

Chapter 31: Faraday's Law!

So far, we've looked at:

- Electric Fields for stationary charges

$$E = \int \frac{k \cdot dq}{r^2} \rightarrow \oint E \cdot dA = \frac{q_{in}}{\epsilon_o}$$

- Magnetic fields of moving charges

$$\oint B \cdot d\ell = \mu_o I$$

Now we turn to the electric field caused by a changing magnetic field.

2.)

One example of Faraday's Law of Induction

One simple experiment demonstrate that a current is produced by a changing magnetic field (naked coil in B-fld attached to galvanometer).

3.)

Faraday's Law of Induction

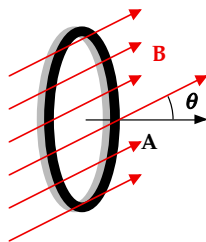
The emf $\mathcal{E}$ induced in a circuit is directly proportional to the time rate of change of magnetic flux through the circuit.

$$\mathcal{E} = \frac{-d\Phi_B}{dt}$$

$$\text{where } \Phi_B = \int \vec{B} \cdot d\vec{A}$$

If we want to induce a flow of current, we need to change the magnetic flux, which we can do by changing B, A, N or θ .

$$\mathcal{E} = \frac{-d}{dt} N(BA \cos \theta)$$



4.)

Example

A coil is wrapped with 200 turns of wire on the perimeter of a square frame of sides 18cm. The total resistance of the coil is 2.0 ohms. A uniform magnetic field is turned on perpendicular to the plane of the coil.

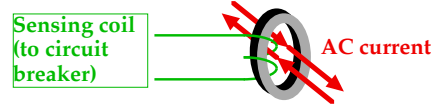
a.) If the field changes linearly from 0 to 0.50Wb/m² in a time of 0.80s, find the magnitude of the induced emf in the coil while the field is changing.

b.) What is magnitude of the current induced in the coil while the field is changing.

- a. $A=0.0324 \text{ m}^2$, $\text{emf}=4.1\text{V}$
b. 2.05A

5.)

Real Life Apps



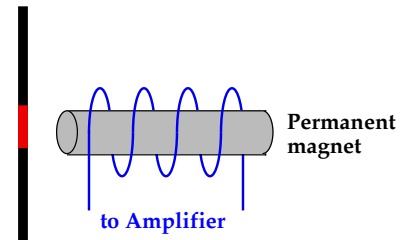
Ground Fault Interrupter
If more current is coming out than going back in, flux changes, inducing an emf which triggers the circuit breaker.



6.)

Real Life Apps

Vibrating guitar string (w/ magnetized portion shown)



Vibrating string produces a change in magnetic flux in the coil, which is transmitted as an emf to the amplifier.

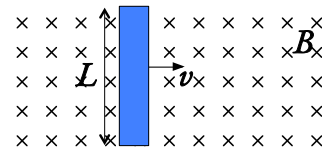


7.)

Motional EMF

When we move a conductor through a magnetic field, a *motional emf* occurs. How so?

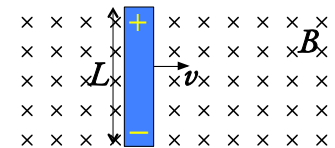
As the charge in the bar moves through the magnetic field, it experiences a force governed by $\vec{F} = q\vec{v} \times \vec{B}$. In this case, that means positive charge will "travel" (we are assuming positive charge *can* travel) to the top of the bar leaving negative charge at the bottom. This will continue until the force generated by the static electric field produced by the clump of charge counteracts the magnetic force due to the charge flow in the B-field. In other words:



$$\begin{aligned} F_B &= F_E \\ \Rightarrow qv \times B &= qE \\ \Rightarrow E &= vB \end{aligned}$$

8.)

As voltages are associated with potential energy functions, and as potential energy functions are associated with conservative force fields, and as static electric fields produce conservative forces fields, we can write the electrical potential difference between the top and bottom of the rod as:



$$\begin{aligned} \Delta V &= -\vec{E} \cdot \vec{d} \\ &= -E L \cos 180^\circ \\ &= -(Bv)L(-1) \\ &= BLv \end{aligned}$$

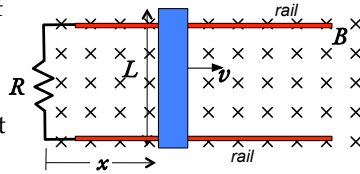
Note: This analysis is associated with *charge in motion* in a magnetic field. The voltage we've identified is induced by the motion. It is not an induced EMF in the "Faraday's Law" sense (that is, there is no *change in flux* going on here).

