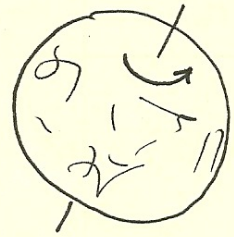
10.1 a) Find angular speed of Earth's rotation about its axis.

Earth rotates ~ once/24 hrs.



$$\frac{1\text{ rev}}{24\text{ hrs}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{2\pi\text{ rads}}{1\text{ rev}}$$

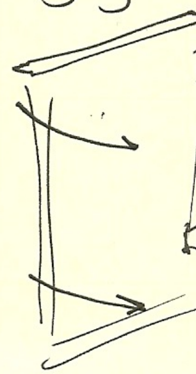
$$= \boxed{7.27\text{e}{-5}\ \text{rad/s}} \text{ or } \boxed{7.27\text{e}{-5}\ \text{s}^{-1}}$$

A dimensionless unit.

b) If the mass of the earth has any elasticity, then there will be a slight bulge at the equator, where a greater centripetal force would be required to keep the matter there moving in a circle. (Due to the greater tangential velocity there).

10.3 $\theta = 5.00 + 10.0t + 2.00t^2$ for a swinging door

Determine θ, ω, & α at $t = 0$s & $t = 3$s.



a) At $t = 0$:

$$\theta = 5 + 10t + 2t^2$$
$$= 5 + 0 + 0 = \boxed{5 \text{ rads}}$$

$$\omega = \frac{\partial \theta}{\partial t} = 10 + 4t$$
$$= 10 + 4(0) = \boxed{10 \text{ rad/s}}$$

$$\alpha = \frac{\partial \omega}{\partial t} = \frac{\partial^2 \theta}{\partial t^2} = \boxed{4 \text{ rad/s}^2}$$

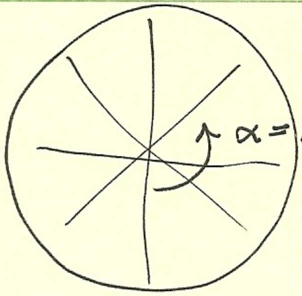
b) At $t = 3$s

$$\theta = 5 + 10(3) + 2(3)^2$$
$$= \boxed{53 \text{ rads}}$$

$$\omega = 10 + 4t$$
$$= 10 + 4(3)$$
$$= \boxed{22 \text{ rad/s}}$$

$$\alpha = \boxed{4 \text{ rad/s}^2}$$

10.5



$\omega_i = 0$

$\omega_f = 12\ \text{rad/s}$

$\alpha = \text{constant}$ $\quad t = 3\,s$

a) $\alpha = \dfrac{\omega_f - \omega_i}{t}$

$= \dfrac{12\,\frac{\text{rad}}{s} - 0}{3\,s} = \boxed{4\ \text{rad/s}^2}$

b) Angle that it rotates through in this time period:

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$= 0 + 0 + \frac{1}{2}(4)(3)^2$

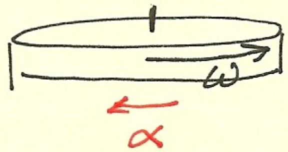
$= \boxed{18\ \text{rads}}$

10.7 $\omega_i = 1.00 \times 10^2 \frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rads}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = \underline{10.5 \text{ rad/s}}$

$\alpha = -2.00 \text{ rad/s}^2$

a) Time to slow down to a halt ($\omega_f = 0$)?

$\alpha = \frac{\omega_f - \omega_i}{t} \rightarrow t = \frac{\omega_f - \omega_i}{\alpha}$



