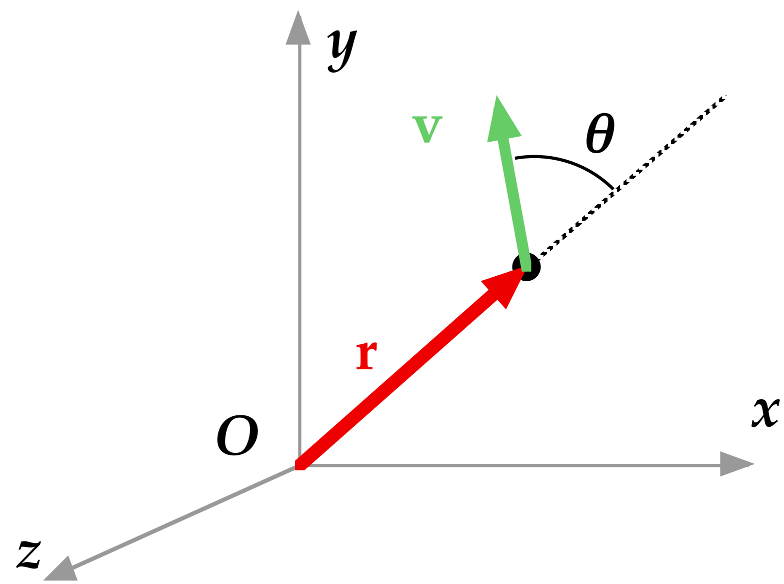
Angular Momentum

Angular Momentum

For a particle traveling with velocity **v** relative to a point *O*, the particle has an angular momentum

$$\boxed{\vec{\mathbf{L}} \equiv \vec{\mathbf{r}} \times \vec{\mathbf{p}}}$$



When is the angular momentum for a particle 0?

When is the angular momentum for a particle = rmv?

Note that even particles traveling in a straight line have an angular momentum relative to a given location!

Example 1

What is the magnitude & direction of **L** relative to the origin for a 2-kg particle traveling at a constant (1**i**+2**j**) m/s at the instant it is at (2**i**-1**j**) m?

$$\mathbf{L} = \mathbf{r} \times m\mathbf{v}$$
$$\mathbf{L} = (2i - 1j) \times (2)(1i + 2j)$$
$$\mathbf{L} = (2i \times 4j) + (-1j \times 2i)$$
$$\mathbf{L} = 8k + 2k = 10k \quad kg \bullet m^2/s$$

What is its **L** 3 seconds later?

$$v = (1i + 2j)$$
$$r_f = r_i + vt = (2i - 1j) + (1i + 2j)3$$
$$r_f = 5i + 5j$$
$$\mathbf{L} = \mathbf{r} \times m\mathbf{v}$$
$$\mathbf{L} = (5i + 5j) \times (2)(1i + 2j)$$
$$\mathbf{L} = (5i \times 4j) + (5j \times 2i)$$
$$\mathbf{L} = 20k + -10k = 10k \quad kg \bullet m^2/s$$

Example 2

A particle with mass *m* moves with a velocity **v** in a counterclockwise circle with radius **r**, centered about the origin.

a) Find the magnitude & direction of its angular momentum relative to the origin.

$$\mathbf{L} = \mathbf{r} \times m\mathbf{v}$$

$$L = rmv \sin\theta = rmv$$

b) Find **L** in terms of ω.

$$\mathbf{L} = rmv$$

$$v = r\omega$$

$$L = rm(r\omega)$$

$$L = (r^2 m)\omega$$

$$L = I\omega$$

Torque & Ang. Mom?

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$

$$\vec{\tau} = \vec{\mathbf{r}} \times \frac{d\vec{\mathbf{p}}}{dt}$$

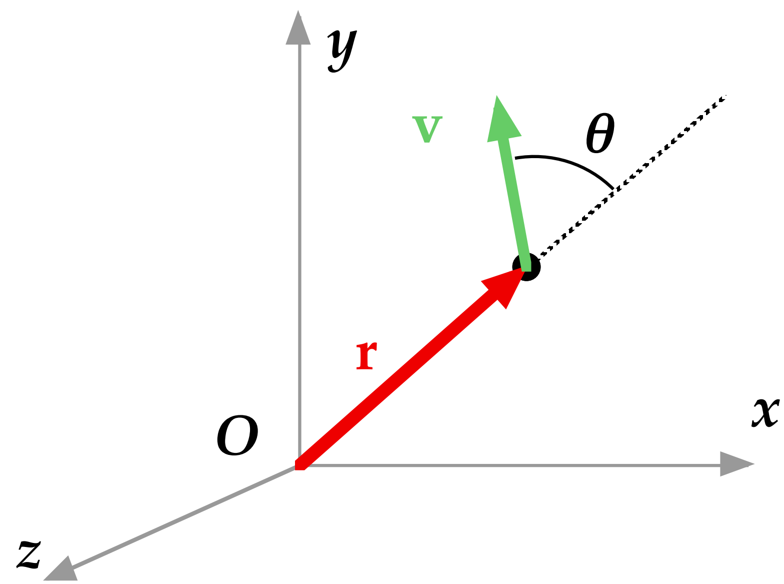
$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$$