9.)

What if the conductor is in a loop?

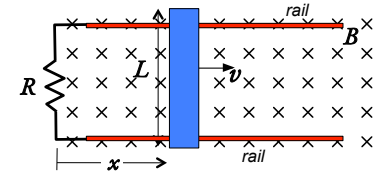
This is a different situation on a whole lot of levels. If we try to extend the previous “motional EMF” situation to this one, we must contend with the fact that a charge build-up at the upper side of the bar doesn’t accumulate as before (charge just keeps moving around the circuit generating an induced current). The problem is that if you make this argument, you end up with the uncomfortable conclusion that because there is no charge build-up, there can be no voltage difference at the top and bottom and, hence, there appears to be no EMF (no voltage) in the system at all (try using Kirchoff’s Second Law on it—you get “ $iR = 0$ ” . . . quack!). Truth be told, the problem resides in the fact that an EMF generated by a changing magnetic flux does *not* generate a conservative force, so we *can’t* define a potential energy, hence voltage function, for the situation.

So what to do?



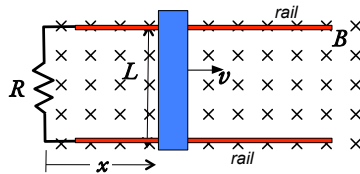
10.)

To get around this difficulty, we need to use Faraday’s Law. In that presentation, the bar’s motion allows more and more magnetic field to pass through the “coil,” which is to say the circuit as a whole. This changes the magnetic flux induces both an EMF and an induced current around the circuit. The math looks like:



11.)

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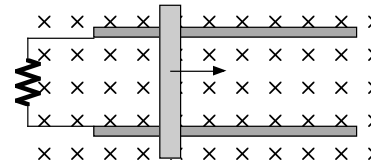


$$\begin{aligned}
 \Phi_B &= \vec{B} \cdot \vec{A} \\
 &= B(Lx)\cos 0^\circ \\
 &= BLx \quad \Rightarrow \quad \varepsilon = -\frac{d}{dt}\Phi_B \\
 &= -\frac{d}{dt}(BLx) \\
 &= -BLv \quad \Rightarrow \quad i_{\text{induced}} = \frac{\varepsilon}{R} \\
 &= \frac{BLv}{R}
 \end{aligned}$$

11b.)

Additionally...

As the current-carrying bar moves through the magnetic field... there’s a magnetic force that acts on it!
If the bar is moving with a constant velocity v , we can calculate the Power delivered by the force acting on the bar.



$$\begin{aligned}
 P &= \frac{W}{t} = \frac{Fx}{t} = F_{\text{app}}v \\
 P &= (I\ell B)v \\
 P &= \left(\left(\frac{B\ell v}{R}\right)\ell B\right)v \\
 P &= \frac{B^2\ell^2v^2}{R} = \frac{V^2}{R}
 \end{aligned}$$

12.)

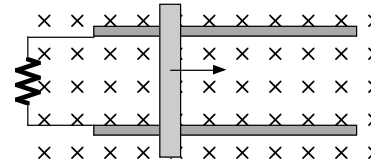
Cool Problem

An airplane flies from LA to Seattle, and due to its motion through the Earth's magnetic field, undergoes motional emf. Which wingtip is positively charged: the left or the right?

13.)

Cool Problem

A bar of mass m and length l is given an initial velocity v_0 and released so that it slides on two parallel frictionless rails in the presence of a magnetic field B as shown. Find v as a function of time.



