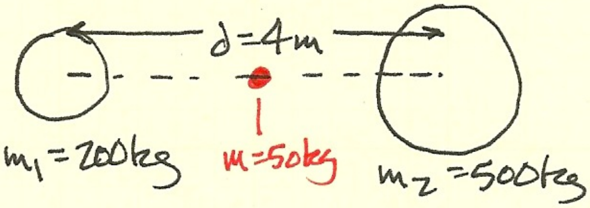
13.3



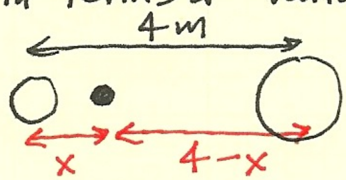
a) $F_g = \frac{G m_1 m_2}{r^2}$; F_{net} on 50kg $= {}^{-}F_{g1} + {}^{+}F_{g2}$

$$F_{net} = -\frac{G m_1 m}{r_1^2} + \frac{G m_2 m}{r_2^2}$$

$$= -\frac{(6.672e\text{-}11)(200)(50)}{2^2} + \frac{(6.672e\text{-}11)(500)(50)}{2^2}$$

$$= \boxed{+2.50e\text{-}7\,N} \quad \text{(to the left)}$$

b) To find the position of 50 kg where $F_{net}=0$, express position r in terms of variables.



$$F_{net} = 0 \rightarrow F_1 = F_2$$

$$\frac{G m_1 m}{x^2} = \frac{G m_2 m}{(4-x)^2}$$

$$\frac{m_1}{x^2} = \frac{m_2}{(4-x)^2}$$

$$\sqrt{x^2 m_2} = \sqrt{(4-x)^2 m_1}$$

$$x\sqrt{m_2} = (4-x)\sqrt{m_1}$$

Solve for x to get $\boxed{1.55m}$ from 200 kg object

13.5

$$F_g = \frac{G m_1 m_2}{r^2}$$

$$= \left(6.672e{-11}\ \frac{Nm^2}{kg^2}\right) \frac{(1.5e{-3}\ kg)(15\ kg)}{(0.045\ m)^2}$$

$$= \boxed{7.41e{-10}\ N}$$

13.10 a_g at $3 \times r_{earth}$?

Use straight numbers, or use ratios.

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$m a_g = G\frac{m_e M}{(r+3r)^2}$$

$$a_g = \frac{(6.672e{-11})(5.98e24)}{(4 \cdot 6.38e6)^2}$$

$$= \boxed{0.613\ m/s^2}$$

$a = 9.8$ at $1\ r_{earth}$

Four times farther away, a will be $\frac{1}{4^2}$ of its surface value:

$$a_g = \frac{g}{4^2} = \frac{9.8}{4^2}$$

$$= \boxed{0.613\ m/s^2}$$

13.12 Find $\frac{\rho_{Moon}}{\rho_{Earth}}$, based on $a_{moon} = \frac{g}{6}$, & $r_{moon} = 0.25\, r_{earth}$.

There are lots of ways to approach this, but here's one.

1) Find a_g based on other quantities:

$$F_g = \frac{GMm}{r^2} = m a_g$$

$$a_g = \frac{GM}{r^2} = \frac{G\rho V}{r^2} = \frac{G\rho \frac{4}{3}\pi r^3}{r^2}$$

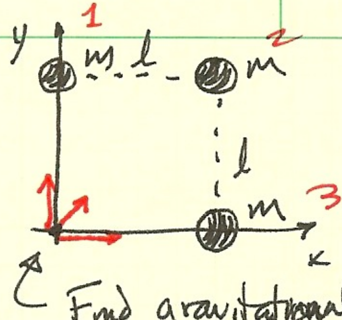
$$a_g = G\rho\pi r \tfrac{4}{3}$$

2) Compare accelerations of moon & Earth:

$$\frac{a_{Moon}}{a_{Earth}} = \frac{\frac{1}{6} a_{Earth}}{a_{earth}} = \frac{1}{6} = \frac{\cancel{G}\rho_m \cancel{\pi} r_m \cancel{\tfrac{4}{3}}}{\cancel{G}\rho_E \cancel{\pi} r_E \cancel{\tfrac{4}{3}}}$$

Rearrange: $\frac{\rho_m}{\rho_E} = \frac{1}{6}\frac{r_E}{r_{moon}} = \frac{1}{6}\frac{r_E}{(0.25 r_E)} = \frac{1}{1.5} = \boxed{0.667}$

13.25



Find gravitational field a at origin.