$$\frac{d}{dt}\vec{\mathbf{L}} = \frac{d}{dt}\left(\vec{\mathbf{r}} \times \vec{\mathbf{p}}\right)$$

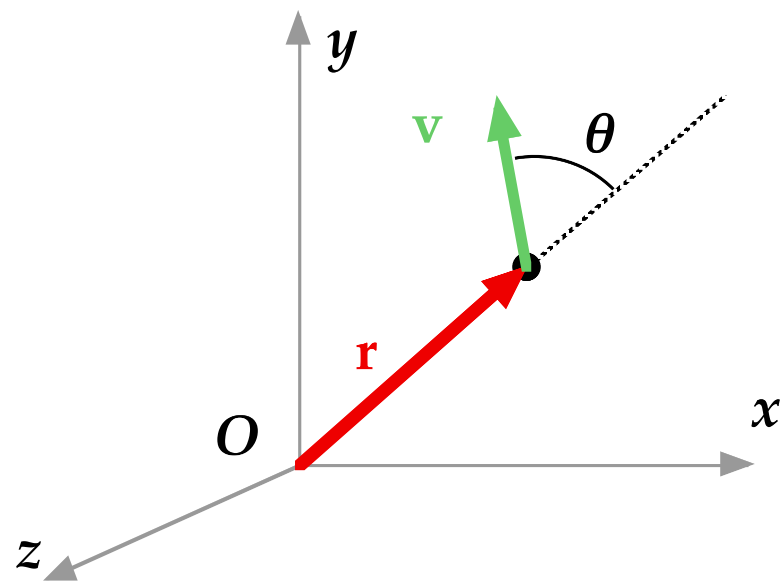
$$\frac{d\vec{\mathbf{L}}}{dt} = \vec{\mathbf{r}} \times \frac{d\vec{\mathbf{p}}}{dt} + \cancel{\frac{d\vec{\mathbf{r}}}{dt} \times \vec{\mathbf{p}}}$$

(**v** and **v** are parallel)

$$\frac{d\vec{\mathbf{L}}}{dt} = \vec{\tau}$$



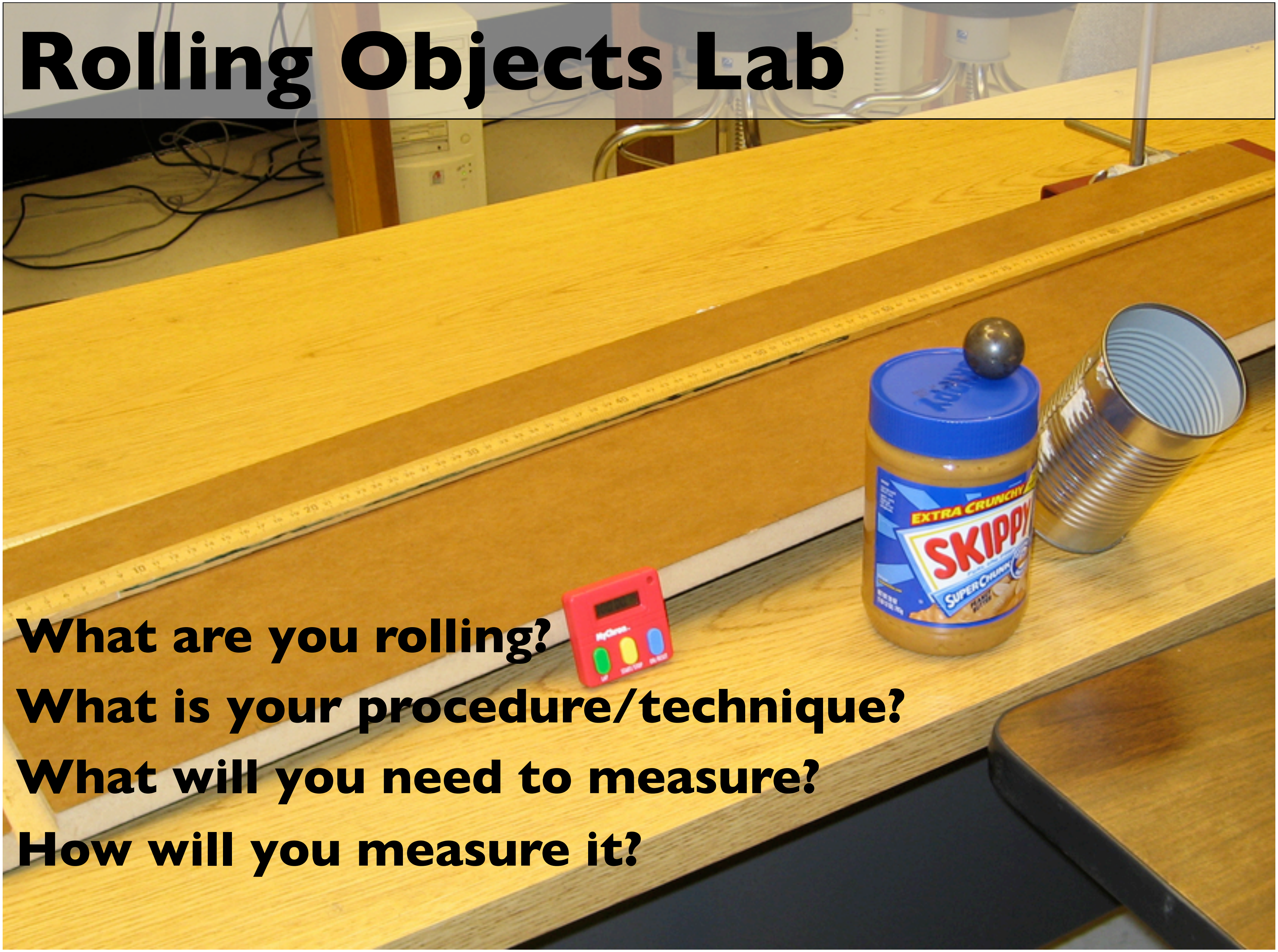
Torque & Ang. Mom!



