CHAPTER 23:
Electrostatics n Static Electricity

Charge

What do you know about the *atom*?

1.) *Its nucleus* is made up of electrically neutral *neutrons* and electrically positive *protons*.

2.) *On its outskirts* in "orbitals" associated with states of energy reside electrically negative *electrons*.

3.) *Outer shell electrons,* called *valence electrons*, determine how atoms will combine with one another to make molecules. Bonding types are:

a.) *Covalent bonding* happens when *valence electrons* are shared between adjacent atoms. Covalent bonding does not allow electrons to migrate throughout a structure. It is characteristic of *insulators*.

b.) *Metallic bonding* happens when *valence electrons* can migrate freely throughout a structure. It is characteristic of *conductors*.

c.) *Ionic bonding* happens when atoms have an unbalanced number of protons and electrons, hence are electrically charged. This kind of bonding is not going to be a player in the materials we will be considering here.

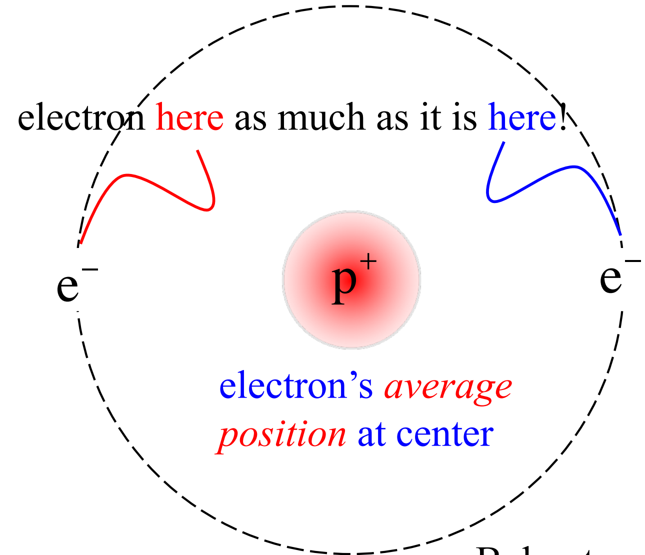
In an electrically neutral atom:

The average position of its protons is where?

(In the nucleus at the center of the atom!)

The average position of its electrons is where?

(Even though the electrons are "moving" around the nucleus at speeds up to 14,000 miles/second, their AVERAGE position is at the center of the atom where reside the protons.)

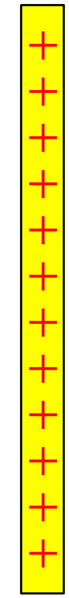


Bohr atom

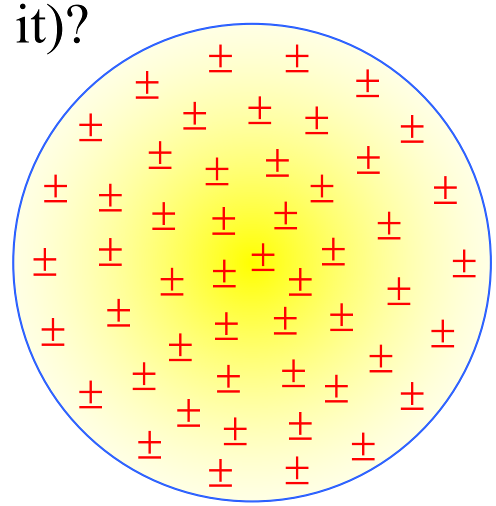
This is why normal atoms are electrically neutral. Essentially covering every positive charge in an electrically neutral atom there sits a negative charge to null the positive out.

Although there will be time when we talk about the movement of *positive charge*, in reality, protons are fixed stationary in the nucleus with electrons ALWAYS being the entities that move onto or off of an object. A *positively charge object* is just an object that has had electrons removed; a *negatively charged object* an object upon which free electrons have been added.

A glass rod, which easily releases *electrons*, is rubbed vigorously by a silk clothe, which easily accepts electrons. This leaves the rod electrically positive.



Consider an electrically neutral conducting sphere (notice that for each proton, there is an electron covering it)?

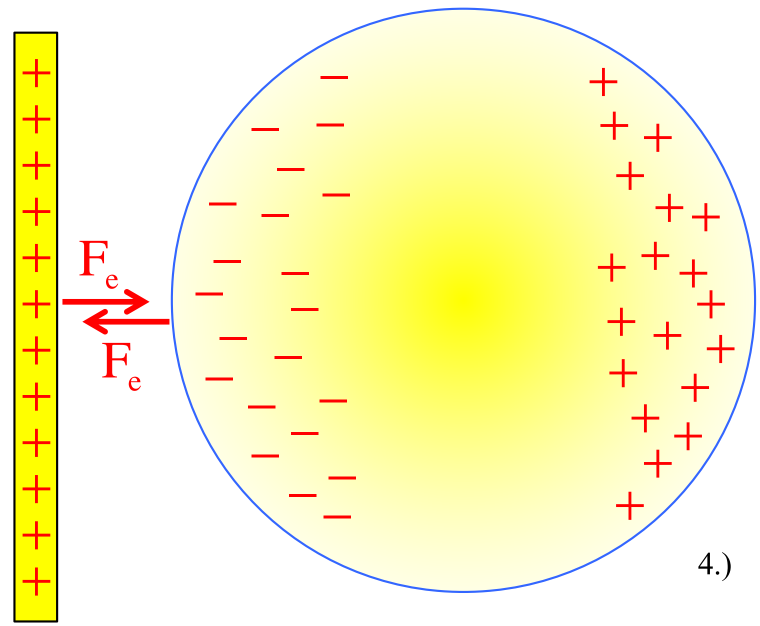


What happens when the two are brought together?

The ball's electrons, which are free to move because the bonding is metallic, will migrate toward the *positive charge on the rod* leaving the far side of the ball *electrically positive* and the near side *electrically negative* (I've omitted all the neutral combinations).

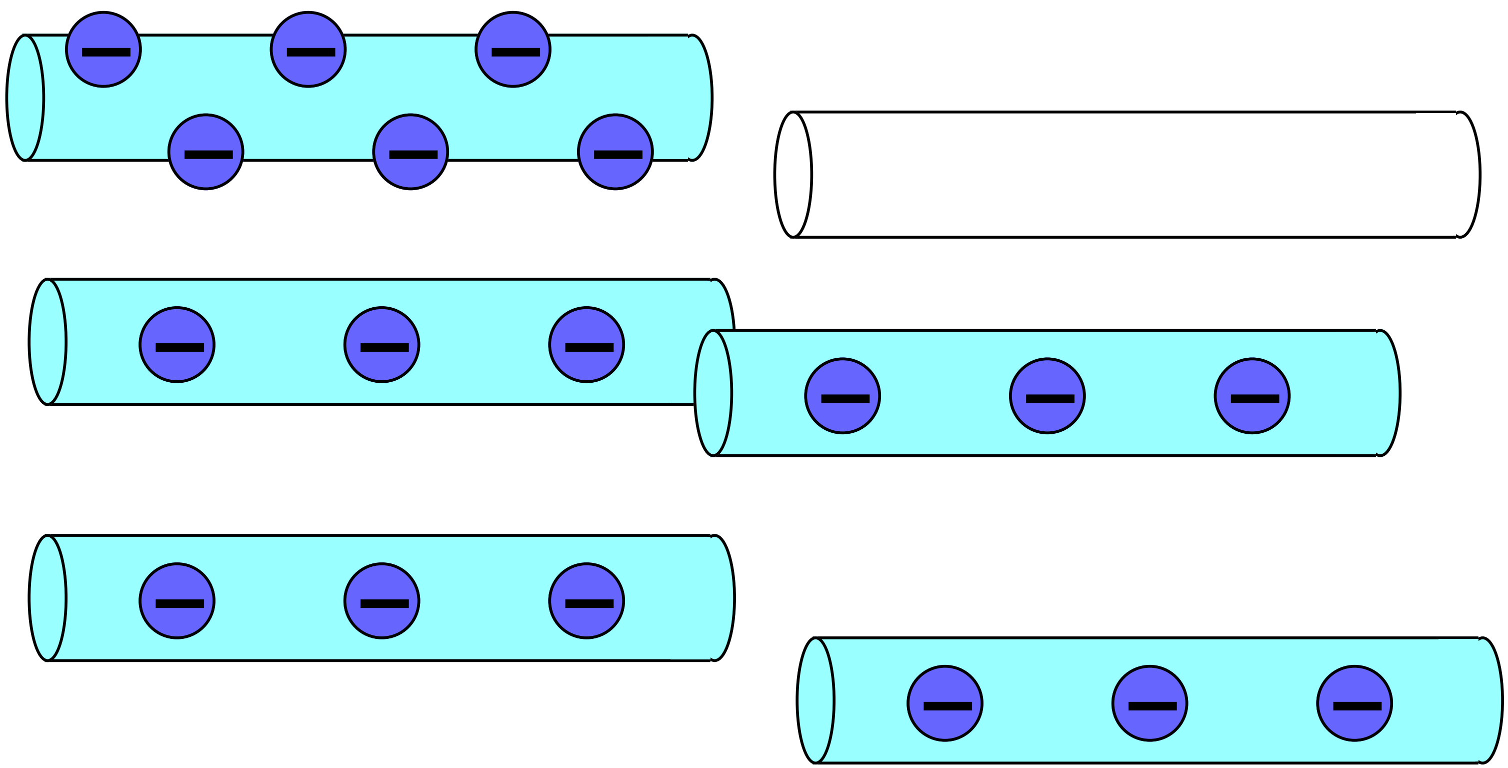
Charge separation like this is called *polarization*.

With this, the electrical repulsion between the rod and ball's far-side positive charge will be overcome by the electrical attraction between the rod and the ball's near-side negative charge, and a net force of attraction will exist.

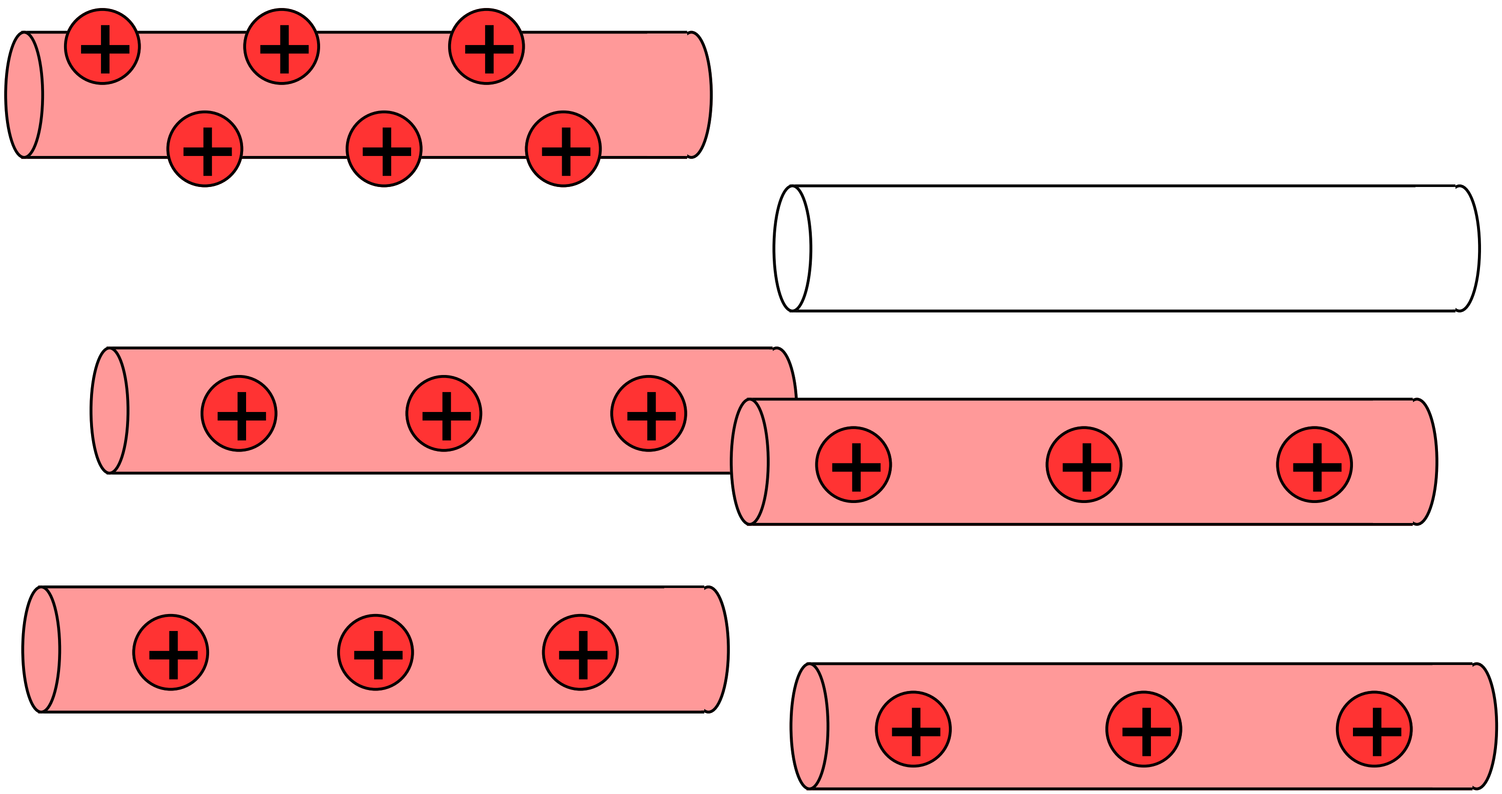


Mr. White's Charging by Conduction Slide 1

"Charging by conduction" occurs when a charged conductor (metal) touches a neutral conductor: some free electrons pass from one object to the other.



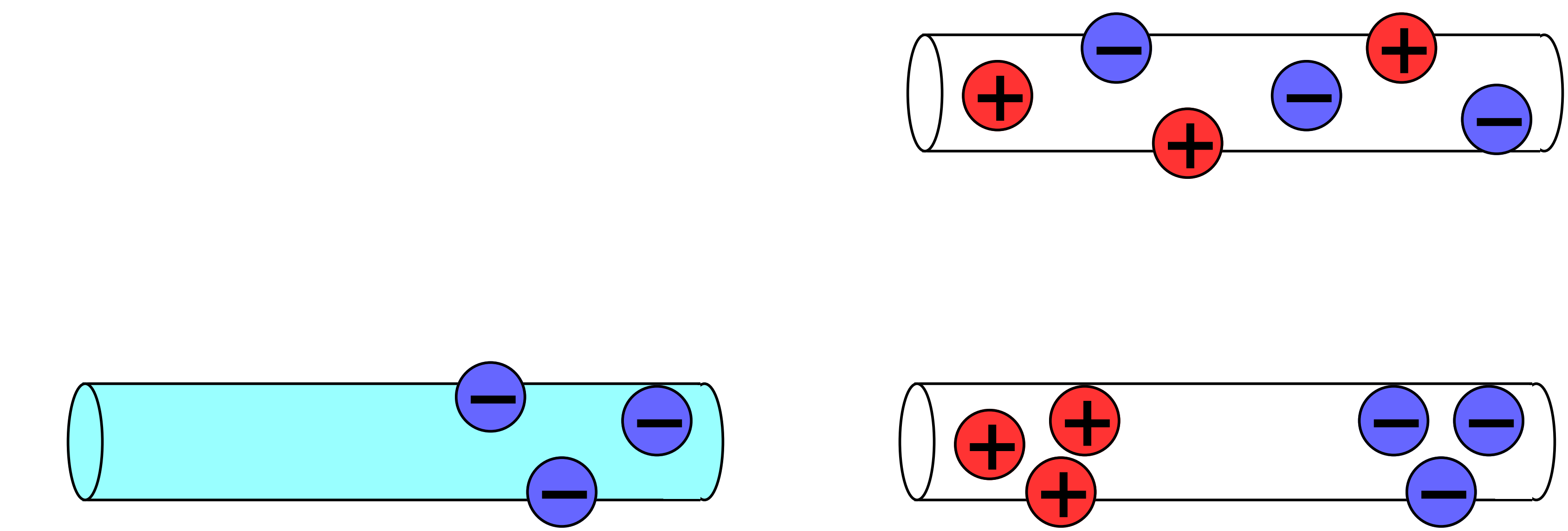
Mr. White's Charging by Conduction Slide 2



Note from Fletch: If both rods are conductors and they are identical, the charge will distribute evenly (as shown) between the two.

