11.1 $\vec{M} = 2\hat{i} - 3\hat{j} + 1\hat{k}$, $\vec{N} = 4\hat{i} + 5\hat{j} - 2\hat{k}$.

$\vec{M} \times \vec{N} = ?$

$\underline{2i - 3j + 1k} \times \underline{4i + 5\hat{j} - 2k}$

$2\hat{i} \times 5\hat{j} = 10k$

$2i \times -2k = -4(-j) = 4j$

$(i \times i = 0)$

$-3j \times 4i = -12(-k) = 12k$

$-3j \times -2k = 6i$

$(j \times j = 0)$

$1k \times 4i = 4j$

$1k \times 5j = 5(-i) = -5i$

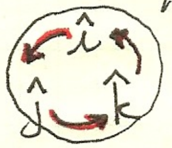
$(k \times k = 0)$

$\vec{M} \times \vec{N} = 6i + -5i + 4j + 4j + 10k + 12k$

$= \boxed{i + 8j + 22k}$

There are a number of different ways to remember the rules for cross-products, where order matters. Some people write determinant matrices, but here's a quick, easy system:

Counterclockwise is +
Clockwise is −



$i \times j = \hat{k}$	$\hat{i} \times \hat{k} = -\hat{j}$
$j \times k = \hat{i}$	$\hat{k} \times \hat{j} = -\hat{i}$
$k \times i = \hat{j}$	$\hat{j} \times \hat{i} = -k$

11.3 $\quad \vec{A} = i + 2j \; ; \; \vec{B} = -2i + 3\hat{j}$

a) $\vec{A} \times \vec{B} = (i + 2j) \times (-2i + 3j)$

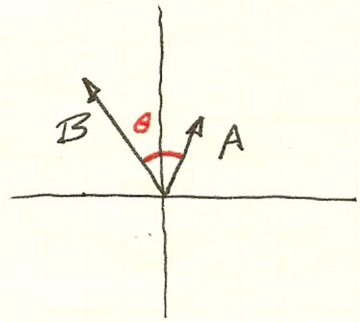
$$= (i \times +3j) + (2j \times -2i)$$

$$= 3k + (-4)(-k)$$

$$= \boxed{7k}$$

b) angle θ between $\vec{A}$ & $\vec{B}$?

$\vec{A} \times \vec{B} = AB \sin\theta$



$$|A| = \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{1^2 + 2^2}$$

$$= \sqrt{5} = 2.24$$

$$|B| = \sqrt{B_x^2 + B_y^2}$$

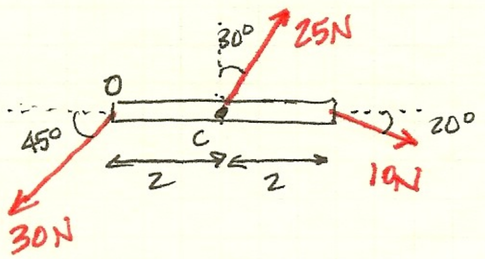
$$= \sqrt{(-2)^2 + 3^2}$$

$$= \sqrt{13} = 3.61$$

$$7\cancel{k} = \sqrt{5}\,\sqrt{13}\,\sin\theta$$

$$\theta = \sin^{-1}\left(\frac{7}{\sqrt{5}\sqrt{13}}\right) = \boxed{60.3^\circ}$$

11.5

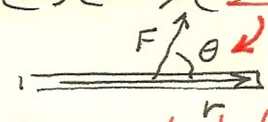


a) Calculate net Torque on beam about axis at O.

$$\tau_{net} = \tau_{30} + \tau_{25} + \tau_{10}$$

$$= r \times F + r \times F + r \times F$$

$$= (0)(30) \; +^{+} (2m)(25N)(\sin 60) \; + \; -(4m)(10N)(\sin 20)$$



Torque is in ccw direction

θ is angle between r & F

Torque in cw direction

τ_{net} (about O) is $\boxed{29.6 \text{ Nm, counterclockwise}}$

b) net Torque about C, center of rod.