$= \frac{0 - 10.5 \text{ rad/s}}{-2 \text{ rad/s}^2}$

$= \boxed{5.24 \text{ s}}$

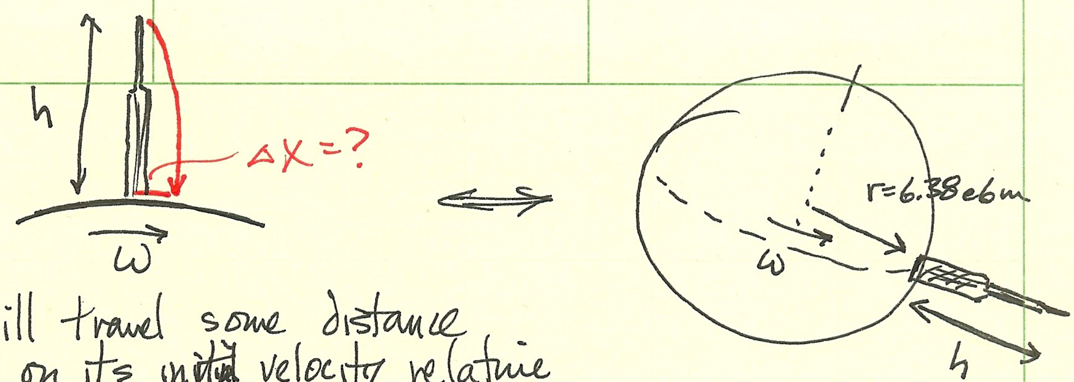
b) θ during this time period?

$\theta_f - \theta_i = \omega_i t + \frac{1}{2} \alpha t^2$

$\theta = (10.5 \text{ rad/s})(5.24) + \frac{1}{2}(-2 \text{ rad/s}^2)(5.24 \text{ s})^2$

$= \boxed{27.6 \text{ rads}}$

10.11



a) Ball will travel some distance based on its initial velocity relative to the earth's surface.

$$V_{earth's\ surface} = r_e \omega_e$$
$$V_{bldg} = (r_e + h)\,\omega_e$$
$$V_{bldg\ relative\ to\ V_{earth}} = r_e\omega_e + h\omega_e - r_e\omega_e$$
$$V_{obj} = \underline{h\omega} \quad \text{(initial horizontal v)}$$

time to fall from building:

$$\Delta y = v_i t + \tfrac{1}{2}at^2 \rightarrow \quad h = \tfrac{1}{2}gt^2$$
$$t = \sqrt{2h/g}$$
$$\Delta X = vt = h\omega\sqrt{\frac{2h}{g}} = \boxed{h^{3/2}\omega\sqrt{\left(\frac{2}{g}\right)}}$$

b) If $h = 50$, $\Delta X = 50^{3/2}(7.27e{-5})\sqrt{\left(\frac{2}{9.8}\right)} = \boxed{\cancel{1.16e{-2}m}}$

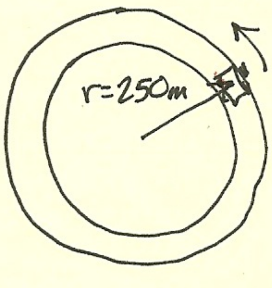
$1.16e{-2}m$

c) $\% = \frac{1.16e{-2}\,m}{50\ \ m} \times 100 = 2.32\,e{-2} = \cancel{0.0023\%}$
$= 0.0223\%$ of height.

(Probably not significant).

d) If angular displacement of earth decreases, the eastward displacement will increase if considering over the time of the fall. The displacement will decrease if considering over a long period of time, with objects dropped at different moments during the acceleration.

10.13



$v = 45.0\,m/s$

a) $\omega = \frac{v}{r} = \frac{45\,m/s}{250\,m} = 0.18\,s^{-1}$

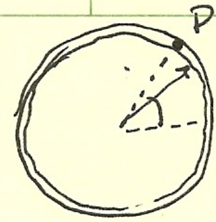
$= \boxed{0.18\,rad/s}$

b) acceleration is only radial, b/c car is traveling w/ constant speed.

$a_c = \frac{v^2}{r} = \frac{(45\,m/s)^2}{250\,m} = \boxed{8.1\,m/s^2}$

toward the center of the circle

10.17



$r = 1.00\,m$ $(d = 2m!)$

$\alpha = 4.00\,rad/s^2$

$\omega_i = 0$ at $t = 0$

$\theta_i = 57.3°$

$\theta_i = \frac{57.3°}{1} \times \frac{2\pi\,rads}{360°} = \underline{1\,rad}$

At $t = 2.00s$, find:

a) angular speed of wheel: $\omega = \omega_i + \alpha t$

$= 0 + \left(4 \frac{rad}{s^2}\right)(2s)$

$= \boxed{8\,rad/s}$

b) tangential speed of wheel: $v = r\omega$

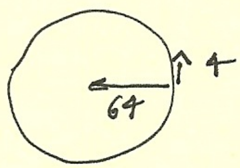
$= (\cancel{2m}\ 1m)(8\,rad/s)$

$= \boxed{8\,m/s}$

c) total acceleration: $\vec{a} = \vec{a}_{radial} + \vec{a}_{tangential}$
(at rim)