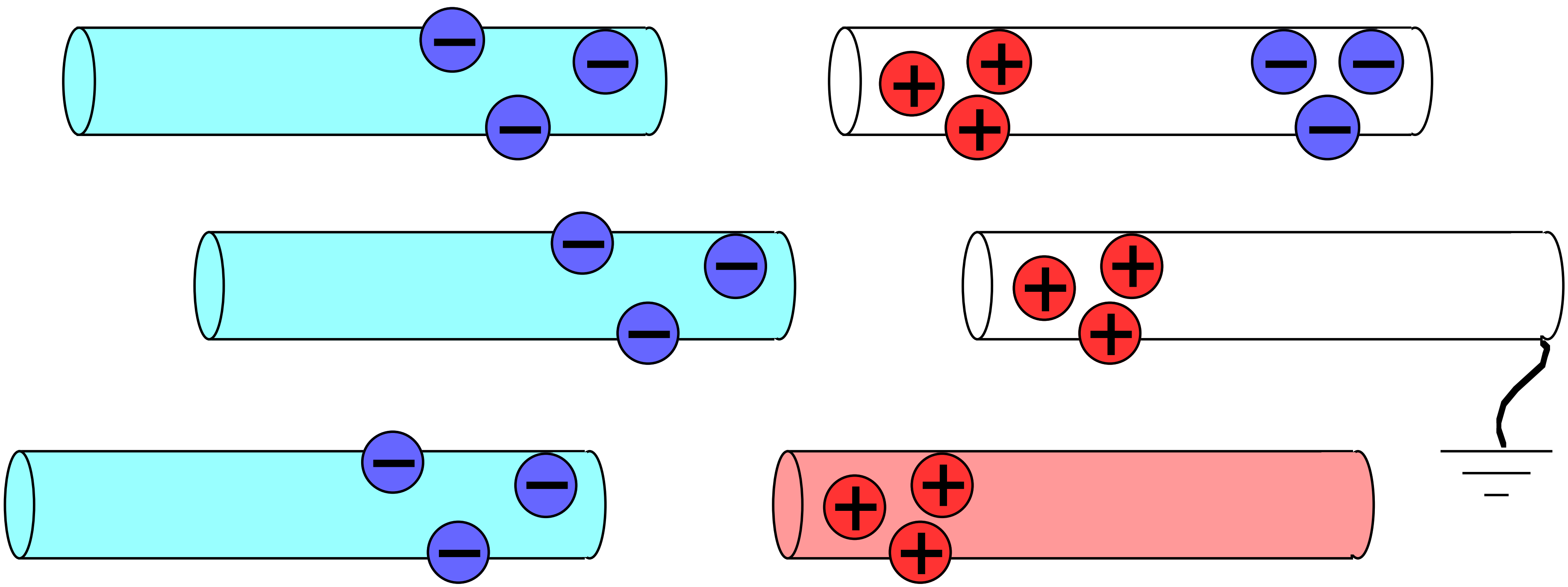
Mr. White's Charging by Induction Slide 1

"Charging by *induction*" occurs when a charged object is brought near a neutral conductor.



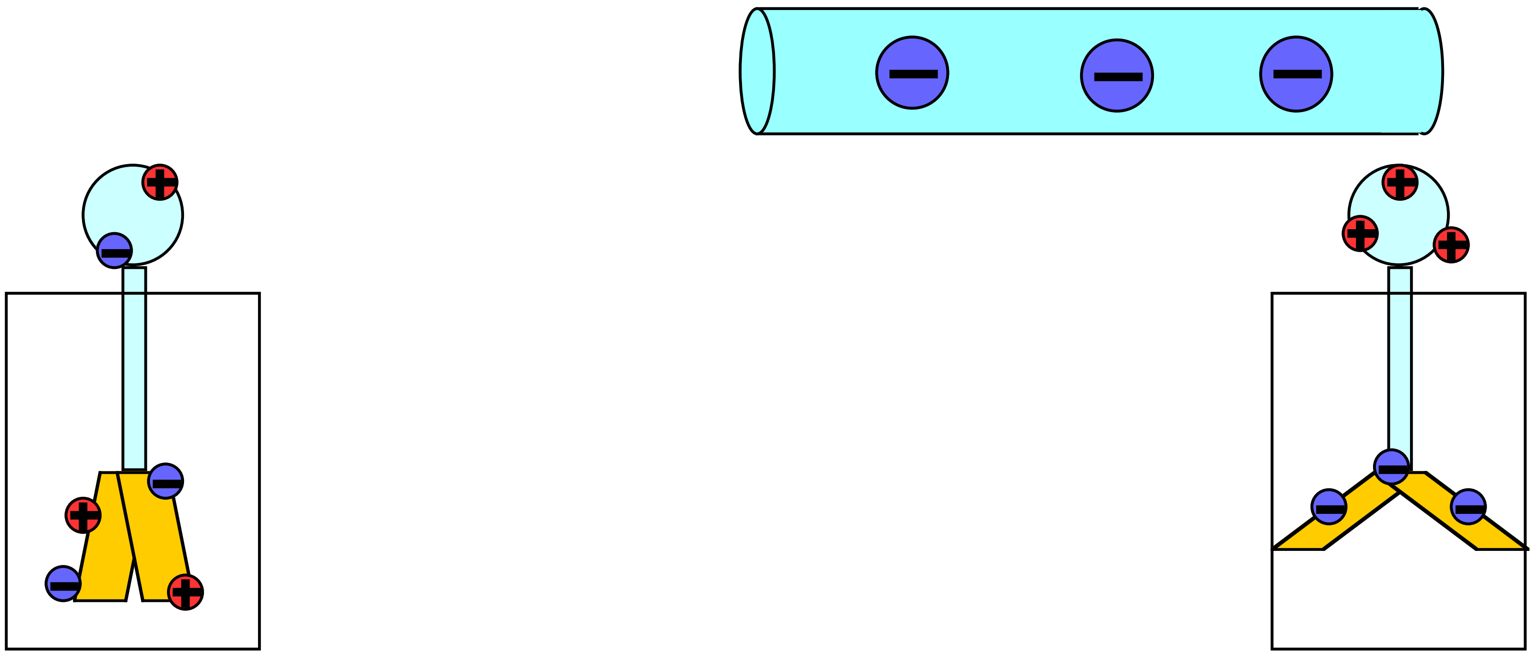
Mr. White's Grounding Slide

The term *grounding* refers to connecting a conductor to the literal ground, ie. the earth. The earth readily accepts or gives up electrons—it has plenty to spare—so grounding a conductor allows for the flow of charges. What effect this has depends on the situation.



Mr. White's Electroscope

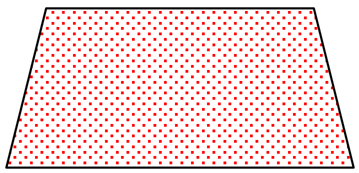
The *electroscope* is a simple device designed to detect the presence of electric charges. Movable leaves (of gold?!) connected to a metal ball separate when a charged object is brought near, or touched to the ball. But why?



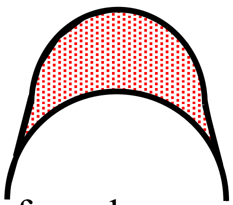
Shielding

When free charge is placed on a conductor, electrical repulsion will motivate electrons to move as far away from other electrons as possible. Consequence:

Force electrons onto a flat conducting surface. At some point, the free electron population already on the surface will provide such a large repulsive force that no additionally placed electrons will make it onto the surface. When that happens, the electrons will be evenly distributed over the surface.

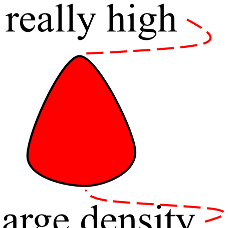


Bend the surface and you can force MORE electrons on, increasing the *surface charge density*. Why? Because there is now material between the electrons, diminishing their repulsive effect on distance electrons. This phenomenon is called SHIELDING.



Oddly shaped conductors will have different charge densities, depending upon the severity of their curvature.

The extreme: the lightning rod, a pointed piece of metal insulated from a house. It accumulates HUGES amount of charge at its end point, attracting potential lightning strikes away from the house.



Coulomb's Law

When two point charges are in the vicinity of one another, they will feel a Newton's Third Law, action/reaction electrical force that will be attractive if the charges are unlike and repulsive if the charges are like.

According to Coulomb, the magnitude of that force will be proportional to the product of the magnitude of the two charges (the symbol for charge is *q*) and inversely proportional to the square of the distance between the two charges.

The proportionality constant is usually defined in terms of another constant called the permittivity of free space ε_o (this is a measure of a vacuum's resistance to forming an electric field—something we'll talk about later). Numerically,

$$\varepsilon_o = 8.85\text{x}10^{-12}\,\text{C}^2/\left(\text{N}\bullet\text{m}^2\right)$$

So mathematically, Coulomb's Law is written as: $\left|\vec{F}_C\right| = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{q_1 q_2}{r^2}$

As $\left(\frac{1}{4\pi\varepsilon_o}\right) = 9\text{x}10^9\,\text{N}\bullet\text{m}^2/\text{C}^2$, this is sometimes defined as *k* and Coulomb's Law is written in some books as: $\left|\vec{F}_C\right| = k\frac{q_1 q_2}{r^2}$

Important Points

Coulomb's Law only works between two *POINT CHARGES*. If you want to deal with more than two charges, you need to execute a vector sum of all the *Coulomb forces* acting on a charge due to *all the point charges in the system*.

Coulomb's Law is only useful in *static situations*. Charge in motion is a whole other kettle of fish.

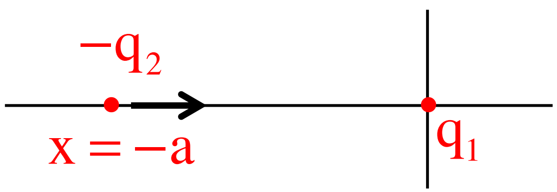
The unit for charge in the MKS system is the *COULOMB*. In that unit:

There is $-1.6\text{x}10^{-19}$ Coulombs worth of charge on an electron;

There is $+1.6\text{x}10^{-19}$ Coulombs worth of charge on a proton;

NEVER include a *charge's sign* when using Coulomb's Law. The direction of force generated by a point charge on another point charge, is determined by *attraction* or *repulsion* between the charges (i.e., whether they are *like* or *unlike* charges) and how the coordinate axis is set up. Beyond determine likeness or unlikeness, the sign of the charges has nothing to do with direction in a coordinate axis setting! To wit:

Example 1: A positive charge q_1 is located at the origin. A negative charge $-q_2$ is located a distance *-a* units along the x-axis. What is the force on q_2 due to the presence of q_1?

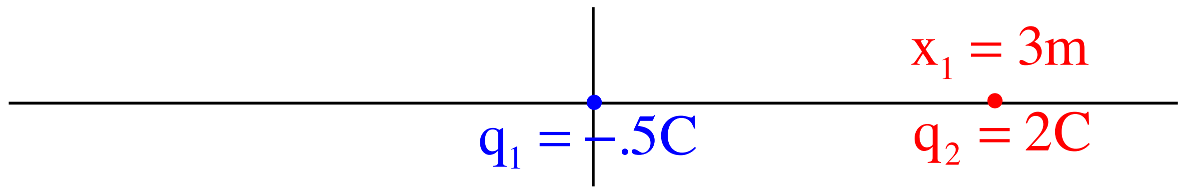


According to Coulomb's Law: $\vec{F}_C = \left(\frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{a^2}\right)(\hat{i})$

Why in the +x-direction? Because opposites attract, and the direction of attraction on q_2 in this case is to the right! If it had been located at $x = a$, the magnitude would have been the same but the direction would have been in the –x-direction!

<u>REGISTER</u>! If you'd included q_2's sign in the *Coulomb expression*, you'd have ended up with a force value that was NEGATIVE, which would have made NO SENSE given the coordinate set-up. USE COULOMB ONLY FOR MAGNITUDES!

Example 2: Consider the charge configuration shown. Where could you put a positive charge q if it is to feel no net electric force?



As a small side point, it doesn't matter whether q is positive or negative—zero force is zero force!

Observation: The point must be along the x-axis. But where?

It can't be between the two charges because the positive charge will push q to the left while the negative charge will also pull q to the left . . . hence no place in between where the net force can be zero.

It can't be in the positive region to the right of the origin because the 2C charge will be both larger and closer to q than will be the case for the -.5C charge . . . hence no place in that region where the net force can be zero.

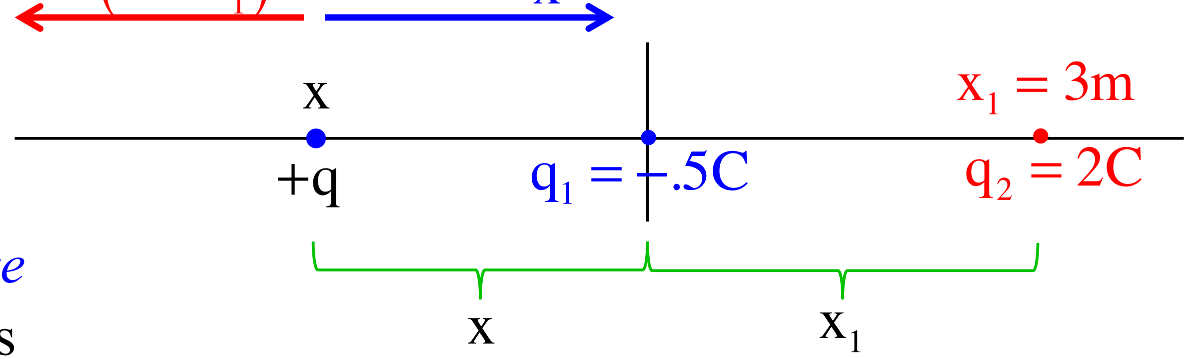
Apparently, the coordinate must be in the *-x-region* to the left of the origin.

Defining x to be the no-force position of *q*, we can write Coulomb's Law between *q* and each of the charges:

Noticing that the *positive charge* is producing a negative force (it's repulsing *q*), and the *negative charge* a positive force, signs need to be added after the fact.