$$\tau_{net} = \tau_{30} + \tau_{25} + \tau_{10}$$

$$= rF\sin\theta + rF\sin\theta + rF\sin\theta$$

$$\stackrel{+}{=} (2m)(30N)(\sin 45) + (0m)(25) + -(2m)(10N)(\sin 20)$$

$$= \boxed{35.6 \text{ N m, counterclockwise}}$$

11.7 If $|\vec{A}\times\vec{B}| = \vec{A}\cdot\vec{B}$, what is angle between $\vec{A}$ + $\vec{B}$?

$|\vec{A}\times\vec{B}| = AB\sin\theta$

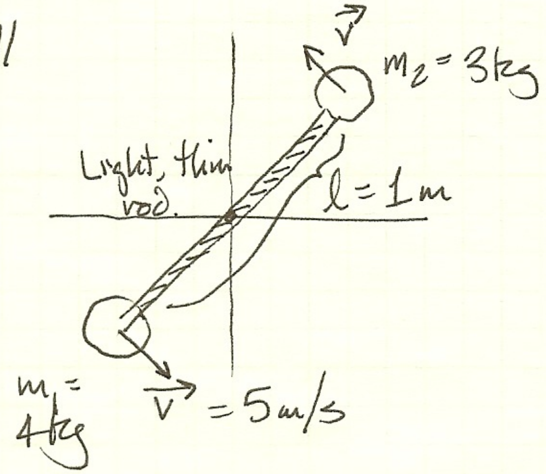
$\vec{A}\cdot\vec{B} = AB\cos\theta$

$$\frac{AB\sin\theta}{AB\cos\theta} = \frac{AB\cos\theta}{AB\cos\theta}$$

$$\tan\theta = 1$$

$$\theta = \tan^{-1}(1) = \boxed{45^\circ}$$

11.11



Determine angular momentum of the system.

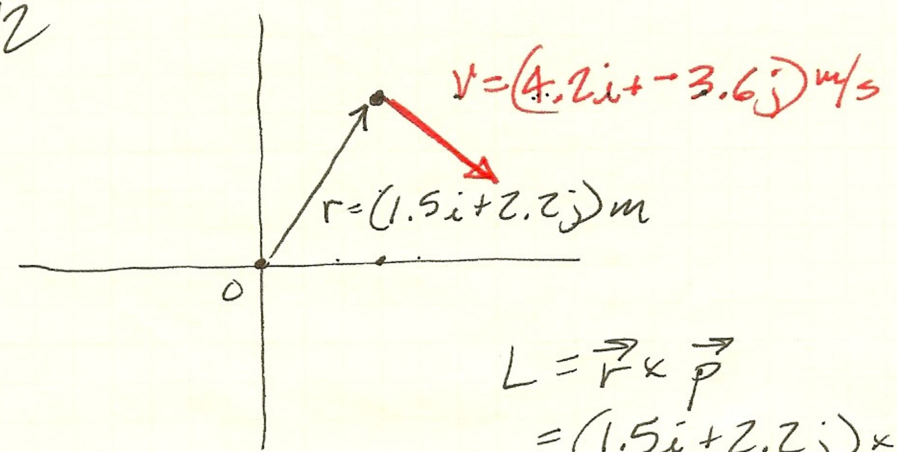
The rod connecting the particles is "light," so mass is negligible. Consider system as two particles then.

Total angular momentum $L_{system} = L_1 + L_2$

$$L_{particle} = \vec{r} \times \vec{p}$$
$$= \vec{r} \times m\vec{v}$$
$$= r m v \sin\theta$$

$$L_{total} = r_1 m_1 v_1 \sin\theta + r_2 m_2 v_2 \sin\theta$$
$$= (0.5m)(4kg)(5 m/s)(\sin 90) + (0.5)(3kg)(5 m/s)(\sin 90)$$
$$= 10 + 7.5$$
$$= \boxed{17.5 \text{ kg m}^2/\text{s, counterclockwise}}$$

11.12



$$L = \vec{r} \times \vec{p}$$

$$= (1.5i + 2.2j) \times (4.2i - 3.6j)(1.5kg)$$

$$= (1.5i \times -3.6j) + (2.2j \times 4.2i)(1.5kg)$$

$$= (-5.4)k + 9.24(-k)(1.5kg)$$

$$= (-14.6k)(1.5kg)$$

$$= \boxed{-22.0\ kg\,\frac{m^2}{s}}$$

11.15 For a particle of mass $m = 2\,\text{kg}$,

$$r = (6\hat{i} + 5t\,\hat{j})\,\text{m}$$

$$
\begin{aligned}
L &= \vec{r} \times \vec{p} \\
&= \vec{r} \times m\vec{v} \\
&= \vec{r} \times m\,\frac{d\vec{r}}{dt} \\
&= (6\hat{i} + 5t\,\hat{j})_{\text{m}} \times (2\,\text{kg})(0i + 5j)\ \text{m/s} \\
&= (6i \times 10j) + (5t\,j \times 0i) \\
&= \boxed{+60\,k\ \text{kg}\,\text{m}^2/\text{s}}
\end{aligned}
$$