14.)

$$F = -ILB \text{ and } F = ma = m \frac{dv}{dt}, \text{ so } -ILB = m \frac{dv}{dt}$$

$$I = \frac{BLv}{R} \text{ (from previous problem)}$$

$$-\frac{BLv}{R}(\ell B) = m \frac{dv}{dt} \text{ Rearrange to get:}$$

$$\frac{dv}{v} = -\frac{B^2 L^2}{mR} dt, \text{ then integrate both sides to get}$$

$$\int_{v_0}^v \frac{dv}{v} = \int_0^t -\frac{B^2 L^2}{mR} dt$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{B^2 L^2}{mR} t, \text{ or (with } \tau = \frac{1}{\frac{B^2 L^2}{mR}} \text{): } \ln\left(\frac{v}{v_0}\right) = -t / \tau$$

$$\frac{v}{v_0} = e^{-t/\tau}, \text{ or } v = v_0 e^{-t/\tau}$$

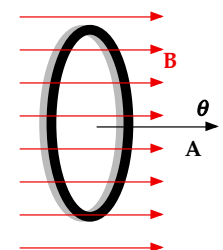
15.)

Lenz's Law

Faraday's Law allows us to calculate the magnitude of an induced emf (and current), but to determine direction, we have to resort to Lenz's Law.

Lenz's Law states that the polarity of the induced emf is such that it tends to produce a current that will create a magnetic flux to *oppose* the change in magnetic flux through the loop.

$$\mathcal{E} = \frac{-d\Phi_B}{dt}$$



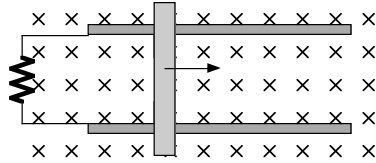
16.)

Lenz's Law

Which direction is current flowing in the following loop?

Procedure:

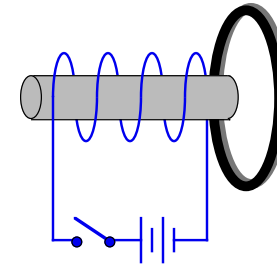
- 1.) Determine direction of the external magnetic field.
- 2.) Determine whether the flux in the area is increasing or decreasing.
- 3.) If the flux is decreasing, the two magnetic fields (the induced and the external) have to be in the *same* direction. If the flux is increasing, the two magnetic fields have to be in the *opposite* direction.
- 4.) To determine the direction of the induced current flow, lay your right hand on coil with fingers curling in the appropriate direction so that the magnetic field matches your conclusion in Part 3.
- 5.) For this example, the flux is increasing so the induced B-field will be in the same direction as the external B-field (i.e., into the page). The direction of current that will produce that B-field is clockwise.



17.)

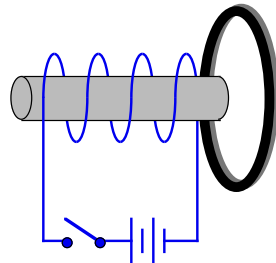
Lenz's Law

Examine the following diagram. Find the direction of the induced current in the core at the instant the switch is closed, after the switch has been closed for several seconds, and when the switch has been opened again.



18.)

The magnetic field in the primary coil will be to the left (second right-hand rule). The flux will increase as the switch is closed, so the B-field through the secondary coil will be to the right. That will be produced by an induced current in the secondary coil that is clockwise.



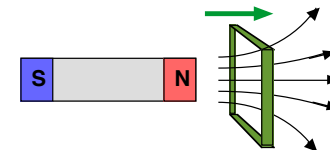
A few seconds after the switch is thrown, the primary coil will have a magnetic field down its axis but the field won't be changing so there will be no induced EMF anywhere in the system.

When the switch is opened, the primary coil will still have a magnetic field down its axis to the left but that field (hence magnetic flux) will be decreasing. As such, the secondary coil's induced magnetic field will be in the *same* direction as the primary field and the induced current in the secondary coil that will generate that circumstance will be counterclockwise.

19.)

Lenz's Law

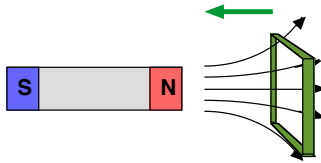
Which direction will the induced current flow if a loop of wire as shown?



20.)

Lenz's Law

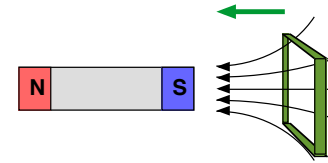
Which direction will the induced current flow if a loop of wire as shown?



