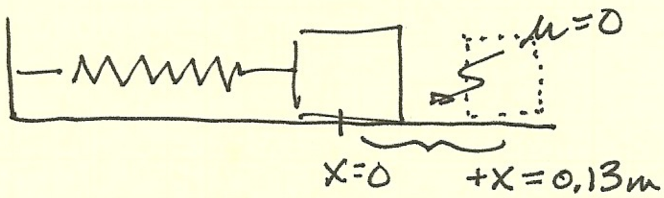
15.1



$m_{block} = 0.60\,kg$

$k = 130\,N/m$

When block is released....

a) Find Force on block.

$F = -kx$

$= (130\,N/m)(0.13\,m)$

$= \boxed{-16.9\,m}$ ← The negative sign just indicates that the direction of the Force is in the negative direction, the opposite of the + displacement.

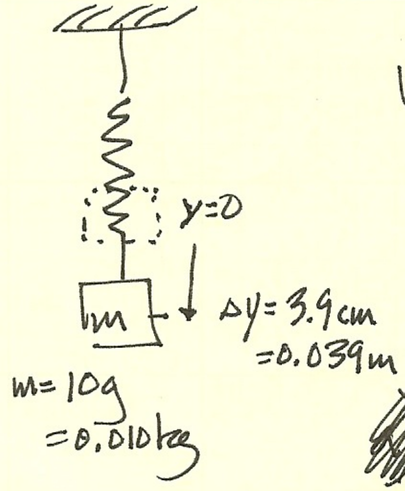
b) Its acceleration?

$\Sigma F = ma$

$-16.9\,N = (0.60\,kg)(a)$

$a = \boxed{-28.2\,m/s^2}$ (in the same direction as the force.)

15.3



Use first situation, hanging a 10g mass from the spring, to determine spring constant k.

$F_{spring} = -kx$

At equilibrium, $F_{spring} = F_{gravity}$.

$F_g = kx$

$mg = kx$

$k = \frac{mg}{x} = \frac{(0.010)(9.8)}{0.039} = \boxed{2.51\ N/m}$

Now, 10g mass is replaced w/ a 25g mass, & put into motion. Period $T = 2\pi\sqrt{\frac{m}{k}}$

$T = 2\pi\sqrt{\frac{0.025\ kg}{2.51\ N/m}} = \boxed{0.627\ s}$

15.5

$$x = \underbrace{4.00}_{\text{Amplitude } A} \cos(\underbrace{3.00\pi}_{\text{Angular frequency } \omega} t + \underbrace{\pi}_{\text{phase constant}})$$

a) $f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{3.00\pi}{2\pi} = \boxed{1.50 \text{ Hz}}$

b) period $T = \frac{2\pi}{\omega} = \frac{2\pi}{3\pi} = \boxed{0.667 \text{ s}}$

c) Amplitude of motion $= \boxed{4.00 \text{ m.}}$

d) Phase constant $= \boxed{\pi \text{ (rads)}}$

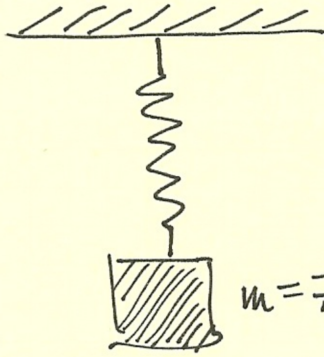
From analyzing equation

e) $x = 4 \cos(3\pi(0.25) + \pi)$

Be sure to use radians when calculating!

$= \boxed{2.83 \text{m}}$

15.9



$m = 7.00\,kg$

Period of oscillation = <u>2.60s</u>

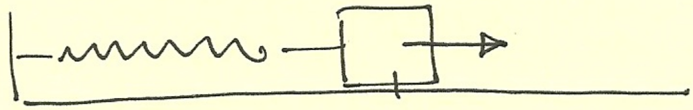
Find k.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T^2 = 4\pi^2\frac{m}{k}$$

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(7)}{(2.60)^2} = \boxed{40.9\ N/m}$$

15.13



$x=0$ at $t=0$

$A = 2.00\text{ cm}$
$f = 1.50\text{ Hz}$

a) Find expression for position of particle as function of time.

Typical form of eqn for position is $x = A\cos(\omega t + \phi)$

Here $T = \frac{1}{f} = \frac{1}{1.50\text{ Hz}} = \boxed{0.667\text{ s}}$

$T = \frac{2\pi}{\omega} \rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{0.667} = \underline{\cancel{4.19}\ 3\pi}\ s^{-1}$

What is ϕ?