$$\vec{\tau} = \frac{d\vec{\mathbf{L}}}{dt} \text{ (for a particle)}$$

$$\sum \vec{\tau}_{ext} = \sum \frac{d\vec{\mathbf{L}}_i}{dt} = \frac{d}{dt}\sum d\vec{\mathbf{L}}_i = \frac{d\vec{\mathbf{L}}}{dt} \text{ (for a system of particles)}$$

Rolling Objects Lab



What are you rolling?

What is your procedure/technique?

What will you need to measure?

How will you measure it?

Rolling Objects Lab

Procedure

Part I. Theoretical Development

Using first Force/Torque concepts, and then Energy (translational and rotational) concepts, develop equations that yield **position**, **velocity**, and **acceleration** as a function of **time** for three different objects rolling down an inclined plane: a hoop, a disk, and a sphere. It won't be surprising to find that at least two other variables—angle of incline θ and acceleration due to gravity **g**—appearing in your equations. Perhaps mass *m* and radius *R* of the objects will be a factor as well?

It shouldn't be surprising to find that the two different approaches yield identical equations. For this reason, you'll obviously you'll need to be *extremely clear* in showing your development in both approaches.

Part II. Experimental Confirmation

Perform experiments and collect data that will allow you to verify or refute your theoretical predictions. Again, because each group will be performing experiments of their own design, you'll need to be *extremely clear* in describing your experimental set-up and procedure.

Part III. Reporting Results

Your results should be reported both as data tables and as a series of three graphs—one for each type of object rolled down the incline—with two lines on each: one predicted by your calculations, and one of your experimental results. These graphs should include the functions that are being displayed in the body of the graph itself, and (obviously) include labels and units on *x* and *y* axes, as well as a title.

You'll also want to report the percent error between your predicted acceleration and your measured acceleration, which is easily determined from a regression of your data's trendline.

We expect that our experimental data will vary somewhat from our theoretical predictions. Is there a systematic variance between the two? How much error will be acceptable to you in this lab? Do you have reasonable explanations for any error?

Rolling Objects Lab

Additional Notes

In **Excel**, use grid lines around your data to help keep it visually organized. Be sure to set units and appropriate significant figures for columns. You can copy/paste Excel data cells into Word documents.

Use **Excel** to create graphs of data for each of your rolling objects. Your graph should include both a best-fit line for your data, and superimposed on that the predicted function, based on your theoretical analysis. See a follow-up document, the online video tutorial, or the instructor in class for info on how to do this.

Regarding *graphs*: larger is better. For this assignment, graphs should take up at least half a page.

Use **Word** for your document. In Word, you can use the Drawing toolbar and the grid to help keep objects in your drawings aligned with each other. Use *text boxes* liberally in your diagrams to label key components. Feel free to paste in photos into your document, where appropriate. Use Equation Editor (it comes with Microsoft Office as an optional install), highlight important equations in your mathematical development, and blurb extensively so that we can identify important steps in your development.