21.)

Lenz's Law

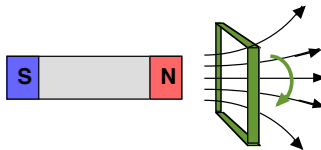
Which direction will the induced current flow if a loop of wire as shown?



22.)

Lenz's Law

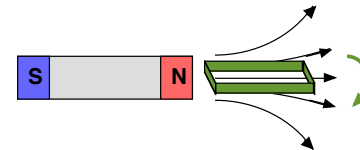
Which direction will the induced current flow if a loop of wire as shown?



23.)

Lenz's Law

Which direction will the induced current flow if a loop of wire as shown?



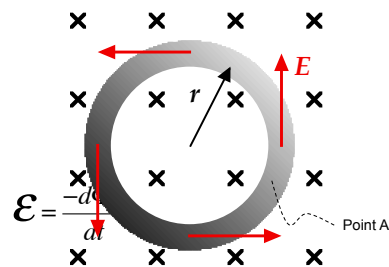
24.)

Induced *emfs* and E Fields

We've determined that a changing magnetic flux induces an *emf* and a resulting current flow in a conducting loop, so... an electric field must be present! The relationship we've used for this before is $\Delta V = -\int \vec{E} \cdot d\vec{s}$.

There is a problem with this, though. The circulation of the electric field around the loop is clearly non-zero, but the voltage difference between Point A and Point A (i.e., around a *closed* path) is zero. So what's the deal?

The deal is that voltages are modified potential energy functions, and potential energy must be associated with conservative forces. The force generated by a changing magnetic flux is *non-conservative*. So what to do?



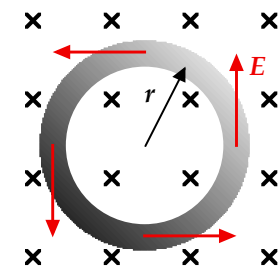
25.)

Induced *emfs* and E Fields

For a conducting loop in a *changing* magnetic field (it has to be changing for this to work), Faradays law ($\mathcal{E} = \frac{-Nd\Phi_B}{dt}$) states that a current-driving EMF will be induced in the coil, thereby setting up the required electric field. In other words:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

where E is a **non-conservative, time-varying field** generated by the **changing magnetic flux through the coil**.



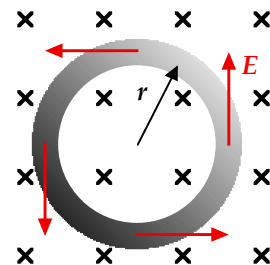
26.)

Induced *emfs* and E Fields

How strong is this electric field? To derive an expression for E :

Noting that the magnetic flux through the coil is $\vec{B} \cdot \vec{A} = B(\pi r^2) \cos 0^\circ$, and assuming there is only one loop in the coil (i.e., $N=1$), we can write:

$$\begin{aligned} \oint \vec{E} \cdot d\vec{s} &= -\frac{Nd\Phi_B}{dt} \\ \Rightarrow \oint E ds &= -\frac{d(B(\pi r^2))}{dt} \\ \Rightarrow E(2\pi r) &= -\frac{(\pi r^2)d(B)}{dt} \\ \Rightarrow E &= -\frac{\pi r^2}{2\pi r} \frac{dB}{dt} \\ \Rightarrow E &= -\frac{r}{2} \frac{dB}{dt} \end{aligned}$$



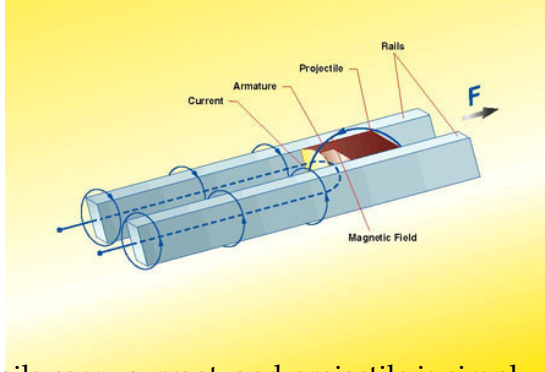
27.)

Electromagnetic Induction Demo

Examine the devices in class, and make sure you understand how they work!