Summing these forces vectorially, we get:

$$\left|\vec{F}_2\right| = k\frac{qq_2}{\left(x+x_1\right)^2} \qquad \left|\vec{F}_1\right| = k\frac{qq_1}{x^2}$$



$\sum F_x:$

$$-\cancel{k}\frac{\cancel{q}q_2}{\left(x+x_1\right)^2} + \cancel{k}\frac{\cancel{q}q_1}{x^2} = m\cancel{a_x}^{\,0}$$

$$\Rightarrow \quad \frac{q_2}{\left(x+x_1\right)^2} = \frac{q_1}{x^2}$$

$$\Rightarrow \quad \frac{(2C)}{(x+3)^2} = \frac{(.5C)}{x^2}$$

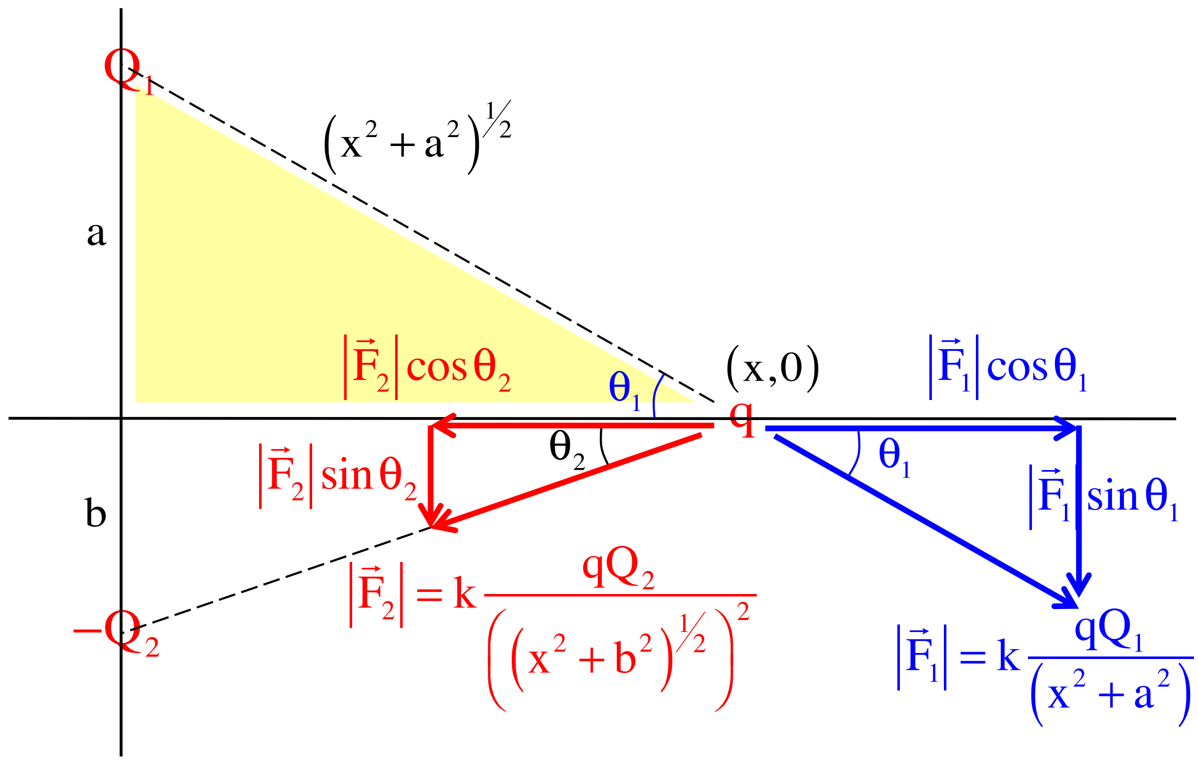
$$\Rightarrow \quad 2x^2 = .5\left(x^2+6x+9\right)$$

$$\Rightarrow \quad -1.5x^2 + 3x + 4.5 = 0$$

$$\Rightarrow \quad x = 3m$$

Example 3: Determine the force on q.

Begin by determining the direction and magnitude of the force on q due to the presence of each of the charges in the system.



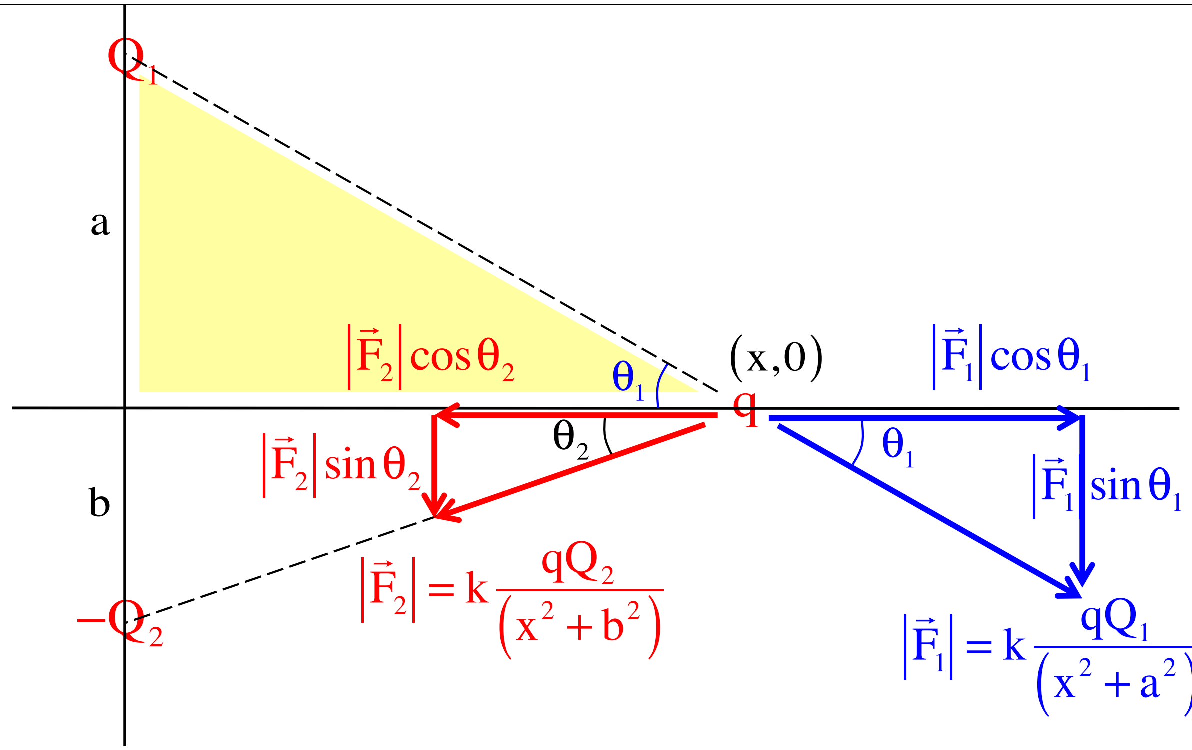
To add the forces vectorially, we need to break the forces into components.

$$\vec{F}_C = \left(|F_1|\cos\theta_1 - |F_2|\cos\theta_2\right)(\hat{i}) + \left(-|F_1|\sin\theta_1 - |F_2|\sin\theta_2\right)(\hat{j})$$

At this point, we need a little trickery. Looking at the shaded triangle, the sine and cosine can be written as:

$$\sin\theta_1 = \frac{a}{(x^2+a^2)^{1/2}} \quad \text{and} \quad \cos\theta_1 = \frac{x}{(x^2+a^2)^{1/2}}$$

Putting it all together:

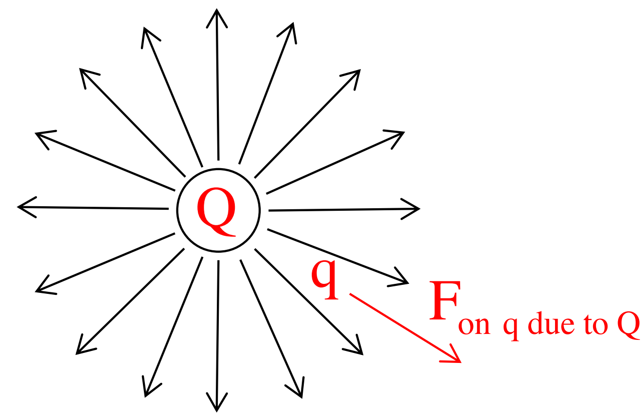


$$\vec{F}_C = \left(|F_1| \cos\theta_1 - |F_2| \cos\theta_2 \right)\left(\hat{i}\right) + \left(-|F_1|\sin\theta_1 - |F_2|\sin\theta_2 \right)\left(\hat{j}\right)$$

$$= \left(\frac{1}{4\pi\varepsilon_o}\frac{qQ_1}{\left(x^2+a^2\right)}\left(\frac{x}{\left(x^2+a^2\right)^{1/2}}\right) - \frac{1}{4\pi\varepsilon_o}\frac{qQ_2}{\left(x^2+b^2\right)}\left(\frac{x}{\left(x^2+b^2\right)^{1/2}}\right)\right)\left(\hat{i}\right) + \ \ldots$$

$$= \left(\frac{1}{4\pi\varepsilon_o}\frac{qQ_1 x}{\left(x^2+a^2\right)^{3/2}} - \frac{1}{4\pi\varepsilon_o}\frac{qQ_2 x}{\left(x^2+b^2\right)^{3/2}} \right)\left(\hat{i}\right) + \left(-\frac{1}{4\pi\varepsilon_o}\frac{qQ_1 a}{\left(x^2+a^2\right)^{3/2}} - \frac{1}{4\pi\varepsilon_o}\frac{qQ_2 b}{\left(x^2+b^2\right)^{3/2}} \right)\left(\hat{j}\right)$$

Electric Fields



Place a "field producing" point-charge out in space.

That charge will produce an *electrical disturbance* in the region around it.

To detect that disturbance, place a test charge q in the region. As long as the disturbance is there, the test charge will feel a force on it.

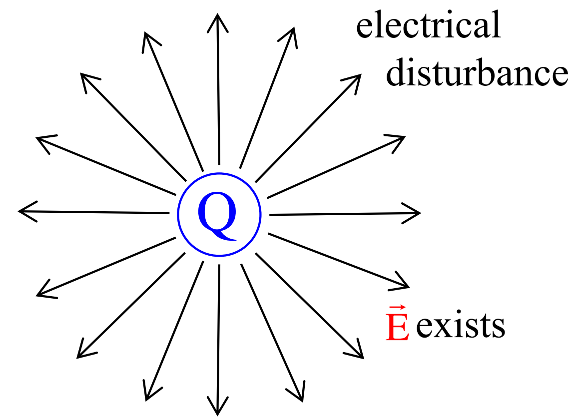
If you measure the force on the *test charge q,* THEN DIVIDE BY THE MAGNITUDE OF THE TEST CHARGE, you end up with a force-related quantity that is dependent upon:

The size of the field producing charge Q, and

The distance from the field producing charge Q.

It will have NOTHING TO DO with the size of the test charge.

Called an ELECTRIC FIELD, this idea of measuring the amount of *force per unit charge AVAILABLE* at a point due to the presence of *field-producing* charge can be (and is) associated with *any* charge configuration.



An ELECTRIC FIELDs exist wherever there is charge. For the electric field to exist, there doesn't need to be present a *secondary charge* to feel the effect of the field-producing charge. And because *E-flds* tell us how much force is available *PER UNIT CHARGE* at a point, the electric field is defined as:

$$\vec{E} = \frac{\vec{F}}{q}$$

Example 3: What force does a 2 C charge feel in an electric field $\vec{E} = (3\ \text{N/C})\hat{j}$?

$$\vec{E} = \frac{\vec{F}}{q} \quad \Rightarrow \quad \vec{F} = q\vec{E}$$

$$= (2\ \text{C})\left[(3\ \text{N/C})\hat{j}\right] = (6\ \text{N})\hat{j}$$

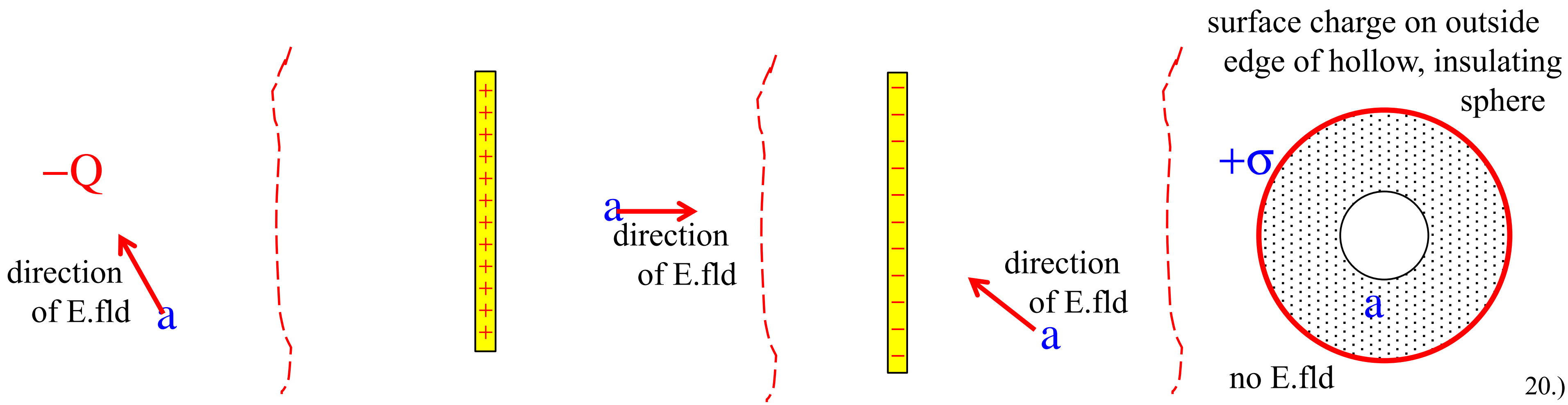
Subtleties Concerning Electric Fields

An electric field, being a *modified force field*, is a vector. So how is its direction defined?

The direction of an electric field is defined as the direction a *POSITIVE TEST CHARGE* will accelerate if released in the field at the point of interest.

DON'T GET AHEAD OF YOURSELF on this. You will see how it all plays out shortly. For now, just take in the definition.

Example 4: At point *a* designated in each of the scenarios below, draw the direction of the electric field generated by the field-producing charge configuration.



Continuing with Subtleties

By definition, positive charges freely accelerate in the *direction* of *electric fields*. As such, negative charges must freely accelerate OPPOSITE the direction of *electric fields*.

So whereas we DIDN'T use a charge's sign when using $\left|\vec{F}_C\right| = k\,q_1 q_2 / r^2$, leaving the force's sign determination to attraction/repulsion characteristics coupled with where the charge was in the coordinate system, we NEED to include charge sign in using $\vec{F} = q\vec{E}$. To see why, reconsider Example 3.

Example 5: So back to *Example 3,* but with a twist. What force does a -2 C charge feel in an electric field $\vec{E} = (3\text{ N/C})\,\hat{j}$?