11.22 If $L = I\omega$, & $K = \frac{1}{2} I \omega^2$

$$\cancel{I = \frac{L}{\omega}}$$

$$\cancel{K = \frac{1}{2}\left(\frac{L}{\omega}\right)\omega^2}$$

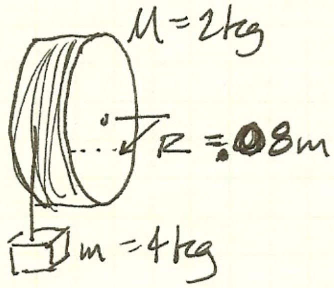
$$\cancel{= \frac{L\omega}{2}}$$

$$\omega = \frac{L}{I}$$

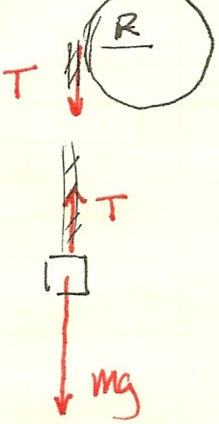
$$K = \frac{1}{2} I \left(\frac{L}{I}\right)^2$$

$$= \boxed{\frac{L^2}{2I}}$$

11.18



a) net τorque about axis of pulley? Have to solve taking into account both masses, & the fact that the mass is "falling". $\tau_{pulley} \neq Rmg$!!!



$\Sigma\tau = I_{hoop}\alpha$

$\Sigma F = ma$

This is true, $\tau_{pulley} = 1.05\,N{\cdot}m$. I misread question, though. They asked for torque "on the system" ie, from external forces (gravity). In that case, yes, $\tau_{ext} = r \times F_g$ $= rmg\sin\theta$ $= \boxed{3.14\,N{\cdot}m}$

$$\Sigma\tau = I_{hoop}\alpha$$

$$R \times T = MR^2\frac{a}{R}$$

solve to get

$$a = \frac{T}{M}$$

$$\Sigma F = ma$$

$$mg - T = ma$$

$$mg - T = m\left(\frac{T}{M}\right)$$

$$mg = \frac{m}{M}T + T$$

$$mg = \left(\frac{m}{M} + 1\right)T$$

$$T = \frac{mg}{\frac{m}{M}+1} = \frac{mMg}{m+M} = \frac{(4)(2)(9.8)}{4+2} = 13.1\,N$$

$$\tau_{pulley} = r \times F = (0.08)(13.1) = \boxed{1.05\,N{\cdot}m}$$

b) $L_{system} = L_{pulley} + L_{mass}$

$$= I\omega + rmv\sin\theta$$

$$= MR^2\left(\frac{v}{R}\right) + Rmv$$

$$= 2\cdot(0.08)v + (0.08)(4)v = \boxed{0.48v\,kg\frac{m^2}{s}}$$

varies w/ v.

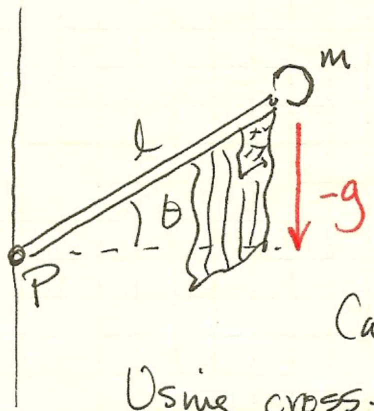
c) Use $\tau = \frac{dL}{dt}$ to find acceleration.

$$\tau_{ext} = \frac{dL}{dt} \longrightarrow r \times F_{ext} = \frac{d}{dt}(0.48v)\ kg\frac{m^2}{s}$$

$$(0.08)(4)(9.8) = (0.48)a$$

$$a = \boxed{6.53\,m/s^2}$$

11.21



ball comes loose, free-falls.

a) L_{ball} about P as a function of time.

Can solve w/ $L = \vec{r} \times \vec{p}$, or $rmv \sin\theta$

Using cross-product:

$$L = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

$$= (l\cos\theta\,\hat{i} + l\sin\theta\,\hat{j}) \times m(v_x\hat{i} + v_y\hat{j})$$

$$v_y = v_i + at = 0 + -gt, \text{ so}$$

$$L = (l\cos\theta\, i + l\sin\theta\, j) \times m(0i - gt\,\hat{j})$$

$$= [l\cos\theta\, i \times m(-gt)\,\hat{j}] + [(l\sin\theta)\, j \times (m0)\, i]$$

$$= \boxed{-mgl\cos\theta\, t \quad kg\cdot m^2/s}$$

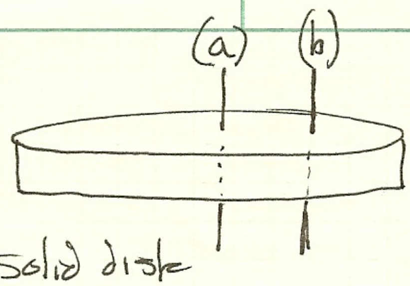
b) Angular momentum of system changes because it is not isolated; an external force (earth's gravity) creates an external torque, & $\tau_{ext} = \frac{\partial L}{\partial t}$.