You may use other applications as well: LibreOffice, Apple's **Numbers** and **Pages**, GoogleDocs, LaTeX, etc., all may be used more or less successfully for analyzing your data, making drawings, and writing your report. The instructor will support you in using these alternatives as far as his experience with them allows.

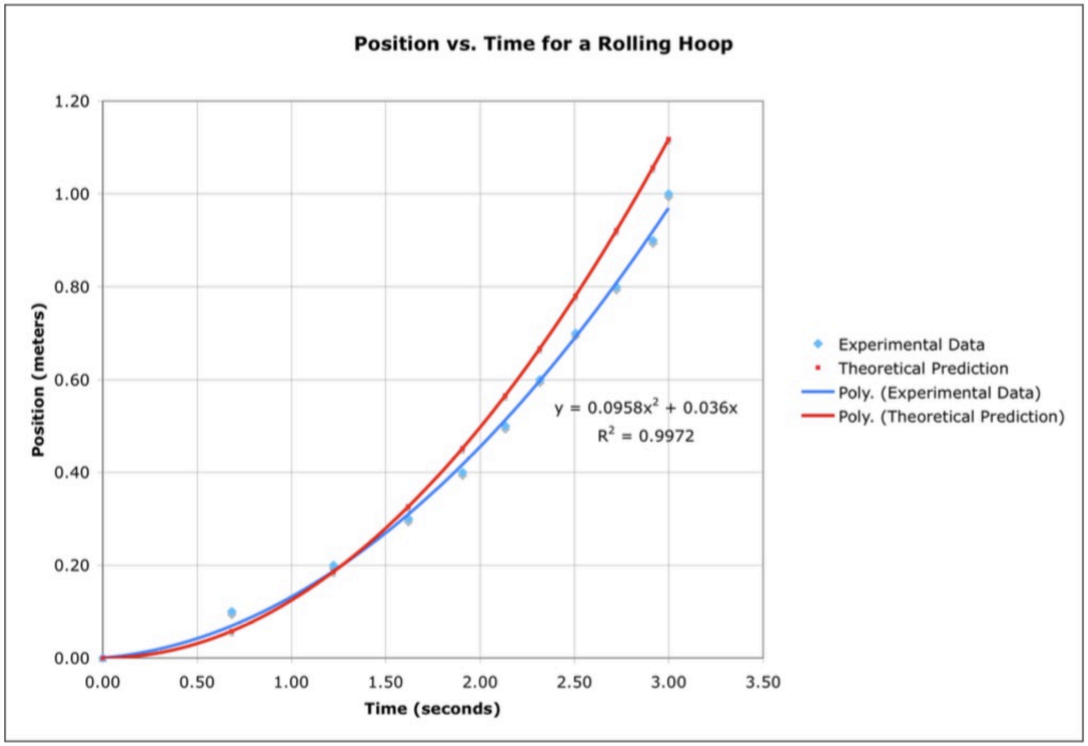
Rolling Objects Lab

Three Technical Challenges

1. Using Equation Editor (Word)

2. Creating graphics (Word)

3. Creating graphs (Excel)



How will you determine your error?
Compare $a_{theoretical}$ with $a_{experimental}$

What do the final results look like?

RO Production Path

Not required, but recommended?

- Develop equations of rolling motion (acceleration) for each of three objects, both by Force/Torque analysis and Energy/kinematics analysis.
- Select objects of three different types to roll.
- Design experiment to collect position-time data for objects. Don't forget to measure θ for your ramp so that you can predict a theoretical acceleration.
- Save/create data tables for use in report.
- Use Microsoft Word to begin creating document with appropriate sections.
- Make backup of Word document!
- Use Equation Editor to enter rolling objects derivations.
- Use Excel to create graphs of both your experimental data *and* your predicted (theoretical) model.