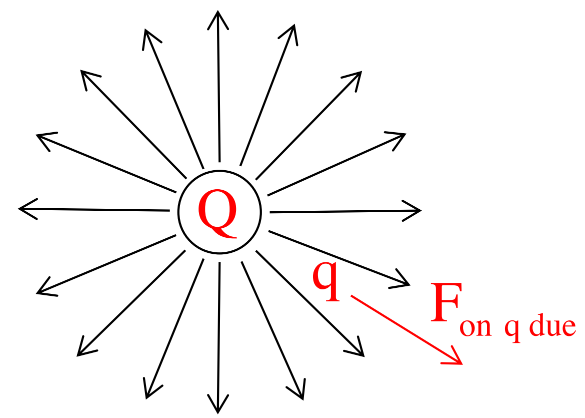
When the charge was positive, the calculation yielded $\vec{F} = (6\text{ N})\,\hat{j}$, so we'd expect the solution to this problem to be *negative* that value. But how to get that negative. Although this is REALLY NOT KOSHER, *IF we use the sign of the charge* in our relationship, we get the right answer (not Kosher because the *sign of a charge* has nothing to do with the sign of a force). Using that, we can write:

$$\vec{F} = q\vec{E} = (-2\text{ C})\left[(3\text{ N/C})\,\hat{j}\right] = -(6\text{ N})\,\hat{j}$$

Point taken?

A Specific Case--The Electric Field Generated by a POINT CHARGE

Example 6: Derive a general expression for the *electric field* generated by a point charge Q?



If we placed a positive test charge q a distance r units from Q, the magnitude of the force on q (according to Coulomb's Law) would be:

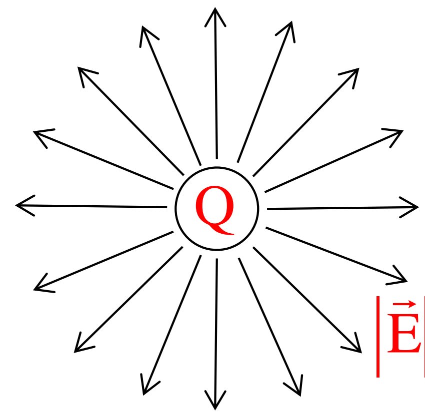
$$\left|\vec{F}_C\right| = k\frac{qQ}{r^2}$$

By definition, the magnitude of the electric field is $\left|\vec{E}\right| = \frac{\left|\vec{F}\right|}{q}$, so:

$$\left|\vec{E}\right| = \frac{\left|\vec{F}\right|}{q} = \frac{\left(k\frac{\cancel{q}Q}{r^2}\right)}{\cancel{q}}$$

$$= k\frac{Q}{r^2}$$

Observations:



$$\left|\vec{E}\right| = k\frac{Q}{r^2}$$

If you are ever working with a field-producing charge that is a POINT CHARGE, you now know its *electric field* will equal:

$$\left|\vec{E}\right| = k\frac{Q}{r^2}$$

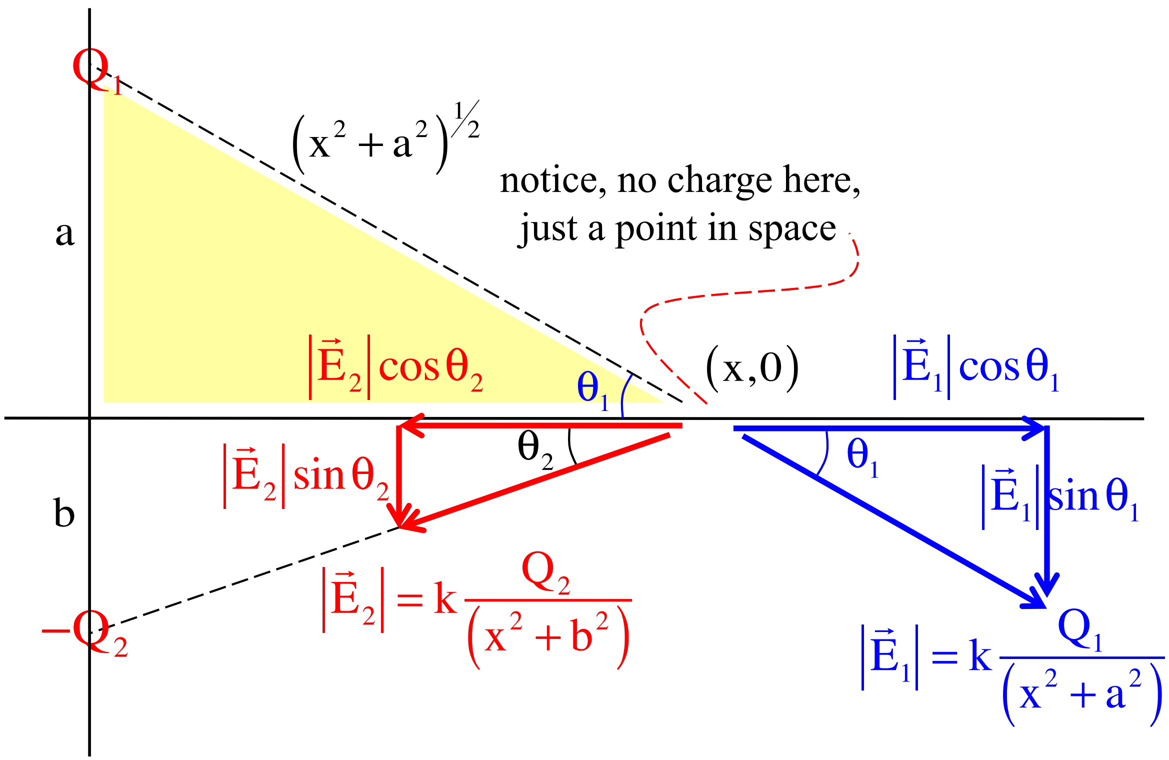
When talking theoretically, the *direction* of this field will be radially outward if the field-producing charge is positive (a positive *test charge* will accelerate away from a positive *field-producing charge*—that's how electric fields are defined . . . the direction a positive test charge will accelerate if put in the field and released) and radially inward if the *field-producing charge* is negative. As such, with the sign of the charge included (again, not kosher, but it gives the correct sign for the vector), the *$\vec{E}$-fld* as a vector is often presented as:

$$\vec{E} = \left(k\frac{Q}{r^2}\right)\hat{r}$$

For most problem solving, though, we work in Cartesian coordinates. In that setting, use the relationship to determine the *magnitude* of the *E-fld* and determine the *direction* using the "acceleration of positive test charge" question.

Example 7: Derive an expression for the electric field at (x,0).

This is very similar to Example 3 (and because the charge q was positive in that problem, the forces on it and the direction of the electric fields will even be the same). The difference? No need to include the test charge q, just work with E:

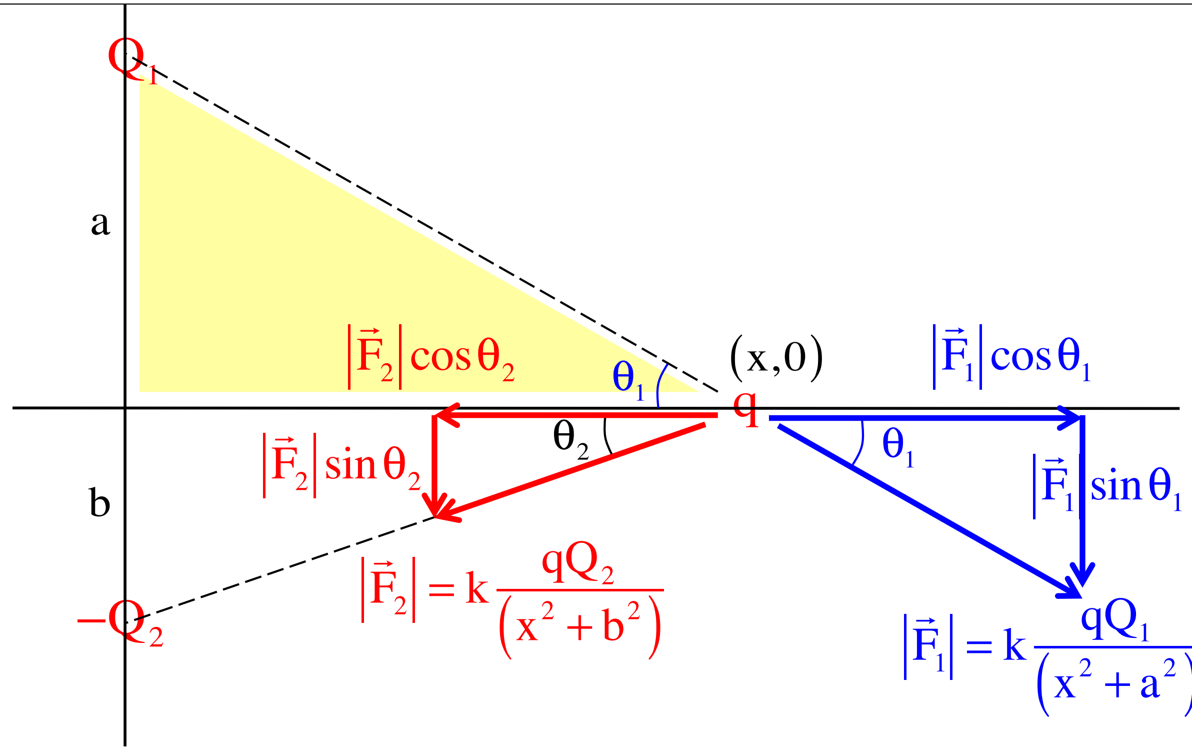


Defining the field directions and magnitudes, then break into components:

$$\vec{E} = \left(\left|\vec{E}_1\right|\cos\theta_1 - \left|\vec{E}_2\right|\cos\theta_2\right)\left(\hat{i}\right) + \left(-\left|\vec{E}_1\right|\sin\theta_1 - \left|\vec{E}_2\right|\sin\theta_2\right)\left(\hat{j}\right)$$

You'd use the same trickery ($\sin\theta_1 = \dfrac{a}{\left(x^2+a^2\right)^{1/2}}$) and finish the problem just like before.

Putting it all together:

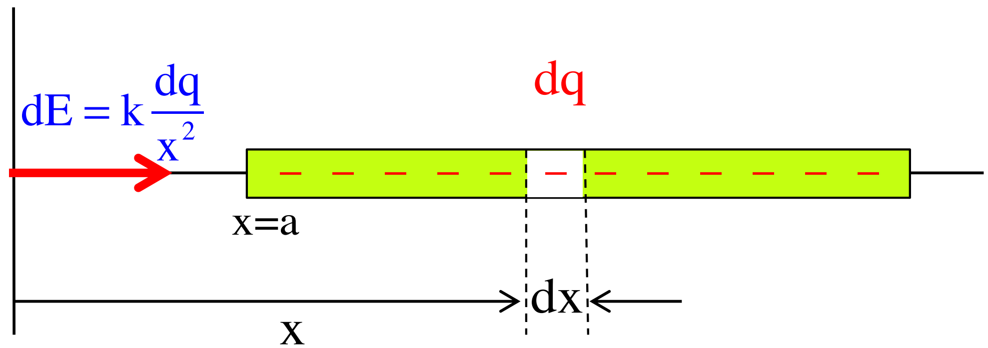


$$\vec{E} = \left(|E_1| \cos\theta_1 - |E_2| \cos\theta_2 \right)(\hat{i}) + \left(-|\vec{E}_1| \sin\theta_1 - |E_2| \sin\theta_2 \right)(\hat{j})$$

$$= \left(\frac{1}{4\pi\varepsilon_o} \frac{Q_1}{(x^2+a^2)} \left(\frac{x}{(x^2+a^2)^{1/2}} \right) - \frac{1}{4\pi\varepsilon_o} \frac{Q_2}{(x^2+b^2)} \left(\frac{x}{(x^2+b^2)^{1/2}} \right) \right)(\hat{i}) + \ \ldots$$

$$= \left(\frac{1}{4\pi\varepsilon_o} \frac{Q_1 x}{(x^2+a^2)^{3/2}} - \frac{1}{4\pi\varepsilon_o} \frac{Q_2 x}{(x^2+b^2)^{3/2}} \right)(\hat{i}) + \left(-\frac{1}{4\pi\varepsilon_o} \frac{Q_1 a}{(x^2+a^2)^{3/2}} - \frac{1}{4\pi\varepsilon_o} \frac{Q_2 b}{(x^2+b^2)^{3/2}} \right)(\hat{j})$$

Example 8: Derive an expression for the electric field at the origin due to a rod with charge *-Q* uniformly distributed over its length *L*.



Being an extended charge distribution, we need to break the rod into a differentially small bit of charge, determine the electric field magnitude and direction as it exists at the origin due to that bit of charge, then sum that differential electric field over the entire rod. To do this, we will need to define a *linear charge density* function $\lambda = Q/L$ (yes, this looks just like the *linear mass density* function . . . that's OK, we can use the same symbol to do the more than one thing):

As always, the *density function* can be written in *differential terms* as $\lambda = dq/dx$. With that, we have:

$$\left|\vec{E}\right| = \int dE = \int_{x=a}^{a+L} \frac{1}{4\pi\varepsilon_o}\frac{dq}{x^2}$$

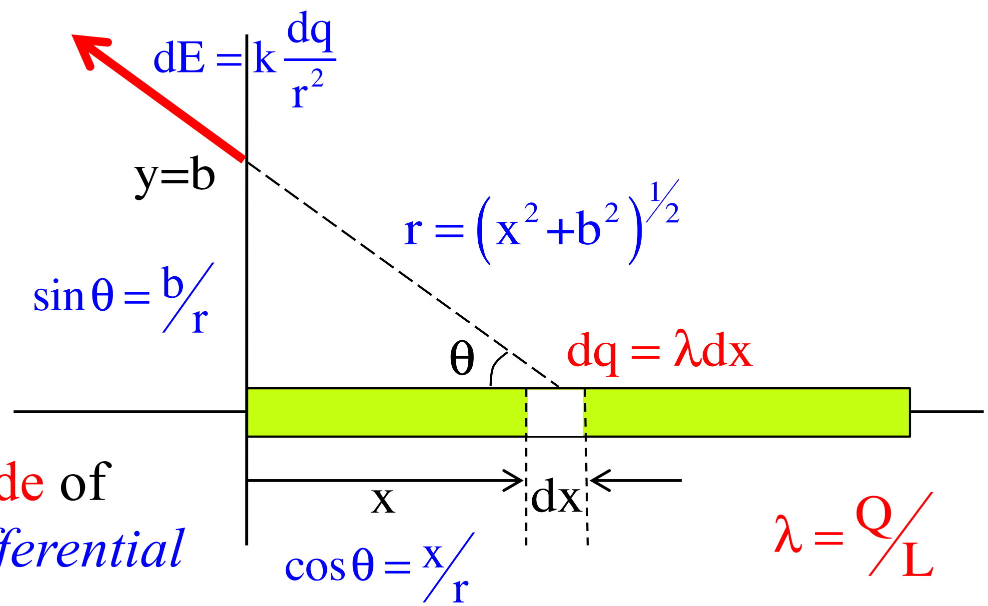
$$= \int_{x=a}^{a+L} \frac{1}{4\pi\varepsilon_o}\frac{(\lambda dx)}{x^2} = \frac{\left(Q/L\right)}{4\pi\varepsilon_o}\int_{x=a}^{a+L} x^{-2}\,dx$$

$$= \frac{\left(Q/L\right)}{4\pi\varepsilon_o}\left(-x^{-1}\Big|_{x=a}^{a+L}\right) = \frac{Q}{4\pi\varepsilon_o L}\left[\left(-\frac{1}{a+L}\right) - \left(-\frac{1}{a}\right)\right]$$

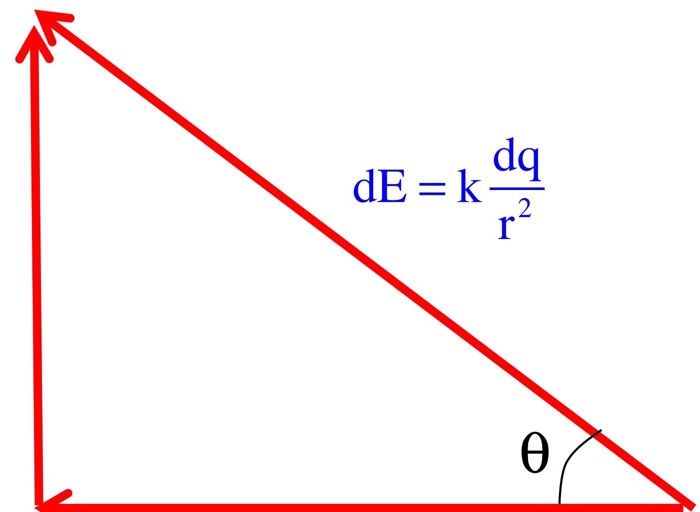
$$= \frac{Q}{4\pi\varepsilon_o \cancel{L}}\left(\frac{(\cancel{a}+\cancel{L})-\cancel{a}}{a(a+L)}\right) = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{a(a+L)}\right)$$

Example 9 (a non-AP question):

Derive an expression for the electric field at an arbitrary point $y = b$ on the y-axis due to a rod with charge Q uniformly distributed over its length L.