$= \frac{-v^2}{r}\hat{r} + r\alpha\hat{\theta}$

$= \frac{-(8)^2}{1}\hat{r} + (1)(4)\hat{\theta}$



$= -64\hat{r} + 4\hat{\theta}$

$= \boxed{64.1\,m/s^2}$

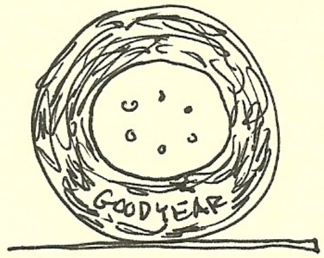
$\theta = \tan\left(\frac{4}{64}\right) = \boxed{3.58° \text{ off radius}}$

d) $\theta_f = \theta_i + \omega t + \frac{1}{2}\alpha t^2$

$= 1\,rad\,(\text{see above}) + (0)(2) + \frac{1}{2}(4)(2)^2$

$= \boxed{9\,rads}$

10.18



$V_{car} = 22\,m/s$
$V_i = 0$
$t = 9\,s.$
tire diameter = 0.58m, so
$r_{tire} = \frac{0.58}{2} = \underline{0.29\,m}$

a) $V_{car} = V_{tan} = r\omega$
$a_{car} = a_{tan} = r\alpha$
$x_{car} = x_{tan} = r\theta$

$$a_{car} = \frac{\Delta V}{t} = \frac{22-0}{9} = 2.44\,m/s^2$$

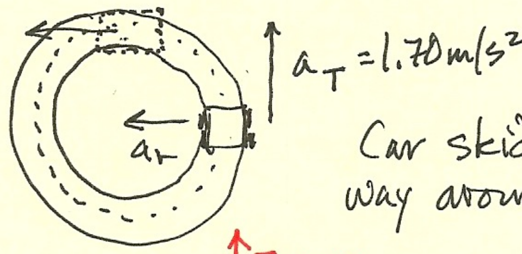
$$\alpha = \frac{a}{r} = \frac{2.44\,m/s^2}{0.29\,m} = 8.43\,rad/s^2$$

$$\theta = \omega_i t + \tfrac{1}{2}\alpha t^2$$
$$= 0t + \tfrac{1}{2}(8.43\,rad/s^2)(9s)^2$$
$$= 341\text{ rads} \times \frac{1\,rev}{2\pi\,rad} = \boxed{54.3\text{ revolutions}}$$

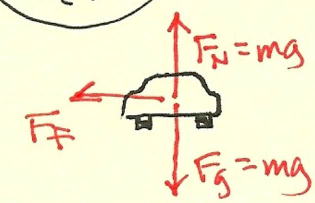
b) $$\omega_f = \omega_i + \alpha t$$
$$= 0 + (8.43\,\tfrac{rad}{s})(9s)$$
$$= 75.9\,rad/s \times \frac{1\,rev}{2\pi\,rad} = \boxed{12.1\,rev/s}$$

There are a number of other ways to solve these problems as well.

10.23



Car skids off after $\frac{1}{4}$ of the way around the track. Calculate μ_s.



$$\mu = \frac{F_f}{F_N} = \frac{F_f}{mg}$$

What's the max F_f the car can handle at that 1/4 circle position?

$\Sigma F = ma$

$F_f = ma$, but what is acceleration at that position?

$$\mu = \frac{ma}{mg} = \frac{a}{g}$$

$$\vec{a} = \vec{a}_{radial} + \vec{a}_{tangential}$$

$$= -\frac{v^2}{r}\hat{r} + 1.7\,m/s^2\,\hat{\theta}$$

So what is velocity at that point?

$$\omega^2 = \omega_i^2 + 2\alpha\theta$$

$$= 0^2 + 2\left(\frac{a}{r}\right)\left(\frac{\pi}{2}\right)$$

$$\omega = \sqrt{2\left(\frac{1.7}{r}\right)\left(\frac{\pi}{2}\right)} = \sqrt{\frac{1.7\pi}{r}}$$

$$v = r\omega = r\sqrt{\frac{1.7\pi}{r}}, \text{ so}$$

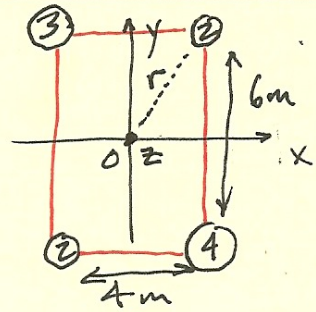
$$a_r = \frac{-v^2}{r} = \frac{(r^2)(1.7\pi)}{r \cdot r} = 1.7\pi$$

$$a_{net} = \sqrt{a_r^2 + a_T^2}$$

$$= \sqrt{(1.7\pi)^2 + (1.7)^2} = 5.60\,m/s^2$$

$$\mu = \frac{5.60\,m/s^2}{9.80\,m/s^2} = \boxed{0.572}$$

10.25



System rotates about the z-axis at $\omega = 6$ rad/s.

