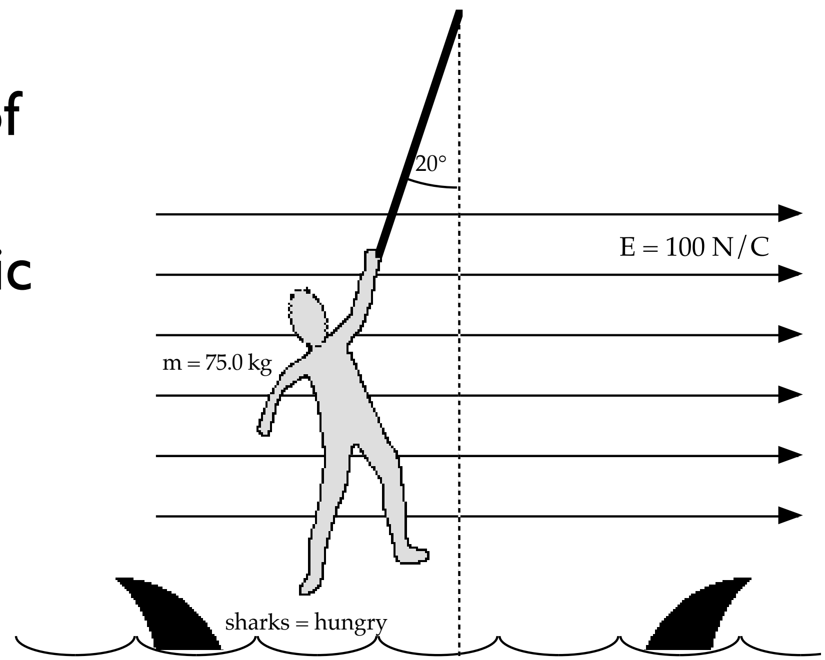
An electron is fired as shown, with v_o=3.0e6 m/s. The **E** field has a strength of 200 N/C, and the width of the field, from left to right, is 0.100m. Find...



a. the acceleration of the e^- in the field
b. how much time it takes to pass through the field.
c. the vertical displacement of the electron while in the field.
d. the final velocity of the e^- as it leaves the field.

Example 10

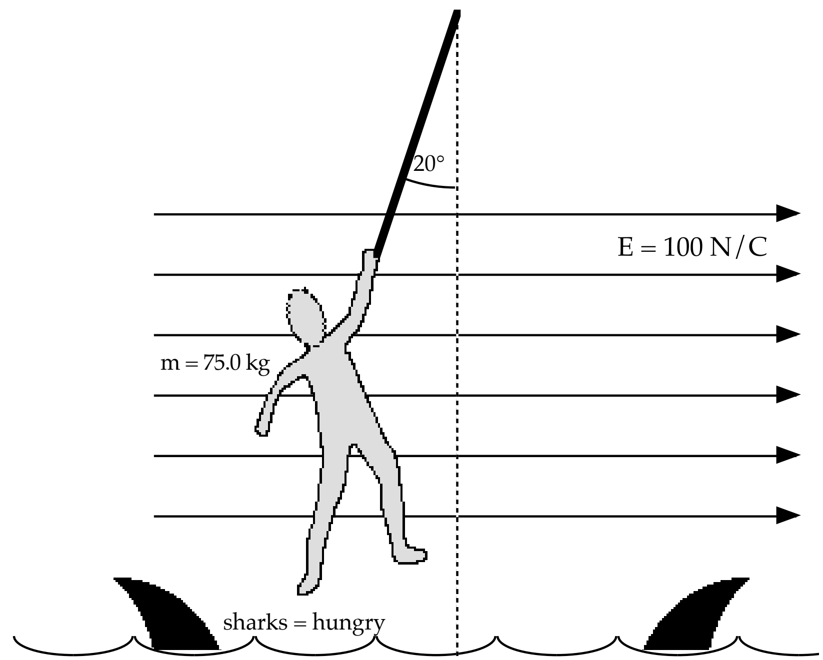
Secret Agent 008 is in a rather sticky situation. A climbing rope of negligible mass suspends our hero over a pool of hungry sharks. The space above the pool is permeated by a uniform electric field of 100 N/C. Luckily, Agent 008 is wearing a special Adjustable Electric Charge Body Suit prepared for him by the ingenious minds at the Imperial Research and Development Lab.



a) In order to stay alive long enough to devise an escape plan, 008 must select a charge for his suit that will enable him to keep the rope at a minimum angle of 20° to the left of vertical, as shown in the drawing above. If he weighs 75 kg, what is the magnitude and polarity of the charge Agent 008 should set the suit for?

Example 10

b) Agent 008 accidentally selects a charge of +10.0 Coulombs for his body suit. To what angle (relative to the vertical) does his body swing now?



c) Unfortunately, Agent 008's rope has been weakened--when the rope reaches this new angle, it snaps. What was the tension in the rope just before it broke?