$x = 0$ at $t = 0$, which means $0 = \cos(\phi)$

$\phi = \cos^{-1}(0) = \underbrace{\frac{\pi}{2}, \frac{3\pi}{2}}$ etc.

which one?

$v = -\omega A \sin(\omega t + \phi)$

$v = -(3\pi)(0.02)\sin(0 + \phi)$

v is positive (stated in problem), so $-\sin(\frac{3}{2}\pi)$ is the solution, &

$\phi = \frac{3\pi}{2}$

So $x = \boxed{0.02\cos(3\pi t + \frac{3}{2}\pi)}$ ← may be simplified to a sin ~~relations~~ relationship if desired.

b) $v = -\omega A \underline{\sin(\omega t + \phi)}$

Max possible value $= 1$, so $v_{max} = -\omega A$

$= 3\pi(0.02) = 0.06\pi$

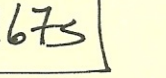
$= \boxed{0.188\text{ m/s}}$

c) Particle has this speed at midpoint, when $t=0$, & $\frac{1}{2}T$ later, or $\boxed{0.333\text{ s}}$

d) Max accel occurs at $a = -\omega^2 A = \boxed{1.78\text{ m/s}^2}$

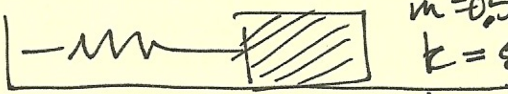
e) Max accel occurs at endpoint, at time $\frac{T}{4} = \boxed{0.167\text{ s}}$

f) In 1 s, number of ~~per~~ cycles is $\frac{1\text{ s}}{T} = \frac{3}{2}$ cycles →



$6 \times 2\text{ cm} = \boxed{12.0\text{ cm}}$

15.15



$m = 0.5\ kg$
$k = 8.00\ N/m$
$A = 0.10\ m$

a) Commonly used
$$\begin{cases} x = A\cos(\omega t + \phi) \\ v = -\omega A \sin(\omega t + \phi) \longrightarrow v_{max} = \omega A \\ a = -\omega^2 A\cos(\omega t+\phi) \longrightarrow a = -\omega^2 x \\ \qquad a_{max} = \omega^2 A \end{cases}$$

$v_{max} = \omega A$; $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8}{0.5}} = 4\ rad/s$

$v_{max} = \left(4\frac{rad}{s}\right)(0.10m) = \boxed{0.4\ m/s}$

b) $a_{max} = \omega^2 A = \left(4\ rad/s\right)^2 (0.10m) = \boxed{1.6\ m/s^2}$

c) At $x = 6.00cm = 0.06m$ from equilibrium,

$v = ?$ $\quad v = -(4)(0.1)\sin(4t + 0)$
$= -0.4\sin(4t)$

t? when is mass at 0.06m?

Here, I've assumed that $x = A$ at time $t=0$. The problem doesn't specify so...

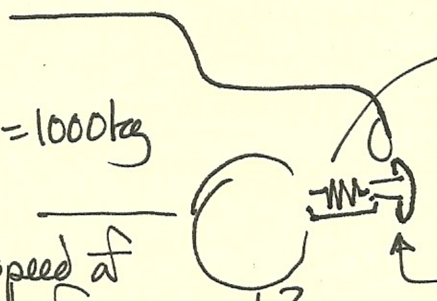
$x = A\cos(\omega t + \phi)$
$0.06 = 0.1\cos(4t + 0)$
Solve for t to get $t = \dfrac{\cos^{-1}\left(\frac{0.06}{0.1}\right)}{4} = \underline{0.23s}$

$v = 0.4\sin(4 \cdot 0.23s) = \boxed{0.32\ m/s}$

d) acceleration $a = -\omega^2 x$
$= (4)^2 0.06 = \boxed{0.96\ m/s^2}$

e) Using same calc as here: $0.08 = 0.1\cos(4t + 0)$
$t = \boxed{0.161s}$

15.17



$m = 1000\,kg$

$k = 5e6\ N/m$

$\Delta x = 3.16\,cm = 0.0316\,m$

Find speed of car before impact?

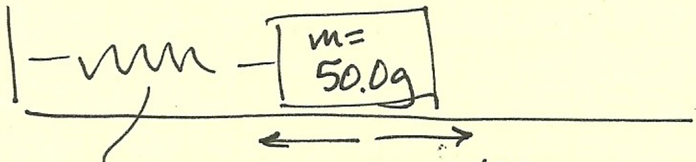
$$K_{car} = U_s$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$v = \sqrt{\frac{k}{m}}\,x = \sqrt{\frac{5e6}{1000}}\ 0.0316$$

$$= \boxed{2.23\ m/s}$$

15.19



$k = 35.0\,N/m$ $\quad A = 0.0400\,m$

a) Total E of the system $= \frac{1}{2}kA^2$

That's when all energy is stored in the screen

$= \frac{1}{2}(35)(0.04)^2$

$= \boxed{0.028\,J}$

b) Speed of block at 1.00 cm displacement = ?