a) Moment of inertia of system = ?

$$I = I_4 + I_3 + 2I_2$$

$I = mr^2$, so

$$I = (4)(2^2+3^2) + 3(2^2+3^2) + 2(2(2^2+3^2))$$

$$= 11(2^2+3^2) = \boxed{143 \text{ kg}\cdot\text{m}^2}$$

$r^2 = x^2 + y^2$, so

$r^2 = 2^2 + 3^2 = \underline{13 \text{ kg m}^2}$

b) $K_{rotational}$ = ?

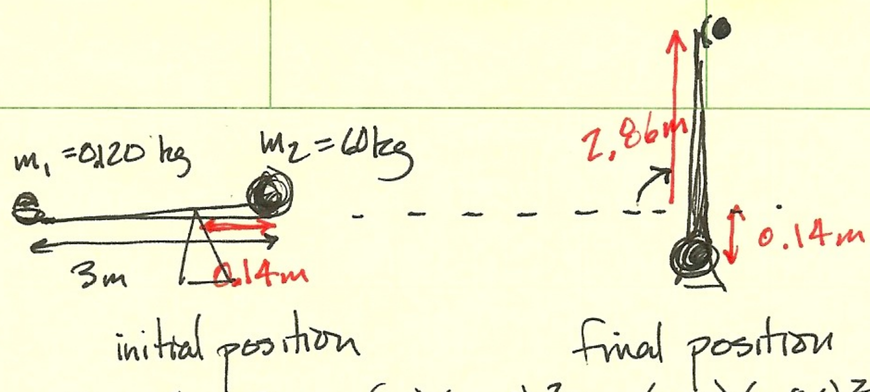
$$K_{rot} = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2}(143 \text{ kg}\cdot\text{m}^2)\left(6 \frac{\text{rad}}{\text{s}}\right)^2$$

$$= \boxed{\ \text{J}}$$

2574

10.27



a) $I = \Sigma mr^2 = (60)(0.14)^2 + (0.12)(2.86)^2 = \underline{2.16 \text{ kg m}^2}$

By cons of E: $U_i = U_f + K$. Consider axle to be $h=0$

(max speed occurs when U_g is least)

$0 = mgh_f + mgh_f + \frac{1}{2}I\omega^2$

$0 = (60)(9.8)(-0.14) + (0.12)(9.8)(2.86) + \frac{1}{2}(2.16)\omega^2$

$\omega = 8.55$ rad/s

$v = r\omega = (2.86)(8.55) = \boxed{24.5 \text{ m/s}}$

b) No, small object does not accelerate constantly — force vectors don't maintain constant angle w/ r.

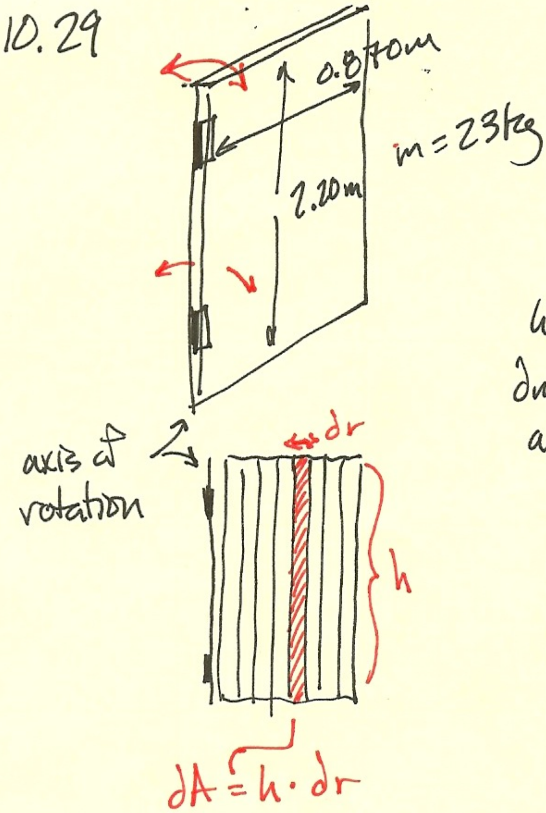
c) No.