As usual, define the direction and magnitude of the *differential E-fld* due to an arbitrary *differential charge dq* on the rod. Once done, break it into component parts using the *sine* and *cosine* trick:



$$dE_y = k\frac{dq}{r^2}\sin\theta = k\frac{dq}{r^2}\left(\frac{b}{r}\right)$$

$$dE_x = k\frac{dq}{r^2}\cos\theta = k\frac{dq}{r^2}\left(\frac{x}{r}\right)$$

Without doing the integrals (the set-up is what is important, and something like this isn't something the AP folks are going to throw at you), the electric field is:

$$\vec{E} = \int dE_x\left(-\hat{i}\right) + \int dE_y\left(\hat{j}\right)$$

$$= \frac{1}{4\pi\varepsilon_o}\left[\int_{x=0}^{L} \frac{(\lambda dx)}{\left(\left(x^2+b^2\right)^{1/2}\right)^2} \frac{x}{\left(x^2+b^2\right)^{1/2}}\left(-\hat{i}\right) + \int_{x=o}^{L} \frac{(\lambda dx)}{\left(\left(x^2+b^2\right)^{1/2}\right)^2} \frac{b}{\left(x^2+b^2\right)^{1/2}}\hat{j}\right]$$

$$= \frac{\lambda}{4\pi\varepsilon_o}\left[\int_{x=0}^{L} \frac{x}{\left(x^2+b^2\right)^{3/2}} dx\left(-\hat{i}\right) + \int_{x=o}^{L} \frac{b}{\left(x^2+b^2\right)^{3/2}} dx\left(\hat{j}\right)\right]$$

Example 10: A ring situated in the y-z plane (as shown) has *–Q's* worth of charge on it. Derive an expression for the E-fld at (x,0).

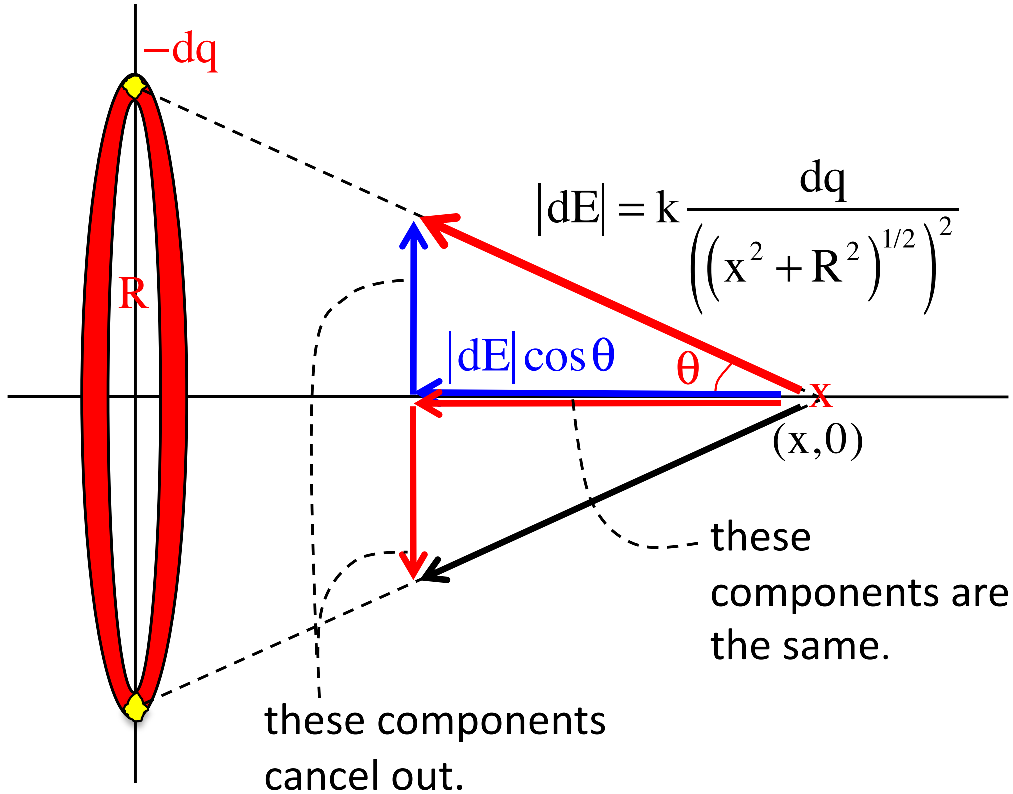
As usual, begin with a differential bit of charge *-dq* and determine the *direction* and *magnitude* of the *differential electric field* at (x,0) due to that bit.

Use any symmetry that exist to simplify the summing task.

Noting that x and *R* are constants, and that the *cosine* is:

$$\cos\theta = \frac{x}{\left(x^2+R^2\right)^{1/2}}$$

we can write:



$$\left|\vec{E}\right| = dE\cos\theta$$

$$= \int\left(k\frac{dq}{\left(x^2+R^2\right)}\right)\left(\frac{x}{\left(x^2+R^2\right)^{1/2}}\right)$$

$$= k\frac{x}{\left(x^2+R^2\right)^{3/2}}\int dq$$

$$= \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{x}{\left(x^2+R^2\right)^{3/2}}Q$$

where the *direction*, by inspection, is $-\hat{i}$

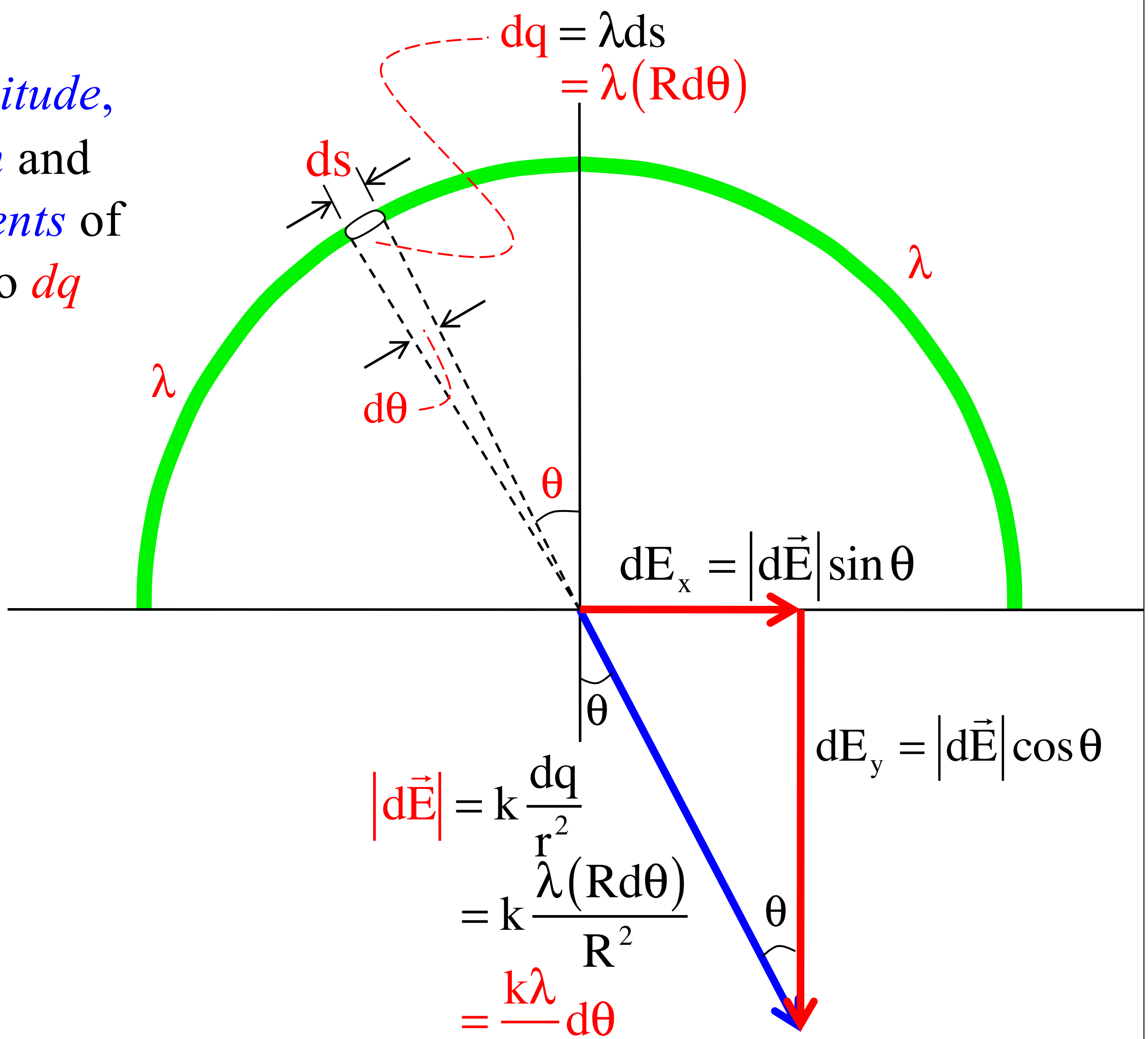
Example 11: Derive an expression for the electric field at the center of a half-hoop of radius *R* with *Q* uniformly distributed over its surface.

This is a little bit tricky as the differential bit of charge *dq* is located on an arc. What needs to be noticed is that the arclength *ds* subtended by an angle $d\theta$ is: $ds = Rd\theta$

Using a linear charge density function, then, we can write: $dq = \lambda ds$

$$= \lambda(Rd\theta)$$

So magnitude, direction and *components* of *dE* due to *dq* become:



$$dE_x = \left|d\vec{E}\right| \sin\theta$$

$$dE_y = \left|d\vec{E}\right| \cos\theta$$

$$\left|d\vec{E}\right| = k\frac{dq}{r^2}$$

$$= k\frac{\lambda(Rd\theta)}{R^2}$$

$$= \frac{k\lambda}{R}d\theta$$

An additional bit of trickery is involved in exploiting the symmetry of the set-up. Notice there is a second *dq* on the right side at an angle θ that will produce a mirror-image *differential electric field* to our original bit of charge. The *x-components* of the two fields will add to zero, so all we really have to deal with is the *y-component*.

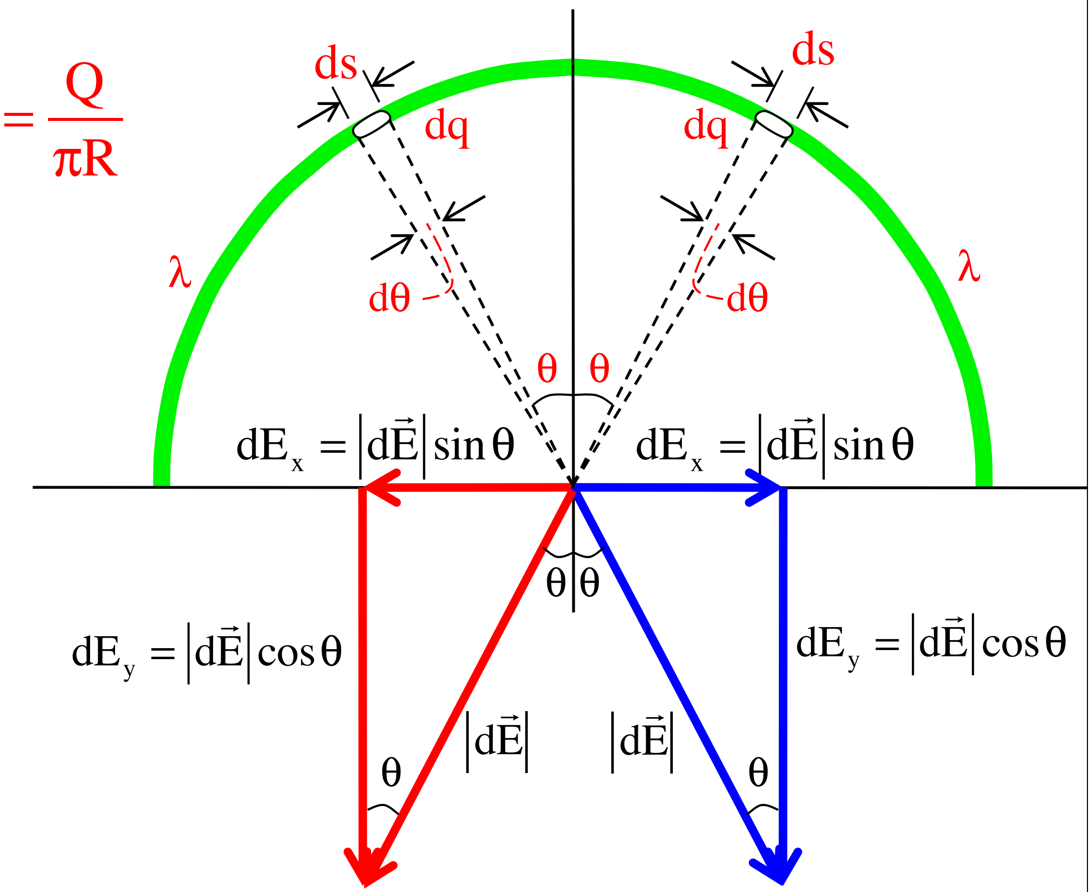
With the *linear charge density* as:

$$\lambda = \frac{Q}{\left(2\pi R/2\right)} = \frac{Q}{\pi R}$$

and *dE* as:

$$dE = \frac{k\lambda}{R}d\theta$$

we can write:



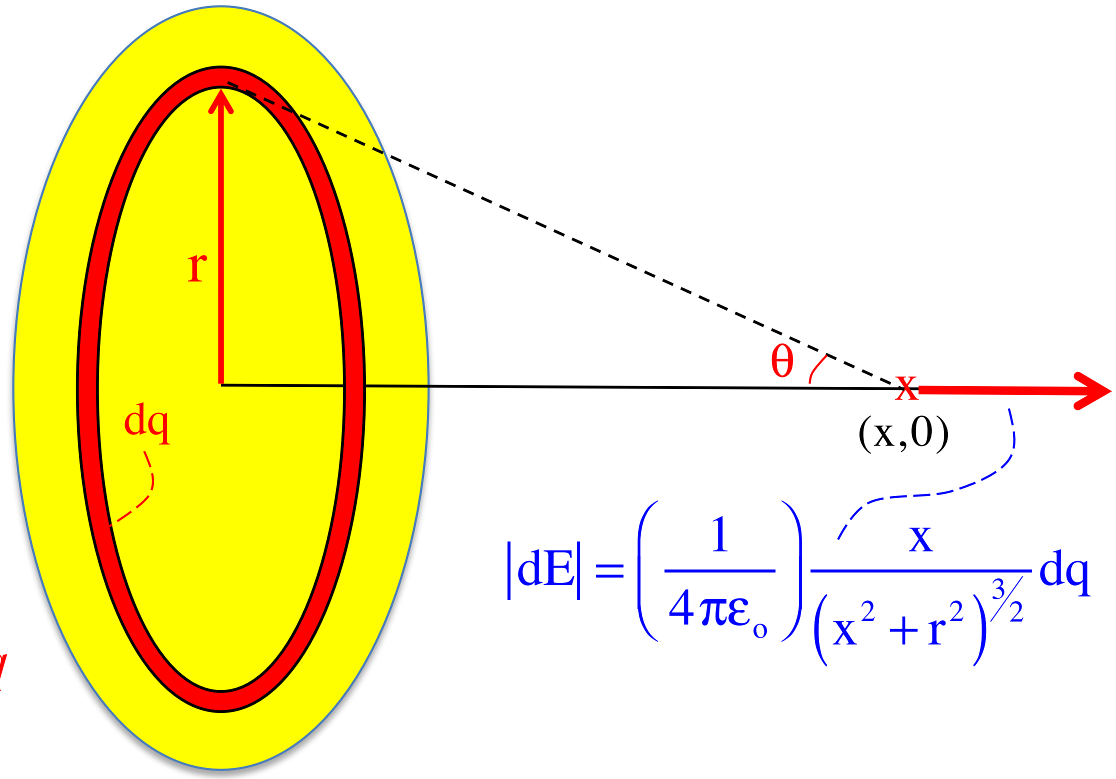
$$\vec{E} = 2\int dE_y\left(-\hat{j}\right) = 2\int dE\cos\theta\left(-\hat{j}\right)$$

$$= \left[-2\left(\frac{1}{4\pi\varepsilon_o}\frac{\lambda}{R}\right)\int_{\theta=0}^{\pi/2}\cos\theta\, d\theta\right]\left(\hat{j}\right)$$

$$= \left[-2\left(\frac{1}{4\pi\varepsilon_o}\frac{\left(Q/\pi R\right)}{R}\right)\sin\theta\Big|_{\theta=0}^{\pi/2}\right]\left(\hat{j}\right)$$

$$= -\frac{Q}{2\pi^2\varepsilon_o R^2}\left(\hat{j}\right)$$

Example 12: A solid disk of radius R situated in the x-z plane (as shown) has Q's worth of charge on it. Derive an expression for the E-fld at (x,0).



$$|dE| = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{x}{\left(x^2+r^2\right)^{3/2}}dq$$

We already know the *E-fld* function along the *x-axis* for a *charged hoop*.

So taking the hoop's charge to be dq and its radius to be r, we can write the magnitude of the *differential E-fld* (in the x-direction) due to a differentially thin hoop on the face of the disk to be:

$$dE = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{x}{\left(x^2+r^2\right)^{3/2}}dq$$

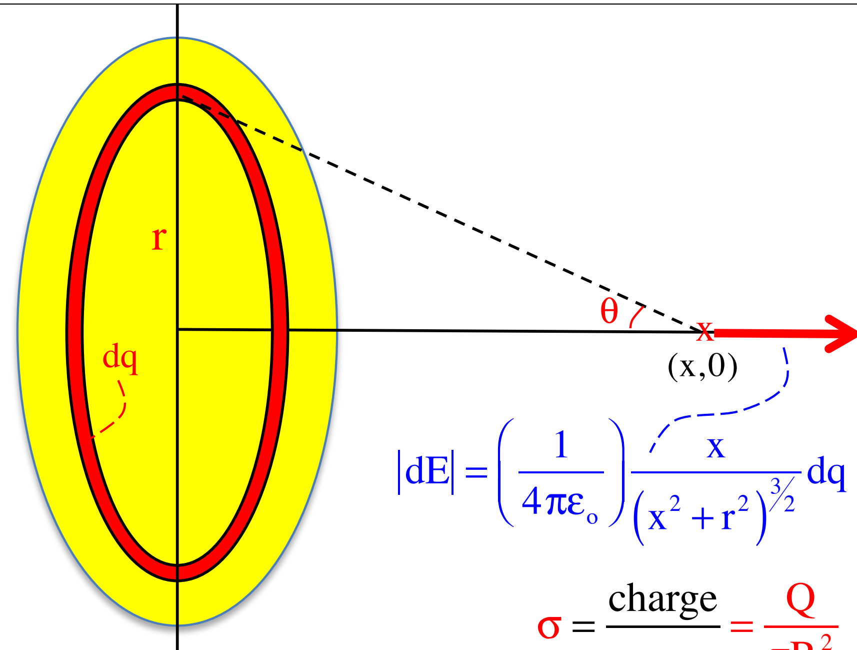
As always, a *surface density function* can be defined as both:

$$\sigma = \frac{\text{charge}}{\text{area}} = \frac{Q}{\pi R^2}$$

and, with the area of a hoop equaling the *circumference of the hoop* times its *thickness*:

$$\sigma = \frac{dq}{dA} = \frac{dq}{(2\pi r)dr} \quad \Rightarrow \quad dq = (2\pi\sigma r)dr$$

Summing over all of the differential hoops yields:



$$|dE| = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{x}{\left(x^2+r^2\right)^{3/2}}dq$$

$$\sigma = \frac{\text{charge}}{\text{area}} = \frac{Q}{\pi R^2}$$

$$dq = (2\pi\sigma r)dr$$

$$|E| = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{x}{\left(x^2+r^2\right)^{3/2}}dq$$

$$= \left(\frac{1}{4\pi\varepsilon_o}\right)\int_{r=0}^{R}\frac{x}{\left(x^2+r^2\right)^{3/2}}(2\pi\sigma r)dr$$

rewriting

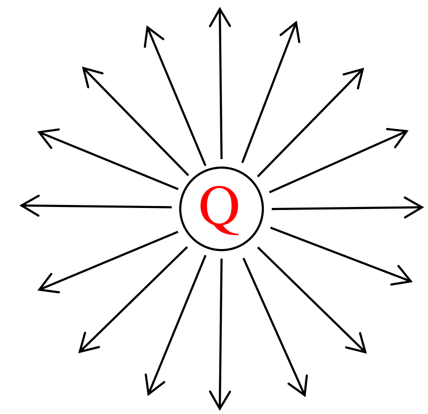
$$= \left(\frac{2\pi x\sigma}{4\pi\varepsilon_o}\right)\int_{r=0}^{R}\frac{r}{\left(x^2+r^2\right)^{3/2}}dr = \left(\frac{2\pi x\sigma}{4\pi\varepsilon_o}\right)\int_{r=0}^{R}r\left(x^2+r^2\right)^{-3/2}dr$$

$$= \left(\frac{2\pi x\sigma}{4\pi\varepsilon_o}\right)(-1)\left(x^2+r^2\right)^{-1/2}\Big|_{r=0}^{R} = \left(\frac{\cancel{2}\cancel{\pi}x\left(\frac{Q}{\cancel{\pi}R^2}\right)}{\underset{2}{\cancel{4}}\pi\varepsilon_o}\right)(-1)\left(\frac{1}{\left(x^2+R^2\right)^{1/2}}-\frac{1}{\left(x^2\right)^{1/2}}\right)$$

$$= \left(\frac{Q}{2\pi R^2\varepsilon_o}\right)(-1)\left(\frac{x}{\left(x^2+R^2\right)^{1/2}}-\frac{x}{x}\right) = \left(\frac{Q}{2\pi R^2\varepsilon_o}\right)\left(1-\frac{x}{\left(x^2+R^2\right)^{1/2}}\right)$$

If you would like to look at more exotic, most-probably non-AP *electric field* problems, look at the Where r You Going PowerPoint on the class Web site.

Electric Fields Lines



electric field lines for positive point charge

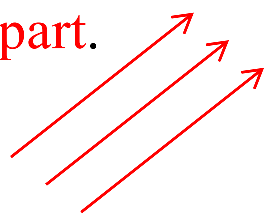
A method to visualize what an *electric field* looks like is wrapped up in what are called *electric field lines*. You saw an example of field lines when I introduced the idea of an electrical disturbance around a charge (see figure to the right). The lines are designed to tell you very specific information about the charge configuration:

As electric field lines move away from positive charge and toward negative charge (remember, the direction of an electric field is defined as the direction a positive test charge would accelerate if released at the point of interest), electric field lines always *leave* positive charges and *enter* negative charges.

The number of line that leave a charge is proportional to the size of the charge.

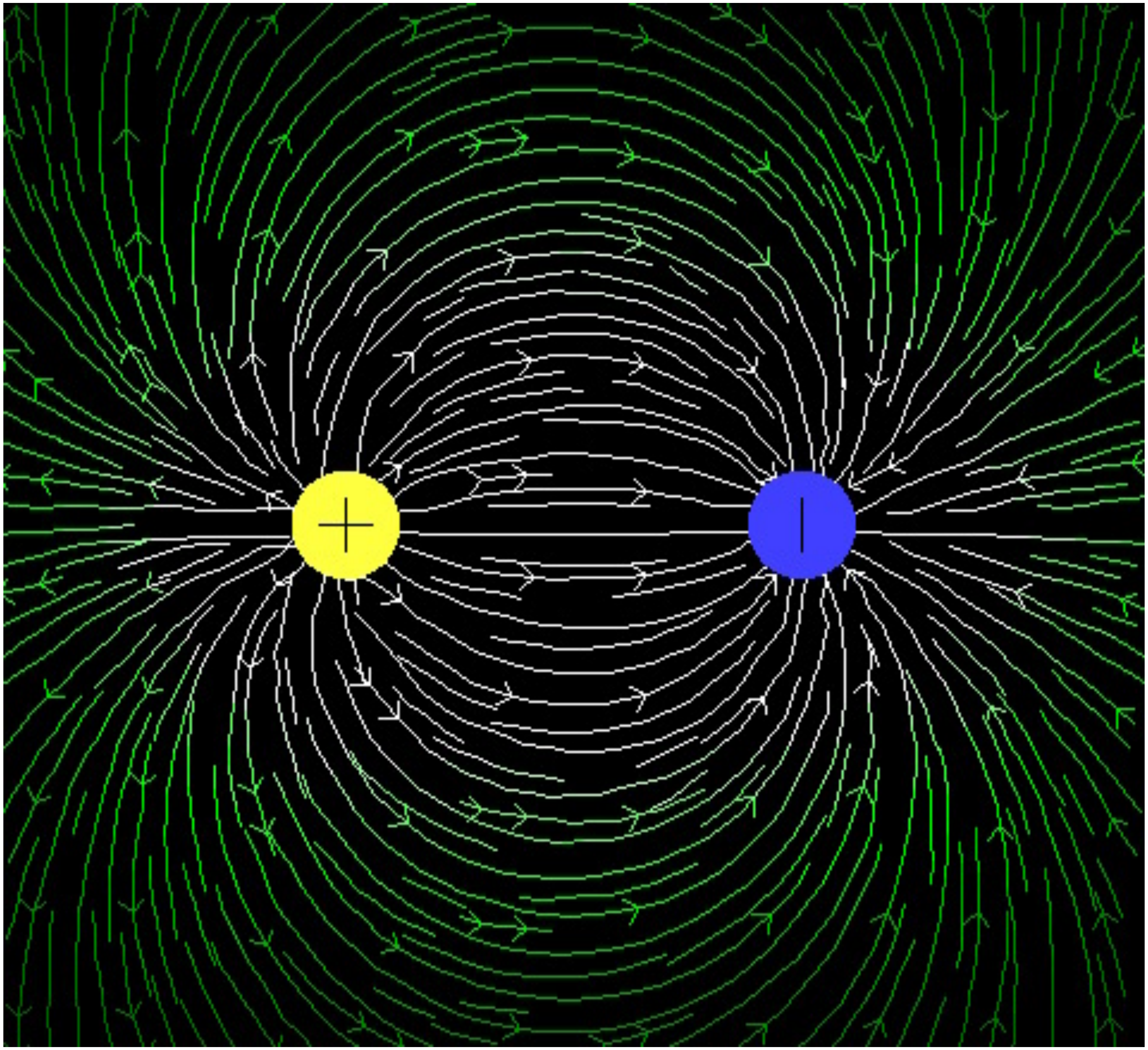
The distance between lines gives you a relative feel for the strength of the field at a particular point—the closer the lines, the stronger the force. That means a constant *E-fld* will have field lines that are *parallel* and *equidistant* apart.

The lines gives you a relative feel for the direction of the field at a given point, and skirt areas where an E.fld is zero.

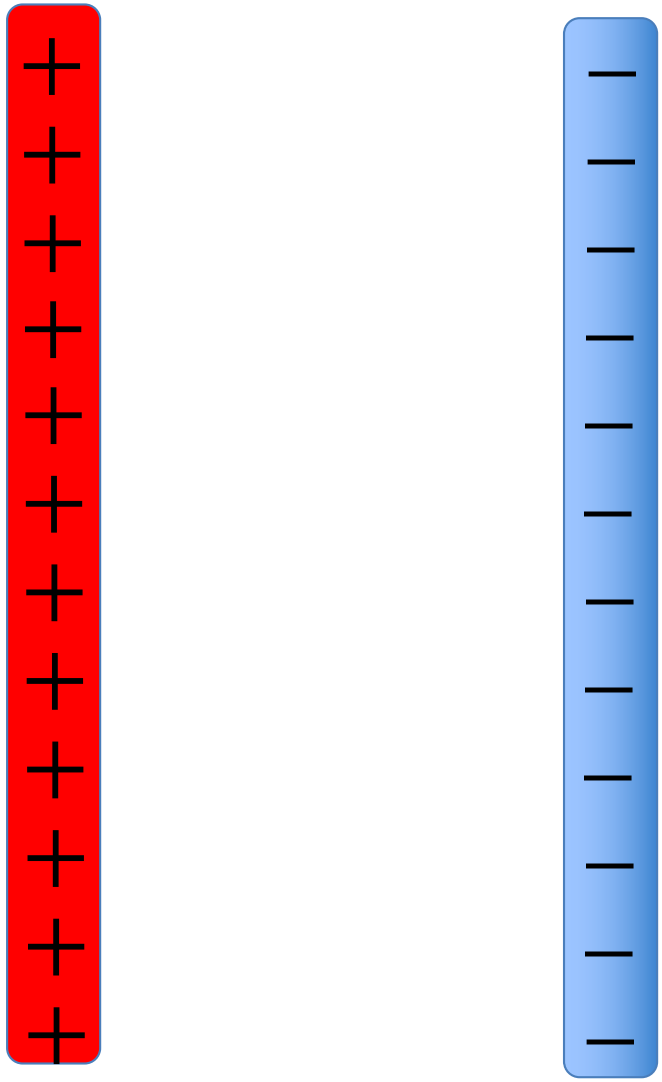

constant E-fld

(next five pictures courtesy of Mr. White)

(courtesy of Mr. White)



38.)