c) Rate of change of angular momentum is $\frac{\partial L}{\partial t}$

$$\frac{\partial L}{\partial t} = \frac{\partial}{\partial t}(-mgl\cos\theta\, t)$$

$$= \boxed{-mgl\cos\theta}$$

11.25



Solid disk

a) Angular momentum of disk when rotating about center.

$$L = I\omega$$

$I_{disk} = \frac{1}{2}MR^2$, so

$$L_{disk} = \frac{1}{2}MR^2\omega$$

$$= \frac{1}{2}(3\,kg)(0.2\,m)^2(6\,rad/s) = \boxed{0.36\,kg\,m^2/s}$$

b) When moving the axis to location B, the moment of inertia of the disk changes. We can calculate the new I using the parallel axis theorem.

$$I = I_{cm} + MD^2$$

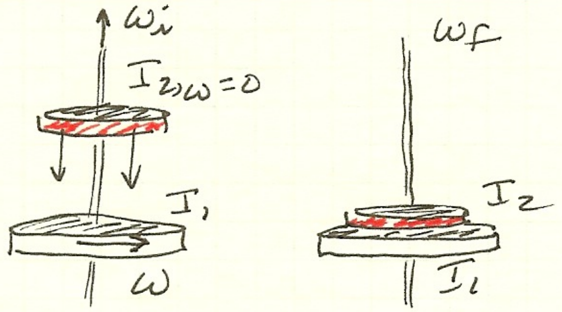
$$= \frac{1}{2}MR^2 + M\left(\frac{R}{2}\right)^2 = \frac{3}{4}MR^2$$

$$L_{disk} = I\omega = \left(\frac{3}{4}MR^2\right)(\omega)$$

$$= \left(\frac{3}{4}\right)(3)(0.2)^2(6) = \boxed{0.54\,kg\,m^2/s}$$

As expected the I & L are higher in the second case because some of the mass is located much farther away from the axis of rotation.

11.30



a) Calulate ω_f after upper disk falls onto lower disk.

Because no external Torques act, $\tau_{ext} = \frac{dL}{dt} = 0$.
Total angular momentum of system remains constant.

$$\cancel{\tau_i = \tau_f}$$

$$\Sigma L_i = \Sigma L_f$$

$$I_1\omega_1 + I_2\omega_2 = I_1\omega_1' + I_2\omega_2'$$

$$I_1\omega + 0 = (I_1 + I_2)\omega'$$

$$\omega' = \boxed{\frac{I_1}{I_1 + I_2}\omega}$$

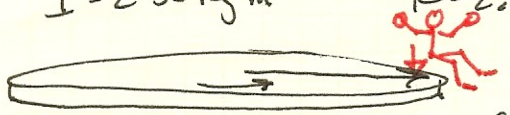
b) We know energy must have been dissipated as heat in the collision. Final to initial K rotational ratio:

$$\frac{K_{rot}'}{K_{rot}} = \frac{\frac{1}{2} I_f \omega_f^2}{\frac{1}{2} I_i \omega_i^2} = \frac{(I_1 + I_2)\left(\frac{I_1}{I_1 + I_2}\omega\right)^2}{(I_1)\omega^2}$$

$$= \boxed{\frac{I_1}{I_1 + I_2}}$$

This value is obviously < 1, confirming our notion that energy must have been "lost" to heat.

11.31 $I = 250\,kg \cdot m^2$ $R = 2.0\,m$



$$\omega = 10\frac{rev}{min} \times \frac{2\pi\,rad}{rev} \times \frac{1\,min}{60\,s} = \underline{1.05\,rad/s}$$

Kid hops onto edge of rotating merry-go-round.

New ω?