d) No

e) No, direction of velocity keeps changing (along w/ magnitude).

f) Yes. We use U_g of Earth – trebuchet system in (a) to solve this problem, so mechanical energy in the system is conserved. (If we'd considered $F_{gravity}$ from "external" earth doing W on isolated trebuchet, then mechanical energy of trebuchet would not be conserved.

10.29



a) $I = ?$

The door has a surface density based on its area.

$$\sigma = \frac{M}{A} = \frac{23\,\text{kg}}{(0.87\,\text{m})(2.2\,\text{m})} = 12.0\ \text{kg/m}^2$$

We can also write $dm = \sigma\, dA$, where dm = the mass of a "strip" of the door with area dA, with $dA = \text{height} \cdot dr$.

$$I = \int r^2\, dm$$
$$= \int r^2 \sigma\, dA$$
$$= \int r^2 \sigma\, h\, dr$$
$$= \sigma h \int_0^{0.87} r^2\, dr$$
$$= \left(\frac{M}{w \cdot h}\right) h \; \frac{1}{3} r^3 \Big|_0^{0.87}$$
$$= \frac{1}{3}\left(\frac{23}{0.87}\right)(0.87)^3 = \boxed{5.80\ \text{kg}\cdot\text{m}^2}$$

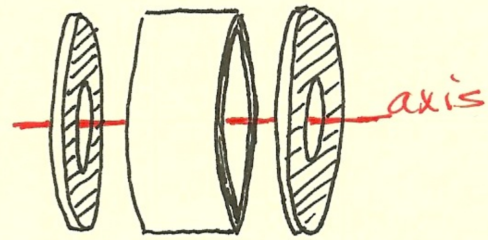
b) Apparently the height of the door is unnecessary. Each section of the "strip" we analyzed is located the same distance from the axis of rotation.

10.31 Easier to understand tire in perspective view:

$\rho_{rubber} = 1.1e3 \, kg/m^3$

$I = ?$

$I = I_{tread} + 2 I_{sidewall}$.



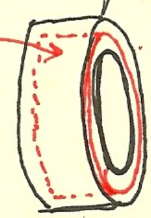
We can use the same formula for both types of segments, based on their width (thickness) & inner & outer radii.

From pg 287 in book, hollow cylinder formula is $I = \frac{1}{2} M (R_1^2 + R_2^2)$

We need to know the mass M of the hollow cylinder:

$M = \int dm = \int \rho \, dV$, where dV is the volume of a small "shell" of the total cylinder:

$\underline{dV = t \cdot 2\pi r \cdot dr}$



$$M = \int_{r_1}^{r_2} \rho t 2\pi r \, dr$$

$$M = 2\pi\rho t \cdot \frac{1}{2} r^2 \Big|_{r_1}^{r_2} = \pi\rho t (R_2^2 - R_1^2)$$

so $I = \frac{1}{2}\pi\rho t (R_2^2 - R_1^2)(R_1^2 + R_2^2)$

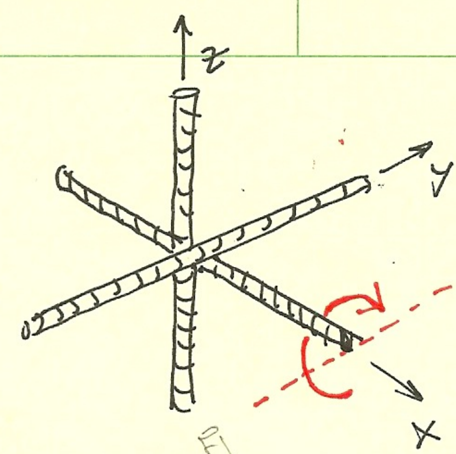
$$\boxed{I = \frac{1}{2}\pi\rho t (R_2^4 - R_1^4)}$$