L for a system

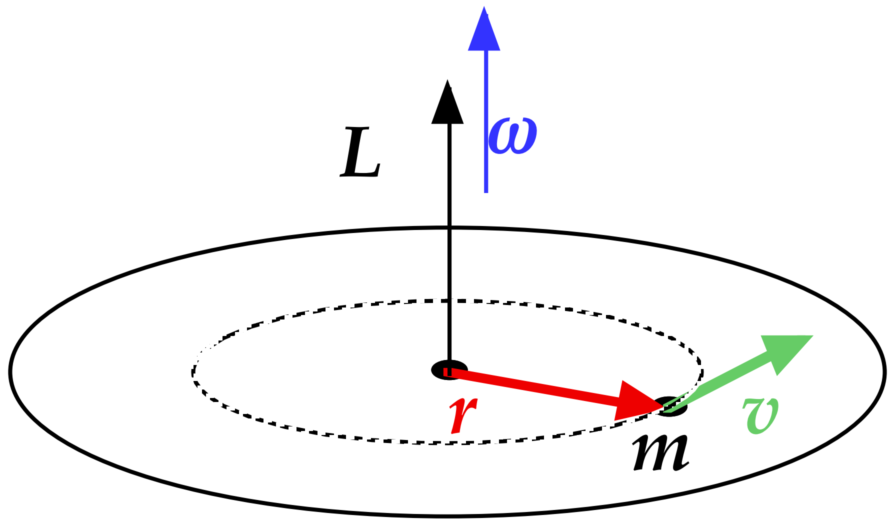
$$L = m_i r_i v_i$$

$$v = r\omega$$

$$L_i = m_i r_i (r_i \omega) = m_i r_i^2 \omega$$

$$L = \sum m_i r_i^2 \omega$$

$$\boxed{L = I\omega}$$



$$L = I\omega$$

$$\frac{dL}{dt} = \frac{d}{dt} I\omega$$

$$\boxed{\tau = I\alpha}$$

Angular Momentum L

$$\vec{\mathbf{L}} \equiv \vec{\mathbf{r}} \times \vec{\mathbf{p}}$$

$$\vec{\mathbf{L}} = I\vec{\omega}$$

$$\tau = \frac{d\vec{\mathbf{L}}}{dt}$$

Example 3

A disk of radius 25 cm and mass 5 kg rotates about a z-axis through its center. What is the angular momentum of the disk when its angular velocity is 2 rad/s?

$$L = I\omega$$

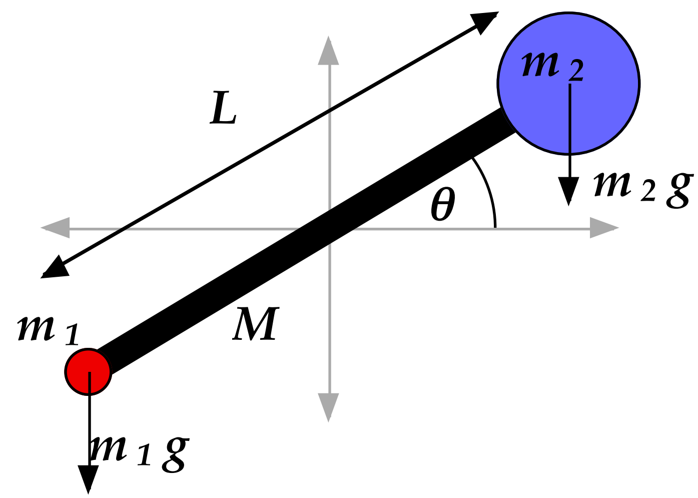
$$L = (\tfrac{1}{2}MR^2)\omega$$

$$= (\tfrac{1}{2}5kg \bullet (0.25m)^2)(2rad/s)$$

$$= 0.313 \;\; kg \bullet m^2/s$$

Example 4

The rotating rod of mass M here is released, and gravity causes it to begin rotating about the rod's midpoint. Find **L** as a function of ω, and α when θ=0.



$$\tau_{net} = I\alpha = \frac{L^2}{4}\left(m_1 + m_2 + \frac{1}{3}M\right)\alpha$$

$$L = I\omega$$

$$I = \sum r^2 m = \left(\tfrac{L}{2}\right)^2 m_1 + \left(\tfrac{L}{2}\right)^2 m_2 + \tfrac{1}{12}ML^2$$

$$= \frac{L^2}{4}\left(m_1 + m_2 + \frac{1}{3}M\right)$$

$$L = \frac{\ell^2}{4}\left(m_1 + m_2 + \frac{1}{3}M\right)\omega$$

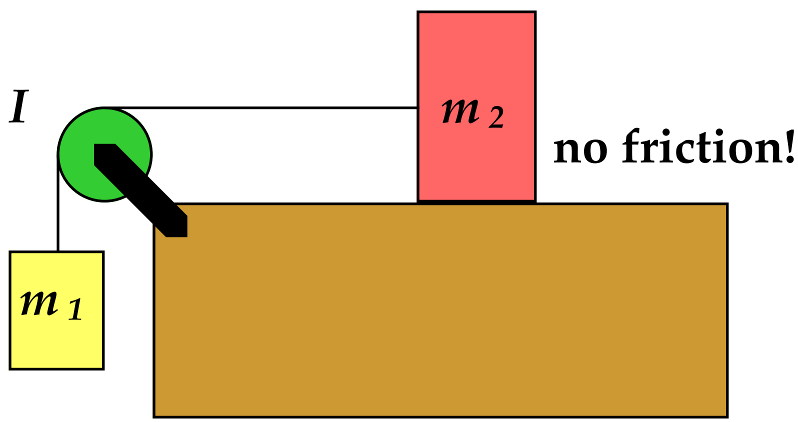
$$\tau_{net} = rF_{g1} + rF_{g2} = \left(\frac{L}{2}m_1 g\right) + \left(-\frac{L}{2}m_2 g\right)$$