For masses along x & y axes (1 & 3)

$$F_g = \frac{G m_1 m_2}{r^2} \rightarrow \cancel{m} a_g = \frac{G M \cancel{m}}{r^2} = \frac{GM}{l^2}$$

For mass along diagonal, (2)

$$a_g = \frac{GM}{(\sqrt{2}\, l)^2} = \frac{1}{2}\frac{GM}{l^2}$$

$$F_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$= \left(0 i + \frac{GM}{l^2} j\right) + \left(\frac{GM}{2 l^2}\right)\left(\frac{\sqrt{2}}{2} i + \frac{\sqrt{2}}{2} j\right) + \left(\frac{GM}{l^2} i + 0 j\right)$$

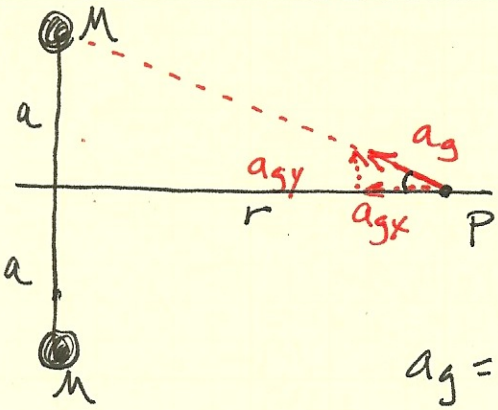
$$= \frac{GM}{l^2}\left(1 + \frac{\sqrt{2}}{4}\right)\hat{i} + \frac{GM}{l^2}\left(1 + \frac{\sqrt{2}}{4}\right)\hat{j}$$

$$= \frac{GM}{l^2}\left(1 + \frac{\sqrt{2}}{4}\right)(\hat{i} + \hat{j})$$

$$\sqrt{1^2 + 1^2} = \sqrt{2}$$

$$= \boxed{\frac{GM}{l^2}\left(\sqrt{2} + \frac{1}{2}\right)}$$ toward opposite corner.

13.26



a) Calculate vector field at P due to masses.

$a_g = \frac{GM}{r^2}$ in general.

Here r is the hypotenuse, so

$r = \sqrt{a^2 + r^2}$, so

$$a_g = \frac{GM}{a^2 + r^2}$$

There are 2 Ms, though, & y-component effects cancel out, so it's just the x-component — two x-components actually, for the 2 masses that produce the gravity field a.

$$a_{net} = 2\left(\frac{GM}{a^2+r^2}\right)\cos\theta$$

$$\cos\theta = \frac{\text{adj}}{\text{hypotenuse}} = \frac{r}{\sqrt{a^2+r^2}}, \text{ so}$$

$$a_{net} = \boxed{2\frac{GMr}{(a^2+r^2)^{3/2}}} \text{ toward origin}$$

b) As $r \to 0$, vertical field components become more significant — fields cancel each other out increasingly.

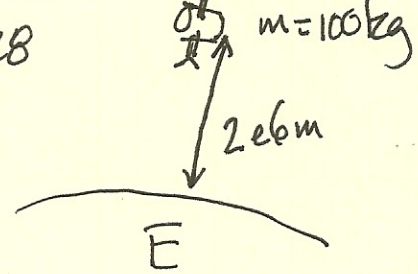
c) As $r \to 0$, a_{net} approaches $\frac{2GM(0)}{a^3}$

d) As $r \to \infty$, vertical components are smaller — more & more of field is horizontal. M & M start to appear as a single M, compared w/ a large horizontal distance.

e) As $r \to \infty$, $a << r$, so $\frac{2GMr}{(a^2+r^2)^{3/2}} \to \frac{2GMr}{(0+r^2)^{3/2}}$

$$= \boxed{\frac{2GM}{r^2}}$$

13.28



a) $U_g = ?$

$$U_g = -\frac{GMm}{r}$$

$$= -\frac{(6.672e{-11})(5.98e24)(100)}{(6.38e6+2e6)}$$

$$= \boxed{-4.76e9\,J}$$

b) F_g on satellite $= ?$

$$F_g = \frac{GMm}{r^2} = \frac{(6.672e{-11})(5.98e24)(100)}{(6.38e6+2e6)^2}$$

$$= \boxed{568\,N}$$

c) Satellite, by Newton's 3rd Law of Motion, exerts an equal force of 568N back on the earth.

13.31 When sun collapses to a white dwarf w/ radius of earth: $m_{sun} = 1.989 \times 10^{30}$ kg (back of textbook)

$r_{earth} = 6.37e6$ m

a) Average density $\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi r^3}$

$$\rho = \frac{1.989e30}{\frac{4}{3}\pi(6.37e6)^3} = \boxed{1.84e9 \text{ kg/m3}}$$

b) Free fall at surface?