$I_{tread} = \frac{1}{2}\pi (1.10e3)(0.20)(0.330^4 - 0.305^4) = 1.1078$

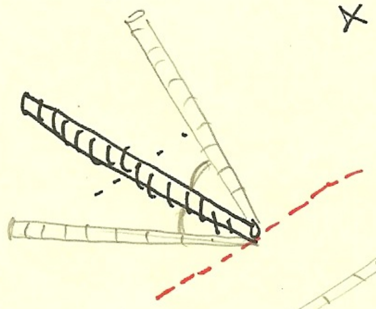
$I_{sidewalls} = 2 \times \frac{1}{2}\pi (1.10e3)(0.00635m)(0.305^4 - 0.165^4) = 0.1736$

$I_{total} = 1.1078 + 0.1736 = \boxed{1.28 \, kg \, m^2}$

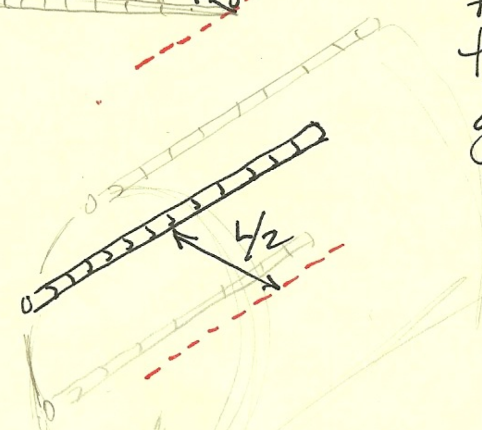
10.33



Find I_{total} for the 3-rod assembly shown, & rotated about a line parallel to the y-axis at one end of the rods.

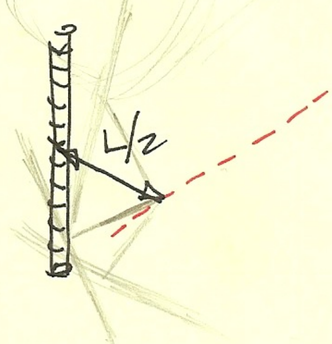


I_x is rotating about one end. We can solve by integrating $I = \int r^2 dm$ from 0 to L, or by using ∥-axis theorem $I = I_{cm} + MD^2$ to get $I_x = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \underline{\frac{1}{3}ML^2}$



I_y is rotating around y-axis w/ all of mass located at $\frac{L}{2}$.

$I_y = \Sigma MR^2 = M\left(\frac{L}{2}\right)^2 = \underline{\frac{1}{4}ML^2}$

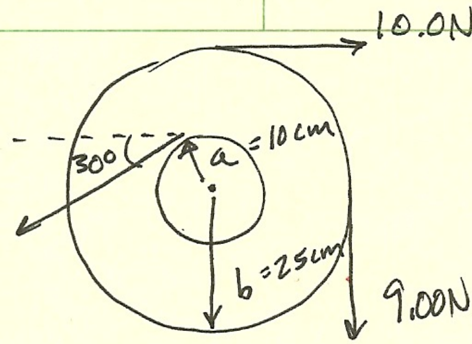


I_z is rotating around y-axis which is also L/2 away from its cm. By parallel-axis theorem:

$$I_z = I_{cm} + MD^2$$
$$= \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$$

$$I_{total} = \frac{1}{3}ML^2 + \frac{1}{4}ML^2 + \frac{1}{3}ML^2$$
$$= \boxed{\frac{11}{12}ML^2}$$

10.35

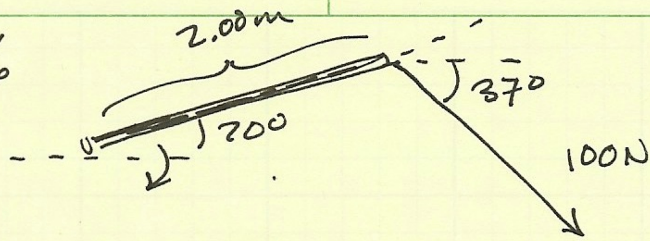


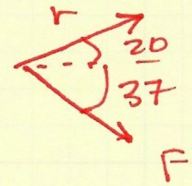
$\tau_{net} = \sum r_i \times F_i$

r & F are ⊥ in each case,
& ccw torques are positive.

$$\tau = r \times F + r \times F + r \times F$$
$$= (0.1m)(12N) - (0.25)(10) - (0.25)(9)$$
$$= \boxed{-3.55\ N \cdot m}$$

10.36





$\theta = 57°$

$$\tau = r \times F$$
$$= rF\sin\theta$$
$$= (2m)(100N)\sin 57°$$
$$= 168\ N \cdot m$$