Total $E = K_f + U_f$

$0.028 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$0.028 = \frac{1}{2}(0.050)v^2 + \frac{1}{2}(35)(0.01)^2$

$v = \boxed{1.02\,m/s}$

d) At 3 cm...

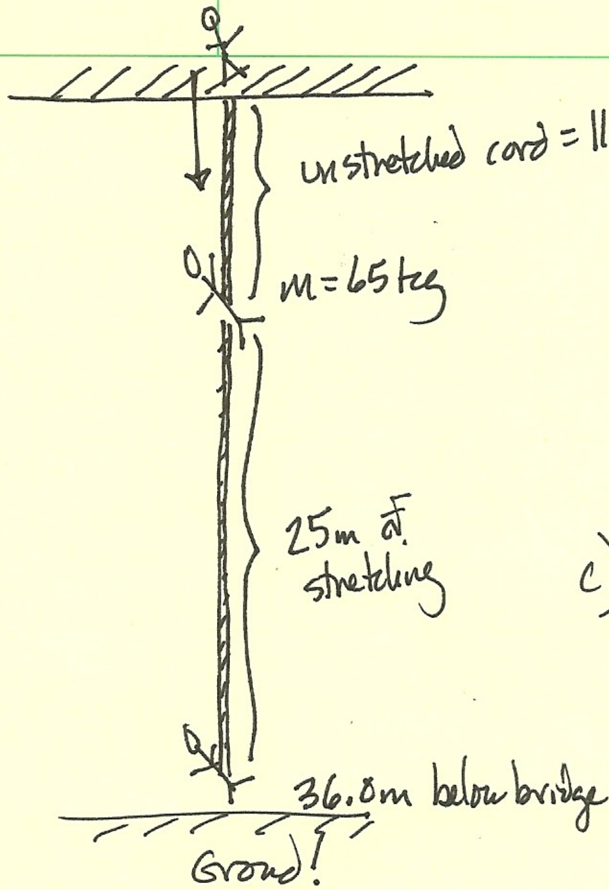
Total $E = K_f + U_f$

$U_f = \frac{1}{2}kx^2 = \frac{1}{2}(35)(0.03)^2 = \boxed{0.0158\,J}$

c) $K_f = \text{Total } E - U_f$

$= 0.028 - 0.0158 = \boxed{0.0122\,J}$

15.22



a) Analysis for first 11 m is based on free-fall.

b) $\Delta y = v_i t + \frac{1}{2}at^2$

$-11 = 0 + \frac{1}{2}(-9.8)t^2$

$t = \boxed{1.50\,s}$

c) jumper, spring & earth account for all the forces in this problem. The system is isolated.

d) To get spring constant of cord, do energy analysis:

$E_{top} = E_{bottom}$

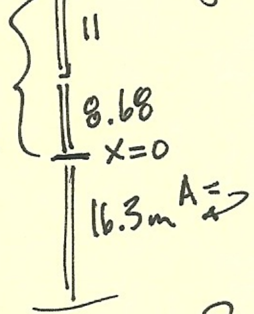
$mgh = \frac{1}{2}kx^2$

$(65)(9.8)(36) = \frac{1}{2}k(25)^2$

$k = \boxed{73.4\,N/m}$

e) Equilibrium point for 65kg climber:

$\boxed{\text{19.7 m below bridge}}$



$F_{spring} = -kx$

$F_g = F_{spring}$ at equilibrium

$mg = kx$

$x = \frac{m}{k}g = \boxed{8.68\,m}$

f) Angular frequency $\omega = ?$ $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{73.4}{65}} = \boxed{1.06\,rad/s}$

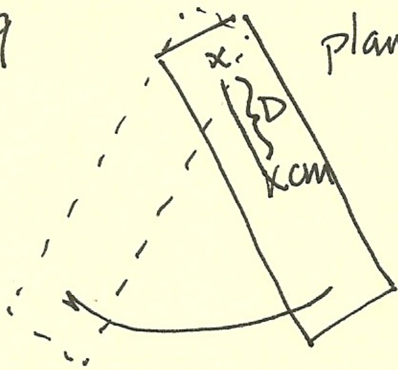
g) $x = A\cos(\omega t + \phi)$

$-8.68 = 16.3\cos(1.06t)$

$t = 2.01\,s$

h) $1.50\,s + 2.01\,s = \boxed{3.51\,s}$

15.29



planar object as a physical pendulum

$f = 0.450\ \text{Hz}$

$m = 2.20\ \text{kg}$

$D = 0.350\ \text{m}$

$I = ?$

Physical pendulum has a period $T = 2\pi\sqrt{\dfrac{I}{mgd}}$.