28.)

What's a Rail Gun?



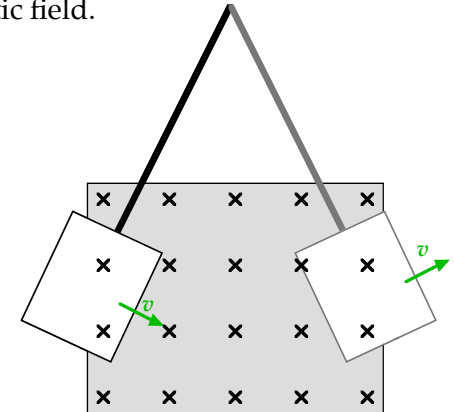
Large rails carry current, and projectile is simply a conductor. Magnetic field created by rails causes a force on the current carrying projectile, according to $F = I \ell \times B$.

29.)

Eddy Currents

If changing magnetic flux produces emf and currents in a circuit, those same changes in magnetic flux can induce *eddy currents*=circulating free charges in a bulk metal moving through a magnetic field.

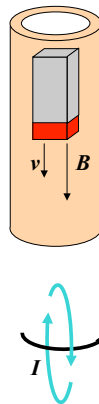
Because eddy currents produce retarding forces, they can be used in as braking systems for mass transit systems. If eddy currents are *not* desired, then the bulk metal is often split into thin layers that are laminated together.



30.)

Eddy Currents

Examine these additional examples of eddy currents in action.



31.)

Maxwell's Equations

At this point in the year, we know all four of Maxwell's equations, which summarize everything we know about electricity and magnetism.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

Gauss's Law-- electric charge distributions produce electric fields

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss's Law for Magnetism-- net magnetic flux through any closed surface is 0 (no magnetic monopoles)

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Faraday's Law of Induction-- relates emf of a closed path to the rate of change of magnetic flux through surface.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \left(+\mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \right)$$

Ampere's Law-- relates currents & magnetic fields

32.)

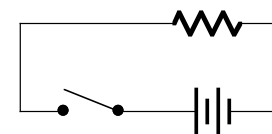
Inductance

- We've already learned that a changing magnetic flux induces emf and currents = *electromagnetic induction*
- Now... what are the consequences of that induction?
- Self-induction, inductors, mutual induction, transformers, RL circuits, RLC circuits...

33.)

Self-Inductance

Examine this simple circuit. When we throw the switch, we don't just get an immediate current flow $I = V/R$ from the battery...



According to Faraday's Law:

1. current begins to flow, increasing w/ time
2. As current increases in circuit, so does magnetic flux through the loop
3. Increasing flux induces an emf that opposes the change in net magnetic flux through the loop (Lenz's Law)
4. The opposing emf results in a *gradual* increase in current.

34.)

Self-Inductance

We can quantify the self-inductance as follows:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{Nd\Phi_B}{dt}$$

$\Phi_B \propto B \propto I$, so $\mathcal{E}$ is proportional to dI , and we can write

$$\mathcal{E} = -L \frac{dI}{dt}, \text{ or } L = -\frac{\mathcal{E}_L}{dI/dt}$$

The unit of inductance is the [Volt-second]/[Ampere], also called a "Henry"!

35.)

What Does It All Mean?

Resistance: A parameter whose magnitude tells you how much resistance there is to charge flow in a circuit. A large resistance means a small current for a given voltage.

$$R = \frac{\mathcal{E}}{I}$$

Capacitance: A parameter whose magnitude tells you how much capacity to store charge a capacitor has. A large capacitance means a large amount of charge can be stored per unit volt across the plates.

$$C = \frac{Q}{\mathcal{E}}$$

Inductance: A parameter whose magnitude tells you the rate at which the magnetic flux changes with a change of current through the coil. A large inductance means a large flux change for a given current change.

$$L = -\frac{\mathcal{E}_L}{dI/dt} = \frac{d\Phi_B}{di}$$

36.)

Inductance of a Coil

For a coil carrying a current I and with N turns:

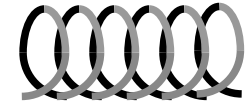
$$\mathcal{E} = -L \frac{dI}{dt}, \text{ and } \mathcal{E} = -N \frac{d\Phi_B}{dt}$$

$$\Rightarrow L_{\text{coil}} = N \frac{d\Phi_B}{dI}$$

37.)