10.38 Grinding wheel, solid disk, so $I = \frac{1}{2}MR^2$

radius = 7 cm = 0.07 m, m = 2.00 kg

τ exerted by motor = 0.600 N·m

a) How long to reach speed of 1200 rev/min?

$$\omega = \frac{1200\,\text{rev}}{\text{min}} \times \frac{1\,\text{min}}{60\,\text{s}} \times \frac{2\pi\,\text{rad}}{1\,\text{rev}} = \underline{126\,\text{rad/s}}$$

$$\tau = I\alpha, \text{ so } \alpha = \frac{\tau}{I} = \frac{0.600\,\text{N·m}}{\frac{1}{2}(2)(0.07)^2} = \underline{122\,\text{rad/s}^2}$$

$$\omega_f = \omega_i + \alpha t \rightarrow t = \frac{\omega_f - \omega_i}{\alpha} = \frac{126 - 0}{122}$$

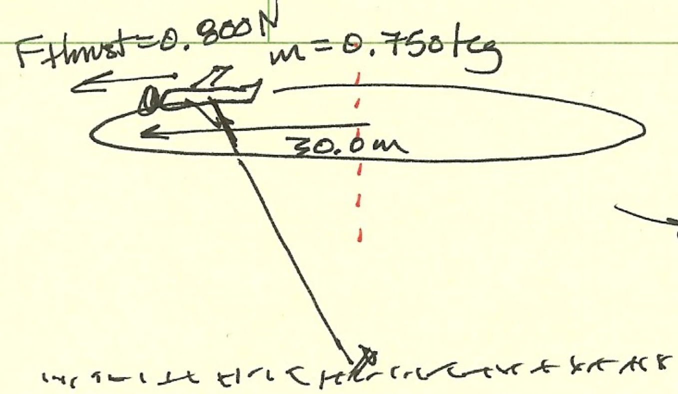
$$= \boxed{1.03\,\text{s}}$$

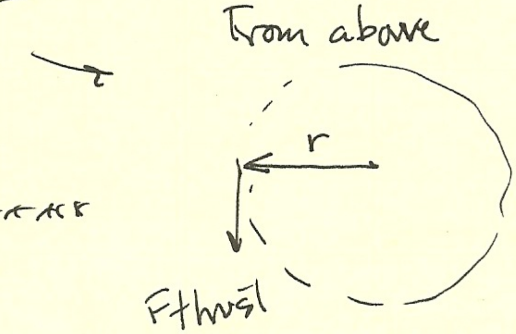
b) $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$$= 0 + \frac{1}{2}(122\,\text{rad/s}^2)(1.03\,\text{s})^2 = \boxed{64.7\,\text{rad}} / 2\pi =$$

$$\boxed{10.3\,\text{revolutions}}$$

10.39





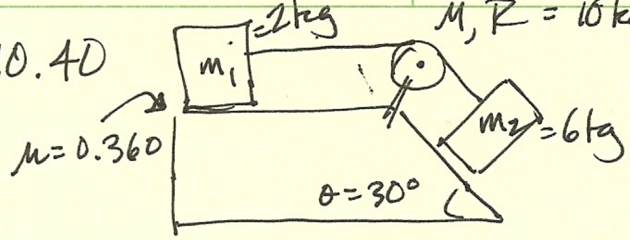
a) $\tau = r \times F$
$= (30\,m)(0.800\,N) = \boxed{24.0\ N\cdot m}$

b) Angular acceleration $\alpha = ?$
$\tau = I\alpha$, so $\alpha = \frac{\tau}{I} = \frac{24\,Nm}{MR^2} = \frac{24\,N\cdot m}{(0.75)(30)^2}$
$\alpha = \boxed{0.0356\ rad/s^2}$

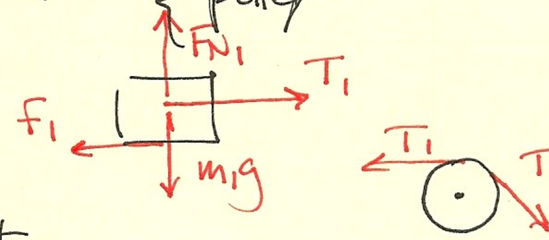
c) Translational (linear) $a = r\alpha$
$= (30\,m)(0.0356\,rad/s^2)$
$= \boxed{1.07\ m/s^2}$