$$T = \frac{1}{f} = \frac{1}{0.450\ \text{Hz}} = 2.22\ \text{s}$$

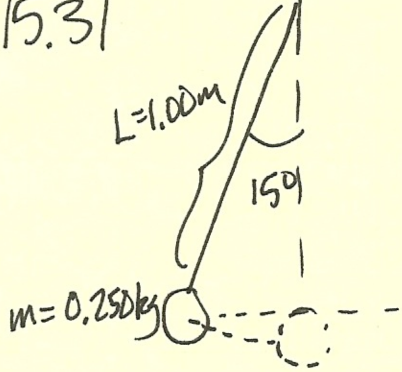
$$T^2 = 4\pi^2 \frac{I}{mgd}$$

$$I = \frac{T^2}{4\pi^2} mgd$$

$$= \frac{(2.22)^2 (2.2\ \text{kg})(9.8\ \text{m/s}^2)(0.350\ \text{m})}{4\pi^2}$$

$$= \cancel{[illegible]}\ \boxed{0.942\ \text{kg m}^2}$$

15.31



Analyze using Simple Harmonic Motion principles:

a) max speed of bob?

$$x = A\cos(\omega t + \phi)$$

$$v = \frac{\partial x}{\partial t} = -\omega A \sin(\omega t + \phi)$$

$$v_{max} = -\omega A$$

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8}{1}} = \underline{3.13 \text{ rad/s}}$$

$$A = r\theta = (1\text{ m})(15°)\left(\frac{2\pi}{360°}\right) = \underline{0.262\text{m}}$$

$$v_{max} = (3.13 \tfrac{\text{rad}}{\text{s}})(0.262\text{m}) = \boxed{0.819 \text{ m/s}}$$

b) $\alpha_{max} = ?$

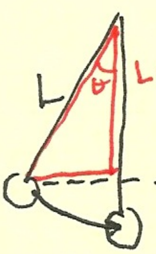
$$a_{max} = -\omega^2 A = (3.13)^2(0.262) = 2.57 \text{ m/s}^2$$

$$\alpha_{max} = \frac{a_{max}}{r} = \frac{2.57 \text{ m/s}^2}{1.00\text{m}} = \boxed{2.57 \text{ rad/s}^2}$$

c) Max $F_{restoring} = m a_{max}$

$$= (0.25\text{ kg})(2.57 \text{ m/s}^2) = \boxed{0.642\text{N}}$$

d) Solve for v_{max} & a_{max} using other (earlier) analysis models.



v_{max} at bottom can be solved using Cons of Energy:

$$h = L - L\cos\theta$$

$$U_g = K \longrightarrow \cancel{m}gh = \tfrac{1}{2}\cancel{m}v^2 \longrightarrow v = \sqrt{2g(L - L\cos\theta)}$$

$$= \sqrt{2 \cdot 9.8 \cdot (1 - 1\cos 15)}$$

$$= \boxed{0.817 \text{ m/s}}$$

$$a_{max} = g\sin\theta = \boxed{2.54 \text{ m/s}^2}$$

$$F_{restoring} = ma = (0.25)(2.54) = \boxed{0.635\text{N}}$$

e) Values in (d) are more accurate. The Simple Harmonic Motion analysis is only approximate when applied to a pendulum.

15.33 A simple pendulum has length $L = 5.00\,m$.

Ordinarily, period $T = 2\pi\sqrt{\frac{L}{g}} = \underline{4.49\,s}$.

What is T in an elevator accelerating

a) upwards at $5.0\,m/s^2$?

The effective gravity is $5.0 + 9.8 = 14.8\,m/s^2$

(Think about the Tension in the string, which is <u>greater</u> under these conditions.)

$$T = 2\pi\sqrt{\frac{L}{(5+9.8)}} = \boxed{3.65\,s}$$

b) downwards at $5.0\,m/s^2$

Now, gravity is effectively reduced to $9.8 - 5 = 4.80\,m/s^2$

$$T = 2\pi\sqrt{\frac{L}{4.8}} = \boxed{6.41\,s}$$

c) For horizontal acceleration, what is effective g?

$a = \sqrt{9.8^2 + 5^2}$

$= 11.0\,m/s^2$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$= 2\pi\sqrt{\frac{5}{11}} = \boxed{4.24\,s}$$