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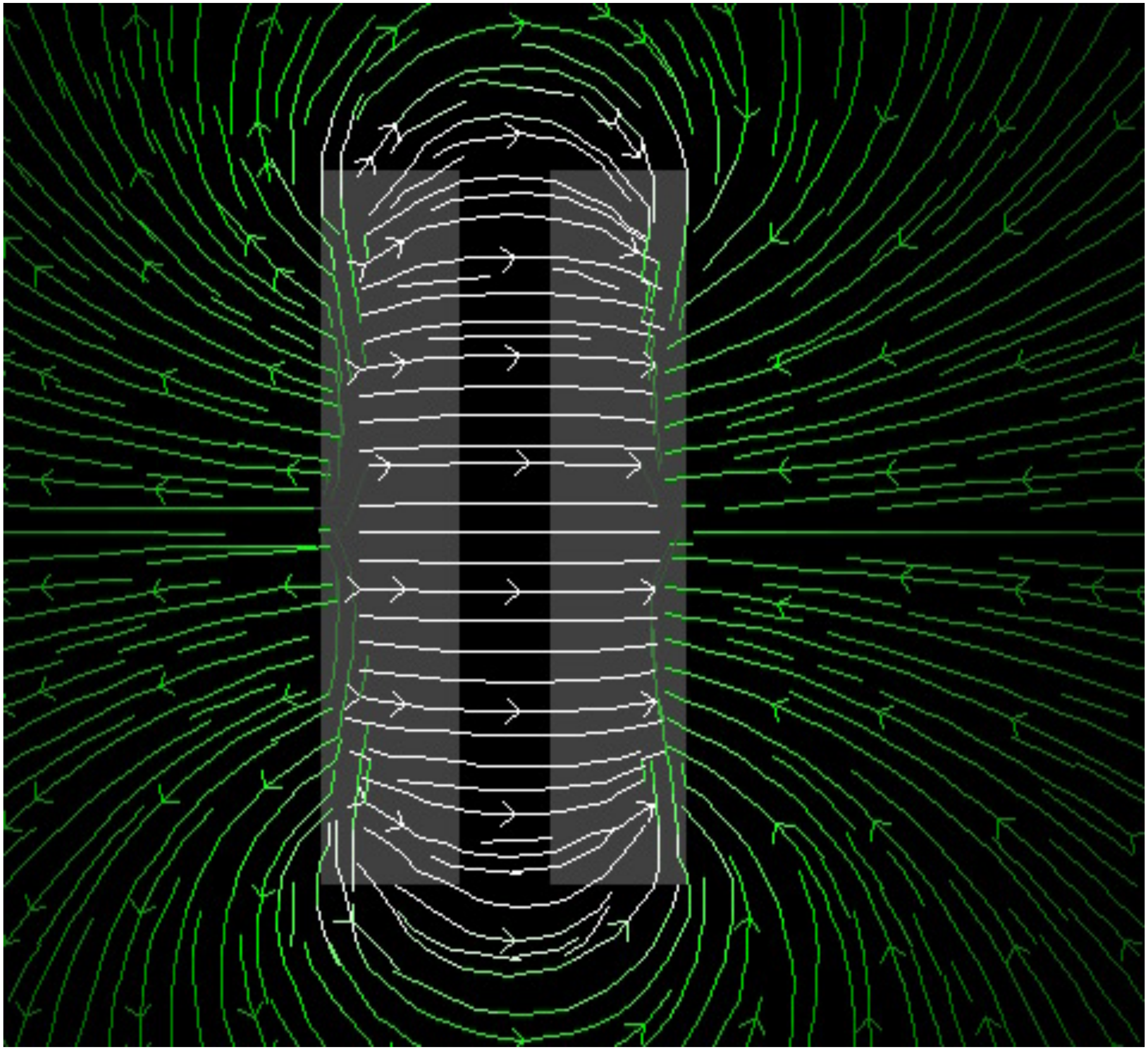
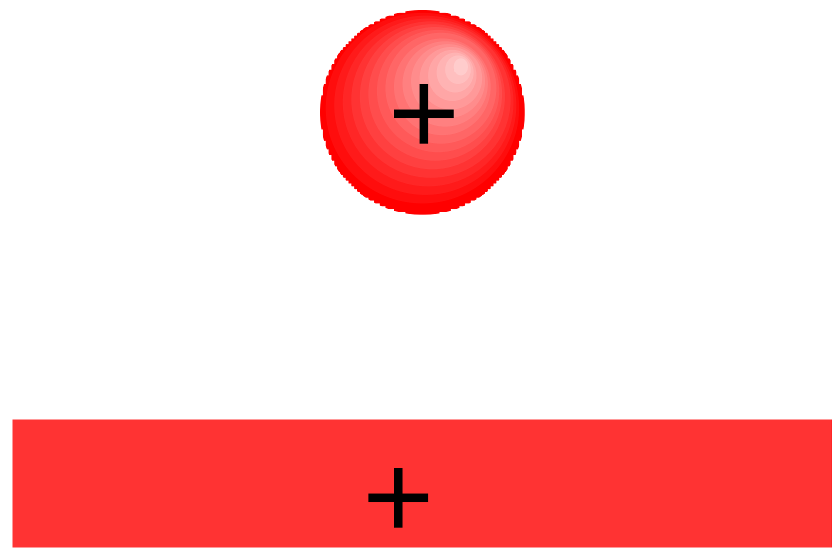


(next five pictures courtesy of Mr. White)



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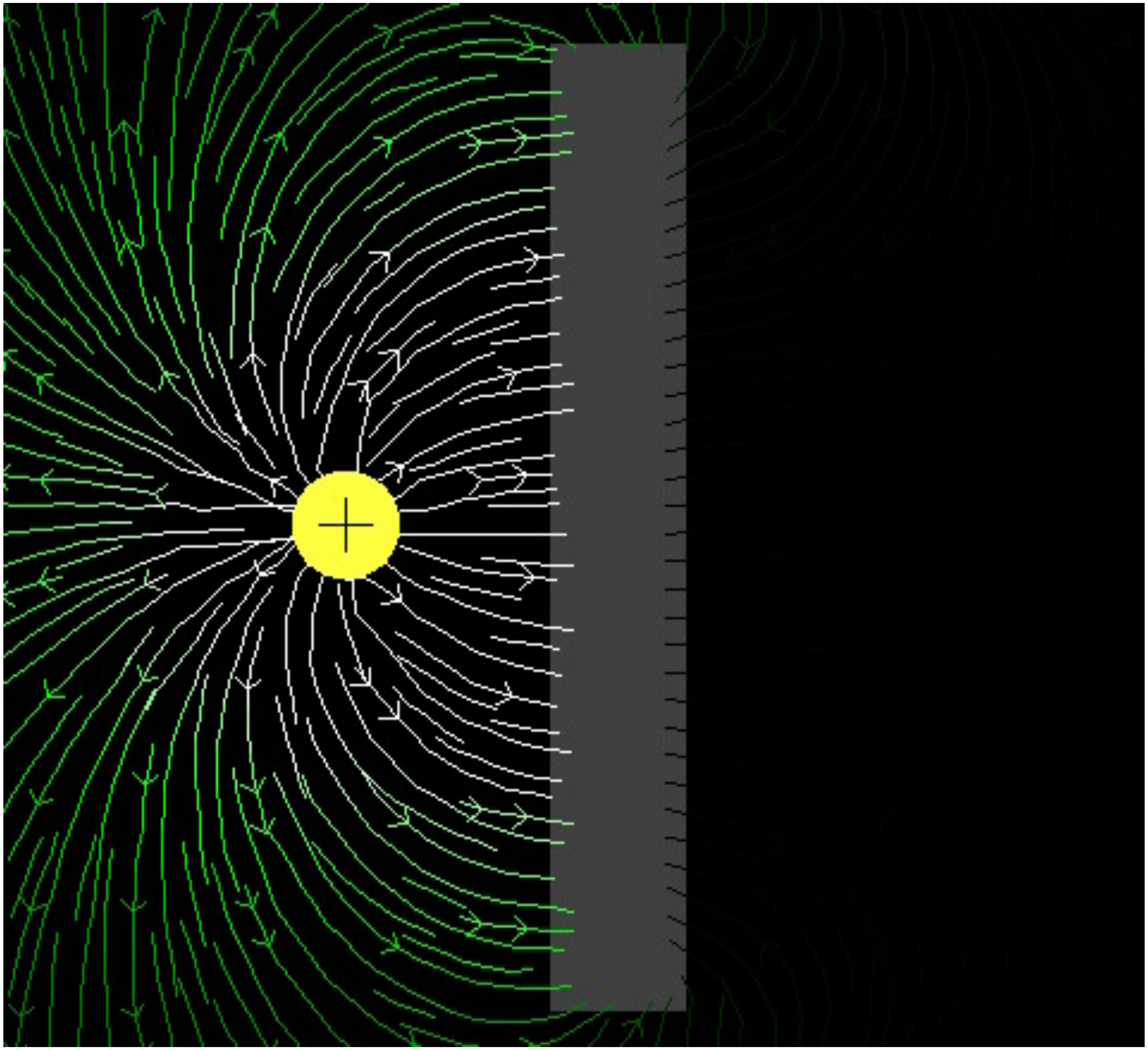

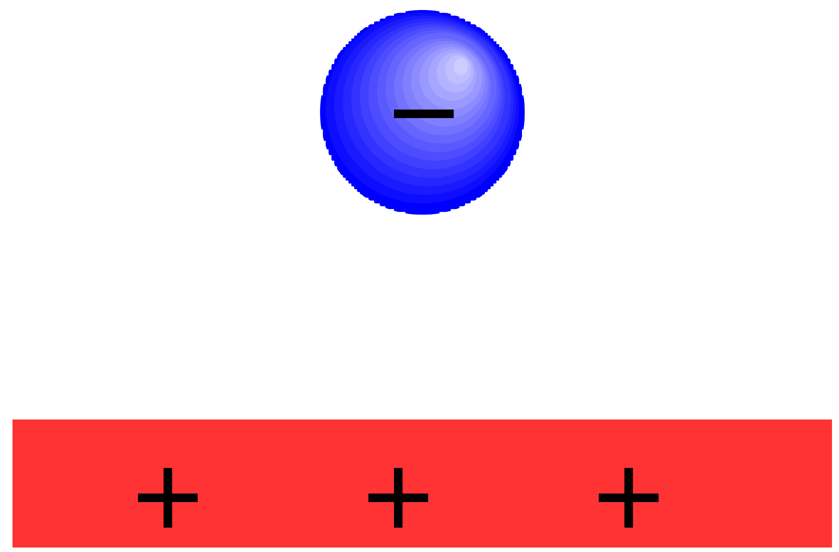

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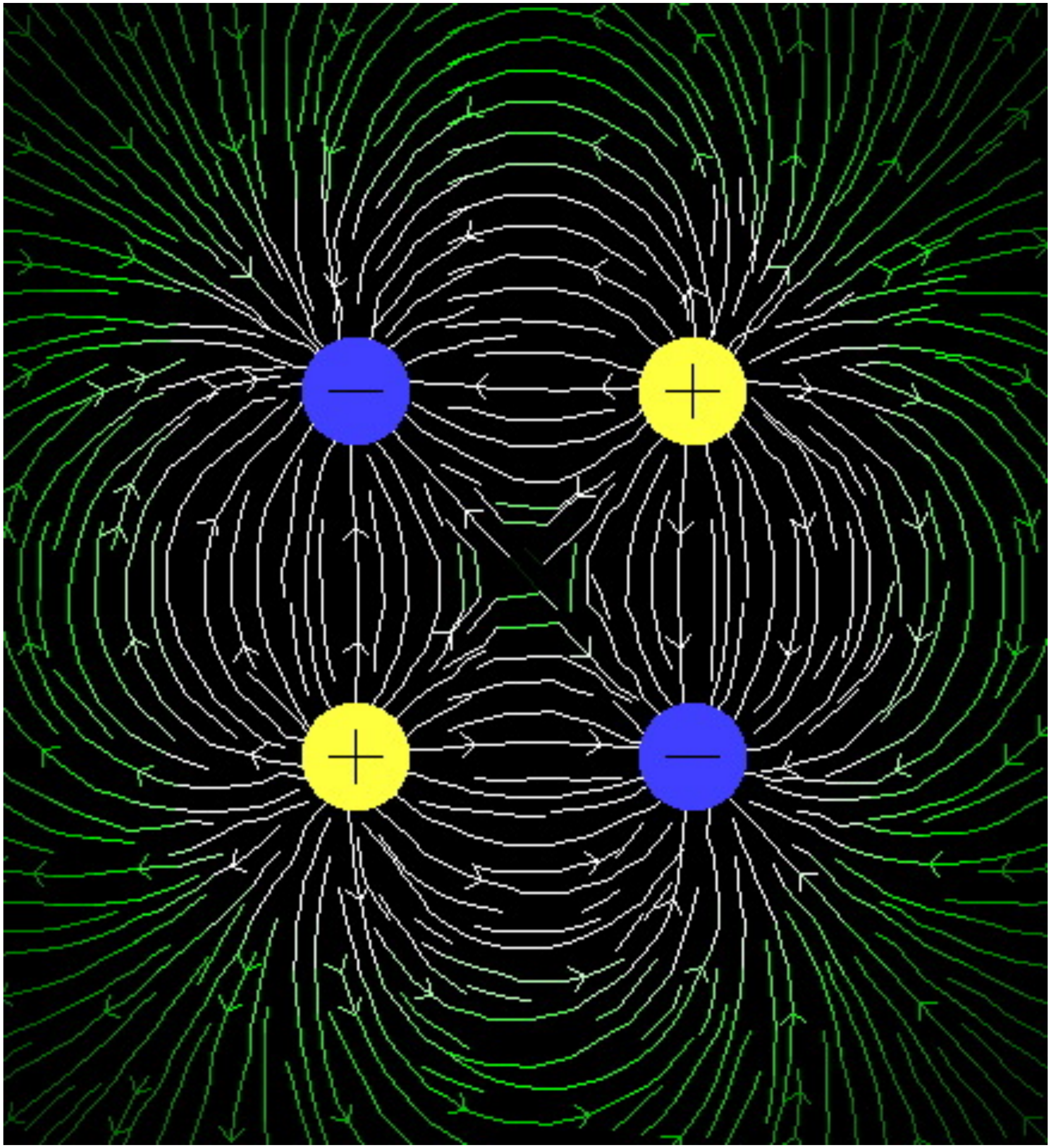