$$\left(\frac{L}{2}m_1 g\right) + \left(-\frac{L}{2}m_2 g\right) = \frac{L^2}{4}\left(m_1 + m_2 + \frac{1}{3}M\right)\alpha$$

$$\alpha = \frac{2g(m_1 - m_2)}{L\left(m_1 + m_2 + \frac{M}{3}\right)}$$

Example 5

Two masses are connected as shown here. Find the acceleration of the system using angular momentum.



$$Pulley: L = I\omega = I\frac{v}{r}$$

$$m_1 : L = rm_1v$$

$$m_2 : L = rm_2v$$

$$L_{total} = I\frac{v}{r} + rm_1v + rm_2v$$

$$\tau_{ext} = \frac{dL}{dt} = \frac{d}{dt}\left(I\frac{v}{r} + rm_1v + rm_2v\right)$$

$$r(m_1g) = \frac{d}{dt}\left(I\frac{v}{r} + rm_1v + rm_2v\right)$$

$$rm_1g = \frac{I}{r}a + rm_1a + rm_2a$$

$$a = \frac{m_1g}{\frac{I}{r^2} + m_1 + m_2}$$

Note that we didn't use Tensions in this analysis because they are *internal* to the system. It' s the external torque due to gravity that changes the angular momentum.

Using Cons. of Ang. Mom.

For a large system of particles:

$$\sum L_n = k \qquad\qquad \sum L_i = \sum L_f = k$$

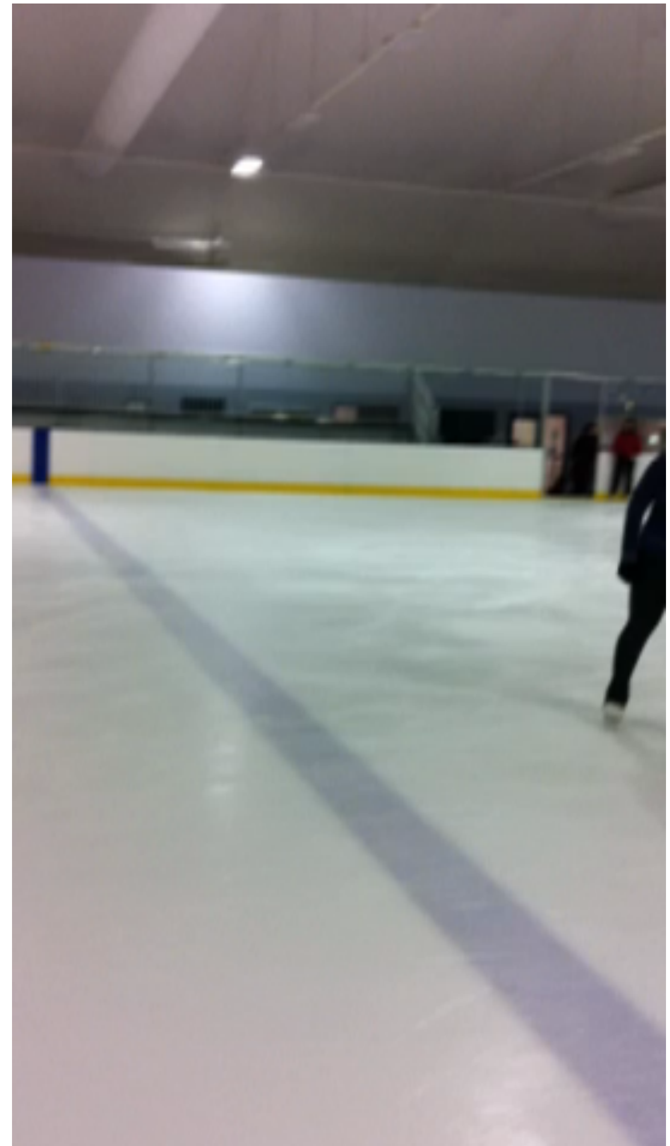
If the system is rotating about a fixed axis, then we can say that

$$I_i \omega_i = I_f \omega_f$$

... even if the distribution of the masses in the system changes.