$$F_g = ma_g$$

$$\frac{GM\cancel{m}}{r^2} = \cancel{m}a$$

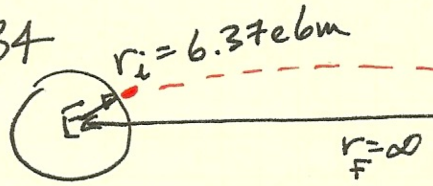
$$a = \frac{GM}{r^2}$$

$$a = \frac{(6.672e{-11})(1.989e30)}{(6.37e6)^2} = \boxed{3.27e6 \text{ m/s2}}$$

c) $$U = -\frac{GMm}{r} = \frac{-(6.672e{-11})(1.989e30)(1)}{(6.37e6)}$$

$$= \boxed{-2.08e13 \text{ J}}$$

13.34



Conservation of Energy analysis:

$$U_i + K_i = U_f + K_f$$

$$-\frac{GMm}{r} + \frac{1}{2}mv_i^2 = 0 + \frac{1}{2}mv_f^2$$

infinitely far away

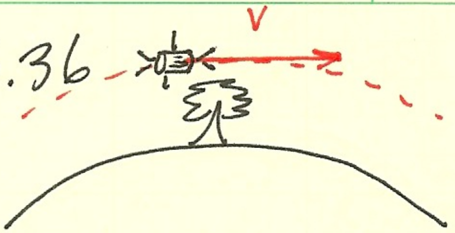
Final K.

$$-\frac{2GM}{r} + \cancel{\frac{1}{2}}v_i^2 = \cancel{\frac{1}{2}}v_f^2$$

$$-2\frac{(6.672e{-11})(5.98e24)}{6.38e6} + (2e4)^2 = v_f^2$$

$$v_f = \boxed{1.66e4 \text{ m/s}}$$

13.36



For a tree-top satellite w/ orbital speed v, show that $v_{esc} = \sqrt{2}\, v$.

"Escape velocity" is the minimum speed necessary to completely escape earth's gravity, theoretically at $r \to \infty$.

For an orbiting satellite: $F_c = F_g$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

This is the orbital velocity currently. $\longrightarrow$ $v^2 = \dfrac{GM}{r}$

To calulate an escape velocity for a given altitude.

$$U_i + K_i = U_f + K_f$$

$$-\frac{GMm}{r} + \frac{1}{2} m v_{esc}^2 = 0 + 0$$

$r \to \infty$

$v = 0$, so no K at end.

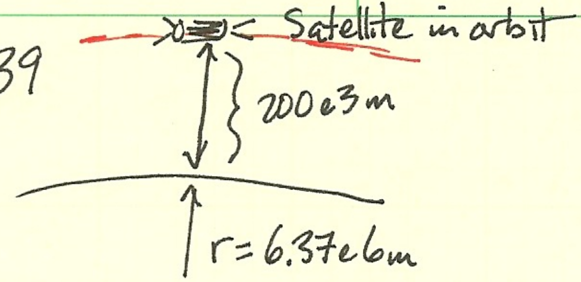
$$\frac{1}{2}\cancel{m} v_{esc}^2 = \frac{GM\cancel{m}}{r}$$

$$\frac{1}{2} v_{esc}^2 = \frac{GM}{r} = v_{orbital}^2$$

$$\sqrt{2 v_{orbital}^2} = \sqrt{v_{esc}^2}$$

$$\boxed{v_{esc} = \sqrt{2}\; v}$$ ✓

13.39



a) Time to complete one orbit?

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$V = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.672e{-11})(5.98e24)}{6.37e6+200e3}}$$

$$V = 7.79e3\ m/s = \frac{\text{circumference}}{\text{time}}$$

$$t = \frac{2\pi r}{7.79e3} = \boxed{5300s = 88\,min}$$

b) Satellite's speed $= \boxed{7.79e3\ m/s}$

c) To put satellite into this orbit from Earth's surface.

$$K_{surface} + \text{Energy input} + U_{surface} = U_{orbit} + K_{orbit}$$

They asked us to include the effect of the earth's rotation, which gives the rocket an initial velocity at launch.

$$\frac{1}{2}mv_i^2 + W_{in} + -\frac{GMm}{r_i} = \underbrace{\frac{-GMm}{r_f} + \frac{1}{2}mv_f^2}$$

$$W_{in} + \frac{1}{2}mv_i^2 + \frac{-GMm}{r_i} = -\frac{GMm}{2r_f}$$ Total E of orbiting satellite

$$v_i \text{ at surface} = \frac{2\pi r}{1\,day} = \frac{2\pi(6.37e6)}{86400s} = 463\ m/s$$

$$W_{in} = \frac{GMm}{r_i} - \frac{1}{2}mv_i^2 - \frac{GMm}{2r_f}$$

$$= \frac{(6.672e{-11})(5.98e24)(200)}{6.37e6} - \frac{1}{2}(200)(463)^2 - \frac{(6.672e{-11})(5.98e24)(200)}{2(6.57e6)}$$

$$= 1.25e10 - 1.07e7 - 6.07e9 = \boxed{6.42e9\,J}$$