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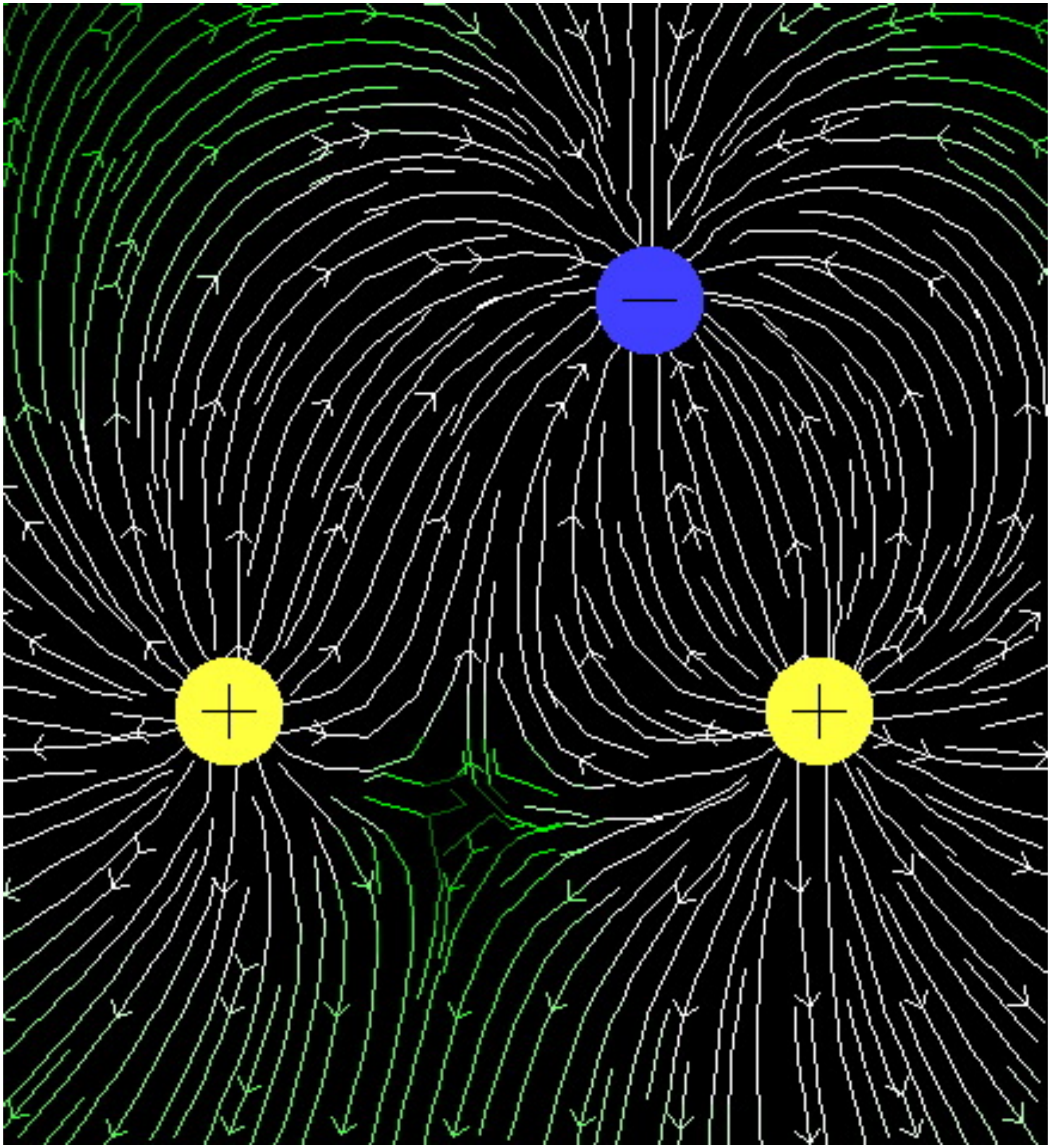


Electric Fields - concept check

From Fletch's book

12.7) What does an *electric field* actually tell you? That is:

a.) Is it a vector? If so, what does its direction signify?

b.) What does its magnitude tell you?

c.) How might electric fields be used in everyday life?

a) it's a vector, which tells you the direction a positive test charge will accelerate if placed in the field (a negative charge would go the other way).
b) Its magnitude is the available **force per unit charge** (like "g"). If you place a charge "q" at any location where you know E, you can calculate the force on the charge to be $F = qE$ (like $F_g = mg$)
c) This is how all our electrical devices work! When you flip a switch, plug something in, whatever, an electric field is the thing that makes electrical devices function.

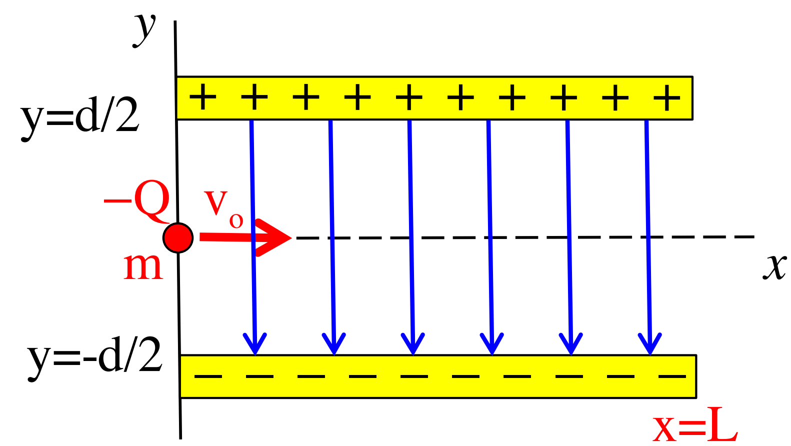
12.8) An electric field is oriented toward the right.

a.) What will an electron do if put in the field?

b.) What will a proton do if put in the field at the same point as mentioned in *Part a*?

a) An electron would accelerate to the **left** because the field indicates how a *positive* charge would accelerate.
b) A proton would accelerate to the right. It would feel the same electrical force as the electron (just in the opposite direction) but would accelerate differently due to its larger mass.

Example 13: A charge $-Q$ and mass m is fired into a cavity bounded by two metallic plates of length L. The plates are d units apart. The charge initially moves along the cavity's central x-axis with velocity v_o (see sketch). Assuming gravity acts downward:

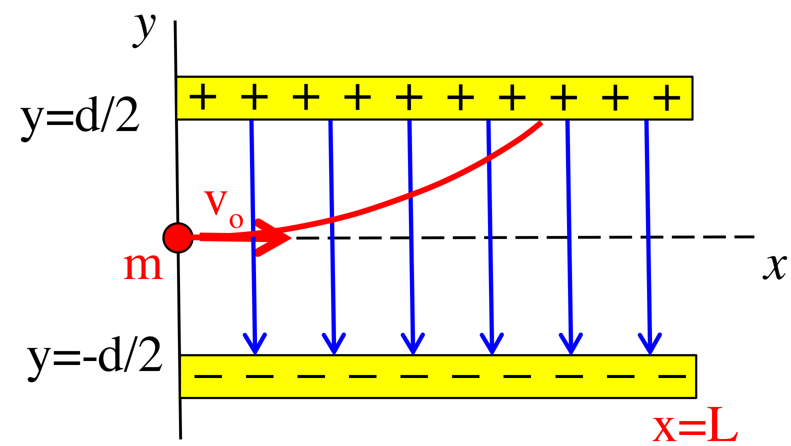


a.) The hope is that Q will move straight along the x-axis even though gravity is acting upon the mass. An *E-fld* is placed between the plates to counteract gravity. In what direction must that field be? Also, what kind of charge must exist on the top and bottom plate to effect that field?

If the electric FORCE must be upward to counteract the downward force of gravity, and if the force on a negative charge is *opposite* the direction of an electric field (think about how the direction of an electric field is defined), the *E-fld* must be downward (hence producing an electrical force upward). (NOTE: The easiest way to think through this is to assume the charge is *positive*, determine the *E-fld* for that particle, then reverse you solution.)

The charge configuration needed to generate a downward electric field, given that field lines *leave* positive charges, is positive charge on the top plate and negative charge on the bottom plate.

b.) What must be the *electric field magnitude* to effect this straight-line motion?



This is a Newton's Second Law problem. Noting that the magnitude of the electric force is $q|E|$, where q does not carry its sign, we can write:

$$\sum F_x :$$

$$Q|E| - mg = m\cancelto{0}{a}$$

$$\Rightarrow \quad |E| = \frac{mg}{Q}$$

c.) Given this E-fld, if this experiment had been carried out in space:

i.) What path would the charge take after the firing? (see sketch)

ii.) Given the electric field as calculated, what velocity would the charge require to just barely clear the cavity without crashing into its wall?

$$\sum F_y :$$

$$Q|E| = ma_y$$

$$\boxed{1} \quad \Rightarrow \cancel{Q}\left(\cancel{m}g/\cancel{Q}\right) = \cancel{m}a_y$$

$$\Rightarrow a_y = g$$

$$\boxed{2} \quad \cancelto{d/2}{y_2} = \cancelto{0}{y_1} + \cancelto{0}{v_{1,y}}\Delta t + \tfrac{1}{2}\cancelto{g}{a_y}(\Delta t)^2$$

$$\Rightarrow \quad (\Delta t) = \left(d/g\right)^{1/2}$$

$$\boxed{3} \quad \cancelto{L}{x_2} = \cancelto{0}{x_1} + \cancelto{v_o}{v_{1,x}}\Delta t + \tfrac{1}{2}\cancelto{0}{a_x}(\Delta t)^2$$

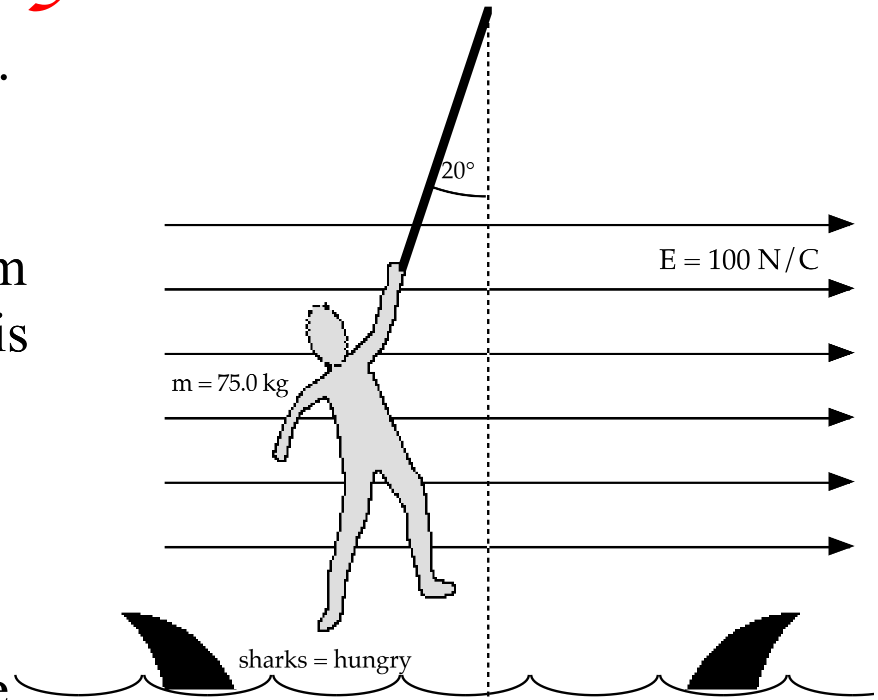
$$\Rightarrow L = v_o\Delta t$$

$$\Rightarrow v_o = L/\Delta t$$

$$= \frac{L}{\left(\frac{d}{g}\right)^{1/2}} = L\left(\frac{g}{d}\right)^{1/2}$$

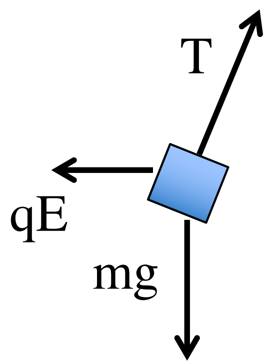
Example 14—courtesy of Mr White:

Secret Agent 008 is in a rather sticky situation. A climbing rope of negligible mass suspends our hero over a pool of hungry sharks. The space above the pool is permeated by a uniform electric field of 100 N/C. Luckily, Agent 008 is wearing a special Adjustable Electric Charge Body Suit prepared for him by the ingenious minds at the Imperial Research and Development Lab.



a) In order to stay alive long enough to devise an escape plan, 008 must select a charge for his suit that will enable him to keep the rope at a minimum angle of 20° to the left of vertical, as shown in the drawing above. If he weighs 75 kg, what is the magnitude and polarity of the charge Agent 008 should set the suit for?

He is going to need an electric force *opposite* the direction of the *E-fld*, so it will need to be a *negative charge*. With the f.b.d.:



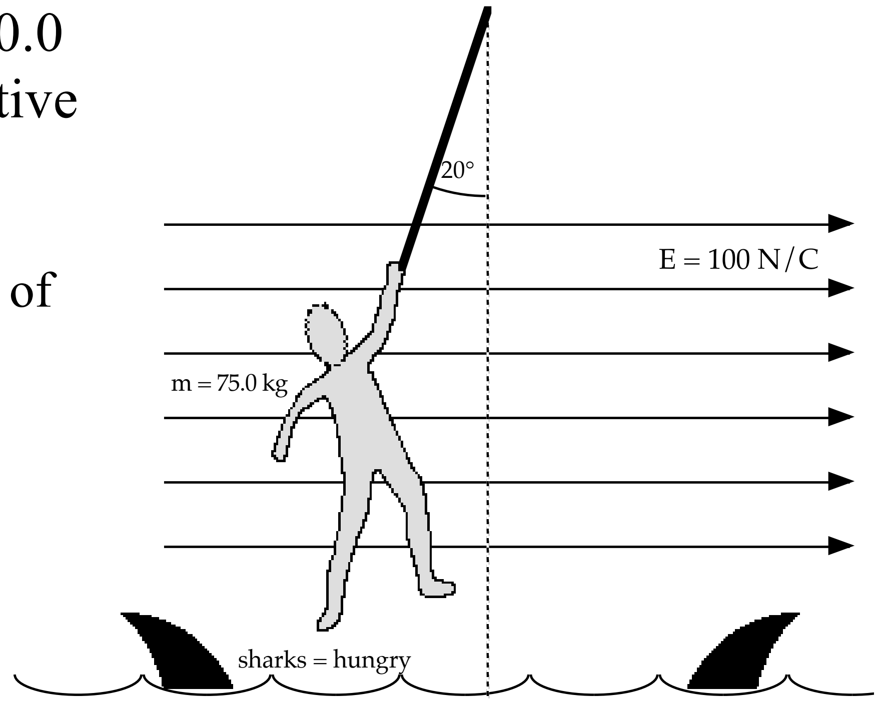
$$T\cos\theta - mg = 0 \quad \text{and}$$

$$T\sin\theta - qE = 0$$

$$\Rightarrow \quad q = \frac{mg}{E}\tan\theta$$

b) Agent 008 accidentally selects a charge of +10.0 Coulombs for his body suit. To what angle (relative to the vertical) does his body swing **now**?

A positive charge will swing him in the direction of the *E-fld*, so the sketch will be a mirror image of what is seen and you'll just have to do the math (it's the same math) to figure out the angle.



c) Unfortunately, Agent 008's rope has been weakened--when the rope reaches this new angle, it snaps. What was the tension in the rope just before it broke?

Go back to either of the equations used to solve *Part b* and solve for *T*.