Demo - Scratch Spin

$$I_i\omega_i = I_f\omega_f$$



http://www.youtube.com/watch?v=NtEnEeEyw_s

Example 6

A star with a radius of 1e4 km rotates about its own axis with a period of 30 days. It then undergoes a supernova explosion and collapses into a neutron star with the same mass, but a radius of only 3km. What is its approximate period?

$$I_1\omega_1 = I_2\omega_2$$

$$\left(\frac{2}{5}mr_1^{\,2}\right)\omega_1 = \left(\frac{2}{5}mr_2^{\,2}\right)\omega_2$$

$$r_1^{\,2}\omega_1 = r_2^{\,2}\omega_2$$

$$(1e4\,km)^2(1rev/30days) = (3km)^2\omega_2$$

$$\omega_2 = 3.7e5rev/day = 1rev/0.23s$$

Example 7

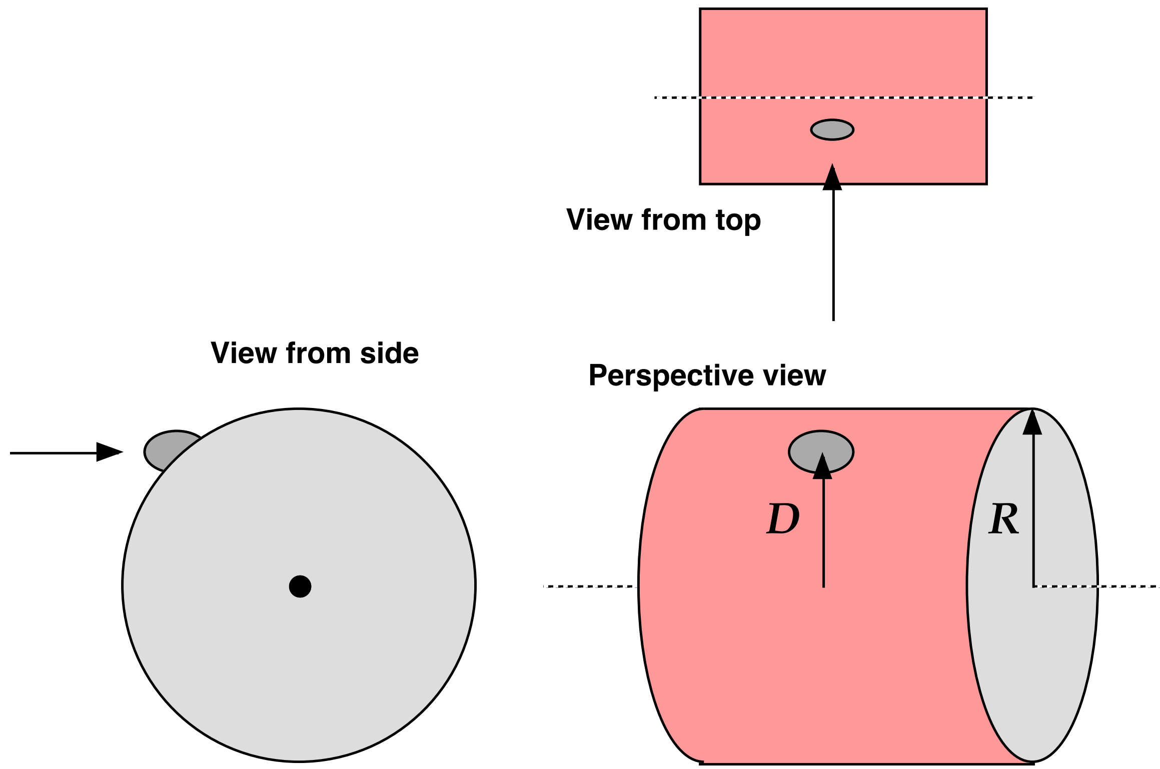
A horizontal platform (disk-shaped) rotates in a horizontal plane about a frictionless vertical axle. The platform has mass 100 kg and a radius R=2m. A student of mass 60 kg stands at the edge of the spinning platform and walks slowly from the rim toward the center. If ω_o=2.0 rad/s when the student is at the rim, calculate ω when student is 0.50 m from the center. Calculate initial and final rotational energies of the system.

ω=4.09 rad/s

K_i=880J, K_f=1.8e3J. K increases due to Work person does in Walking across the platform.

Example 8

A projectile of mass m and velocity v_o is fired at a solid cylinder of mass M and radius R. The cylinder is initially at rest, and mounted on a fixed horizontal axle that runs through its center of mass. The line of motion of the projectile is perpendicular to the axle, and at a distance $D < R$ from the center.



a) Find the angular speed of the system after the projectile strikes and adheres to the surface of the cylinder.

b) Is mechanical energy conserved in this example? Give quantitative evidence to support your answer.