Inductance of a Solenoid

In some circuits, this “resistance to a change in current flow” is actually desired. Such circuits include an “inductor,” a coil that’s often in the shape of a solenoid.



How do we calculate the inductance of a solenoid, with N turns of wire in a length ℓ ?

$$L = N \frac{\Delta\Phi_B}{\Delta I} = N \frac{BA}{I}$$

as $B_{\text{solenoid}} = \frac{\mu_0 NI}{\ell}$, we can write:

$$L = N \frac{(\frac{\mu_0 NI}{\ell})A}{I} = \frac{\mu_0 N^2 A}{\ell}$$

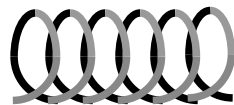
continuing with $L = \frac{\mu_0 N^2 A}{\ell}$
and noting that $N = n\ell$, we can write

$$L = \mu_0 n^2 \ell A = \boxed{\mu_0 n^2 V}$$

38.)

Example

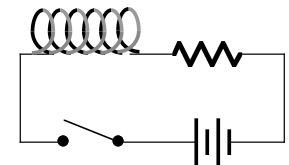
Calculate the inductance of a solenoid that has 300 turns of wire in a 25.0 cm length, w/ a cross-sectional area of 400 cm². Then calculate the self-induced emf in the solenoid if the current is decreasing at a rate of 50.0 A/s.



39.)

RL Circuits

Circuits with a coil in them, as the one shown here, have a self-inductance due to the presence of the coil--the self inductance due to the geometry of the circuit, in these cases, is considered negligible.



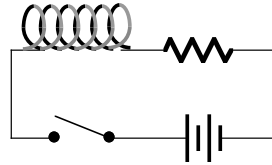
When the switch is thrown, current begins to flow in the circuit. As current increases with time, the inductor resists this increase with a self-induced *back emf* according to $\mathcal{E}_L = -L \cdot dI/dt$. Note that there is a *drop* in potential from left to right as current flows across the inductor.

40.)

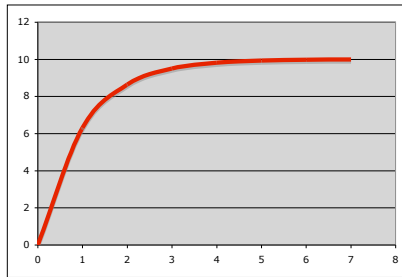
Kirchoff's Analysis

Insert messy derivation here to get:

$$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}), \text{ where } \tau = \frac{L}{R}$$



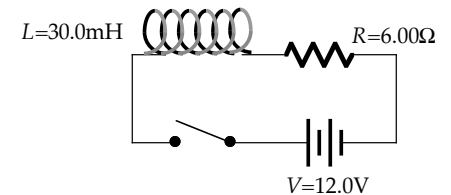
The time constant here represents the amount of time it takes for the current to reach 0.632 of final I ($=V/R$).



41.)

Example

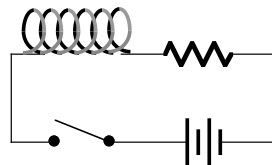
The switch is closed at time $t=0$ s. Find the time constant of the circuit, the current at $t=2.00$ ms, and the current and V across the resistor after one *time constant* has passed.



42.)

Energy in Inductors?

When an inductor causes an induced *emf*, some of the battery's energy is dissipated in the resistor, and some gets stored in the inductor. How much?



$$\mathcal{E} - IR - L \frac{dI}{dt} = 0$$

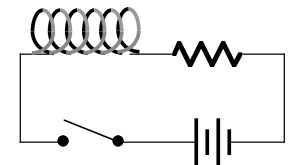
$$\Rightarrow \mathcal{E} = IR + L \frac{dI}{dt}$$

$$\Rightarrow I\mathcal{E} = I^2R + LI \frac{dI}{dt}$$

43.)

Energy in Inductors?

So $LI(dI/dt)$ represents the rate that energy is getting stored in the inductor. How can we determine the *amount* of energy stored in the inductor, U_B ?