No external Torque outside of merry-go-round/kid system, so $\tau_{ext} = 0 = \frac{dL}{dt}$, & angular momentum is conserved.

$$\Sigma L_i = \Sigma L_f$$

$$L_{merry\text{-}go\text{-}round} + L_{kid} = L_{merry\text{-}go\text{-}round}' + L_{kid}'$$

$$I\omega + 0 = I\omega_f + rmv_f$$

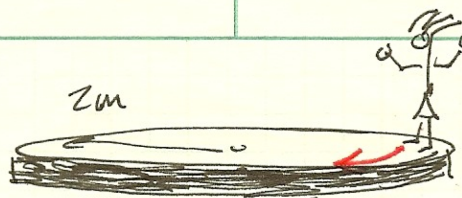
$v = r\omega$, so

$$I\omega = I\omega_f + MR^2\omega_f$$

$$\omega_f = \frac{I\omega}{I + MR^2} = \frac{(250)}{(250 + 25 \cdot 2^2)} 1.05\,rad/s$$

$$= \boxed{0.75\,rad/s} \text{ or } 7.16\,rev/min$$

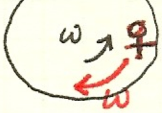
11.35



$m = 60\,kg$ ♀

$I = 500\,kg\,m^2$

$\omega_i = 0$



$v = 1.5\,m/s$ relative to earth.

a) Mechanical energy of the system is not constant — by pushing w/ her legs, she's doing work on the system. — that's why it now has K!

b) Momentum is constant for system. No external forces or torques are being applied.

c) Yes, angular momentum is constant — no external torques are being applied.

d) Turntable rotates counterclockwise in response to woman's motion.

$$\Sigma L_i = \Sigma L_f$$

$$0 = L_{woman} + L_{turntable}$$

$$0 = -rmv + I\omega$$

$$0 = -(2)(60)(1.5) + (500)\,\omega$$

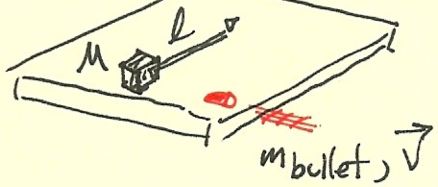
$$\omega = \boxed{0.36 \text{ rad/s}}, \text{ counterclockwise}$$

e) Work done by women comes from food metabolised. How much energy added? Initial K was 0, so energy burned by women created all $K_{rotational}$ + K_{trans}.

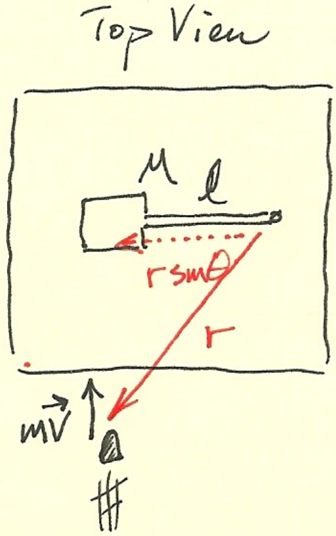
$$\Sigma K_{rot} = \tfrac{1}{2} I\omega^2 ; \; K_{trans} = \tfrac{1}{2} mv^2$$

$$\tfrac{1}{2}(500)(0.36)^2 + \tfrac{1}{2}(60)(1.5)^2 = \boxed{99.9\,J}$$

11:37



Bullet embeds in block, causing the block-rod-bullet system to start rotating.



a) Find angular momentum L of system about the pivot.

$$L_{system} = L_{bullet} + L_{block\text{-}rod}$$
$$L_{initial} = \vec{r} \times m\vec{v} + I\omega$$
$$= \ell m v + 0$$
$$= \boxed{\ell m v}$$

Note that $r\sin\theta = \ell$, regardless of m & θ in this system.

b) What fraction of K converted to ΔE_{int}.

Strategy: find ω of system after collision so we can compare energies.

$$\Sigma L_i = \Sigma L_f$$

$v = r\omega$

$$m\ell v \text{ (from above)} = I\omega + rmv$$
$$= I\omega + mr^2\omega = (I + mr^2)\omega$$

$I = M\ell^2$ for block, so...

$$m\ell v = (M+m)\ell^2\omega$$
$$\omega = \frac{m}{(M+m)\ell} v$$

Fraction of K converted to $\Delta E_{int} = \frac{\Delta E_{int}}{K_i} = \frac{K_i - K_f}{K_i}$

$$\frac{\frac{1}{2}mv^2 - \frac{1}{2}I\omega^2}{\frac{1}{2}mv^2} = \frac{\cancel{\frac{1}{2}}m\cancel{v^2} - \cancel{\frac{1}{2}}\left[(M+m)\cancel{\ell^2}\right]\left(\frac{m}{(M+m)\cancel{\ell}}\cancel{v}\right)^2}{\cancel{\frac{1}{2}}m\cancel{v^2}}$$

$$= 1 - \frac{m}{M+m}$$

$$= \frac{M+m}{M+m} - \frac{m}{M+m} = \boxed{\frac{M}{m+M}}$$