Example 8

a) Find the angular speed of the system after the projectile strikes and adheres to the surface of the cylinder.

$$\sum L_i = \sum L_f$$

$$L_{object} + L_{cylinder} = L_{object+cylinder}$$

$$rmv \sin\theta + 0 = I\omega$$

$$Dmv_0 = \left(\frac{1}{2}MR^2 + mR^2\right)\omega$$

$$\omega = \frac{Dmv_0}{\frac{1}{2}MR^2 + mR^2}$$

b) Is mechanical energy conserved in this example? Give quantitative evidence to support your answer.

$$K_i = \frac{1}{2}mv_0{}^2;\ K_f = \frac{1}{2}I\omega^2$$

$$K_f = \frac{1}{2}\left(\tfrac{1}{2}MR^2 + mR^2\right)\left(\frac{Dmv_0}{\tfrac{1}{2}MR^2 + mR^2}\right)^2$$

$$K_f = \frac{1}{2}\frac{\left(Dmv_0\right)^2}{\left(\tfrac{1}{2}MR^2 + mR^2\right)} = \frac{1}{2}\frac{\left(Dm\right)^2}{\left(\tfrac{1}{2}MR^2 + mR^2\right)}v_0{}^2$$

So, is $m > \dfrac{\left(Dm\right)^2}{\left(\tfrac{1}{2}MR^2 + mR^2\right)}$?

Is $1 > \dfrac{D^2}{\left(\tfrac{1}{2}M/m + 1\right)R^2}$? Yes! So K not conserved.

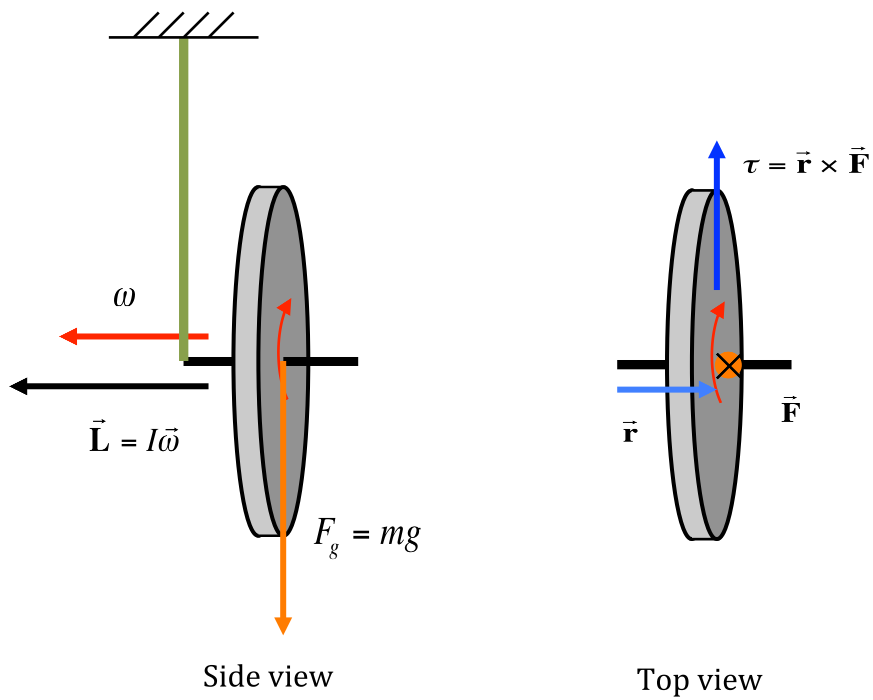
Example 9

What happens when a mass flying through empty space hits a stick floating there?

Is energy conserved?	Yes. *Always!*
Is kinetic energy conserved?	Probably not.
Is linear momentum conserved?	Yes, assuming no $F_{external}$.
Is angular momentum conserved?	Yes, assuming no $\tau_{external}$.

Example 10

What happens when a vertically-oriented spinning wheel is suspended by one end of its horizontal axis?



Example 11

A favorite physics demonstration involves sitting on a stool while holding a spinning bicycle wheel, and twisting the wheel to one side or another.

Which way will the student spin if he tilts the wheel (spinning as shown) to the left?

Example: Angular Momentum

Along with the "spinning ice skater," another favorite example of conservation of angular momentum is the "bicycle wheel on a rotating stool" demonstration.

$$\sum L_i = \sum L_f$$

$$L_{wheel} = L_{wheel}' + L_{student+stool}$$

After wheel is flipped over:

$$L_{wheel} = -L_{wheel} + L_{student+stool}$$

$$2L_{wheel} = L_{student+stool}$$