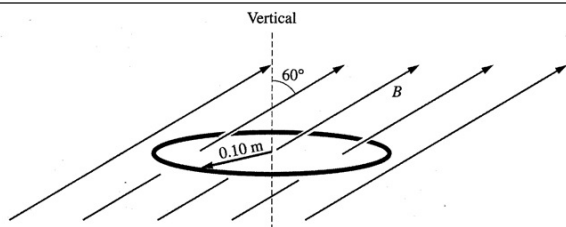


$$\frac{dU_B}{dt} = LI \frac{dI}{dt}$$

$$\Rightarrow \int_0^U dU_B = \int_0^I (LI) dI$$

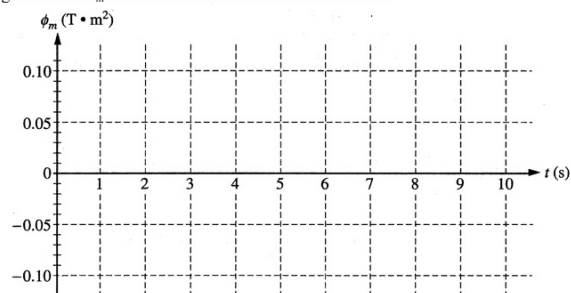
$$\Rightarrow U_B = \frac{1}{2} LI^2$$

44.)

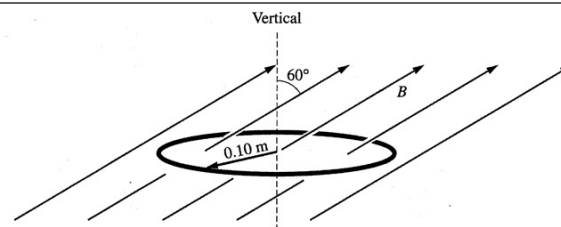


2002E3. A circular wire loop with radius 0.10 m and resistance 50 Ω is suspended horizontally in a magnetic field of magnitude B directed upward at an angle of 60° with the vertical, as shown above. The magnitude of the field in teslas is given as a function of time t in seconds by the equation $B = 4(1 - 0.2t)$.

- Determine the magnetic flux Φ_m through the loop as a function of time.
- Graph the magnetic flux Φ_m as a function of time on the axes below.



45.)



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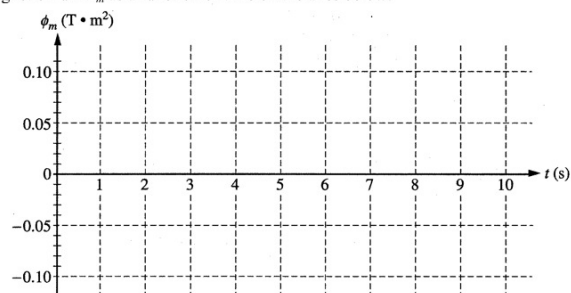
- Determine the magnetic flux Φ_m through the loop as a function of time.

$$\begin{aligned}\Phi_B &= \vec{B} \cdot \vec{A} \\ &= [4(1 - .2t)](\pi r^2) \cos 30^\circ \\ &= [4(1 - .2t)](\pi r^2) \cos 30^\circ\end{aligned}$$

45.)

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- Graph the magnetic flux Φ_m as a function of time on the axes below.

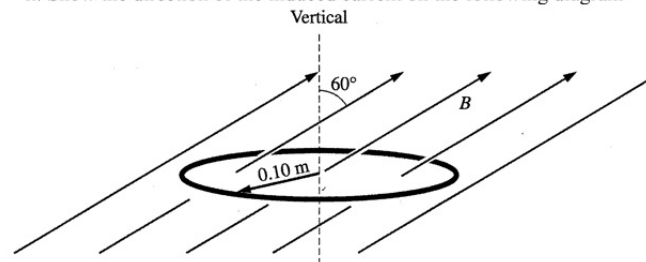


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45.)

2002 E&M3

- Determine the magnitude of the induced emf in the loop.
- Determine the magnitude of the induced current in the loop.
 - Show the direction of the induced current on the following diagram



- Determine the energy dissipated in the loop from $t = 0$ to $t = 4$ s

46.)