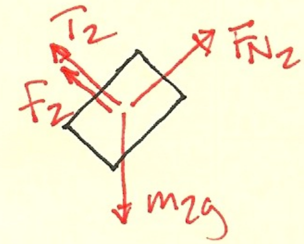
10.40

$M, R = 10\,kg,\ 0.250\,m$



a) Force diagrams for blocks & pulley





b) Acceleration of blocks (ie system). Use shortcut of $\Sigma F = ma$ for system? or Separate analyses?

mass 1: $\Sigma F_1 = m_1 a_1$

$T_1 - f_1 = m_1 a$

$T_1 - \mu m g = m_1 a$

$T_1 = 2a + (0.36)(2)(9.8) = 2a + 7.056$

mass 2: $\Sigma F_2 = m_2 a_2$

$-f_2 + m_2 g \sin\theta - T_2 = m_2 a$

$T_2 = m_2 g \sin\theta - \mu m_2 g \cos\theta - m_2 a$

$= (6)(9.8)(\sin 30) - (0.36)(6)(9.8)(\cos 30) - 6a$

$T_2 = 11.07 - 6a$

pulley: $\Sigma \tau = I\alpha$ $\qquad I_{disk} = \frac{1}{2}MR^2$

$rT_2 - rT_1 = I\frac{a}{R} = \frac{1}{2}MR^2\frac{a}{R}$

$T_2 - T_1 = \frac{1}{2}Ma = \frac{1}{2}(10)a = 5a$

$(11.07 - 6a) - (2a + 7.056) = 5a$

$4.014 = 13a$

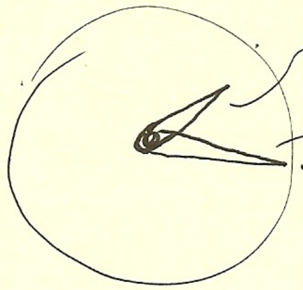
$a = \boxed{0.309\ m/s^2}$

c) Sub back in to get

$T_1 = 2(0.309) + 7.056 = \boxed{7.67N}$

$T_2 = 11.07 - 6(0.309) = \boxed{9.22N}$

10.45



Hour hand, $m_{hour} = 60\text{ kg}$, $L_h = 2.70\text{ m}$

Minute hand, $m_{minute} = 100\text{ kg}$, $L_m = 4.50\text{ m}$

$K_{rotational} = ?$

$$K_{rotational} = K_{hour} + K_{minute}$$

$$= \frac{1}{2} I \omega^2 + \frac{1}{2} I \omega^2$$

I for a long thin rod at one end $= \frac{1}{3} ML^2$, so

$$K = \frac{1}{2}\left(\frac{1}{3} M_h L_h^2\right)\omega_h^2 + \frac{1}{2}\left(\frac{1}{3} M_m L_m^2\right)\omega_m^2$$

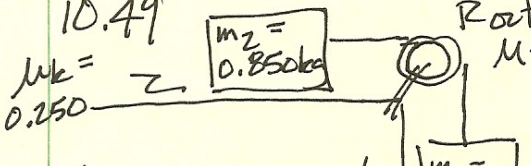
$$\omega_h = \frac{1\text{ rev}}{12\text{ hrs}} \times \frac{2\pi\text{ rads}}{1\text{ rev}} \times \frac{1\text{ hr}}{3600\text{ s}} = 1.45e{-4}\text{ rad/s}$$

$$\omega_{min} = \frac{1\text{ rev}}{60\text{ min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = 1.75e{-3}\text{ rad/s}$$

$$K_{rot} = \frac{1}{2}\left(\frac{1}{3} 60 \cdot 2.70^2\right)(1.45e{-4})^2 + \frac{1}{2}\left(\frac{1}{3} 100 \cdot 4.50^2\right)(1.75e{-3})^2$$

$$= 1.53e{-6} + 1.03e{-3} = \boxed{1.03e{-3}\text{ J}}$$

10.49



$R_{inner} = 0.020m$
$R_{outer} = 0.030m$
$M = 0.350kg$

At x_i, $v_i = 0.820 m/s$
Find v_f at $\Delta x = 0.700m$

a) Use Energy considerations to find v_f after block has moved 0.700m.

$$\Sigma E_i = \Sigma E_f$$

$I_{hollow\ cylinder} = \frac{1}{2}M(R_i^2 + R_o^2)$
$= \frac{1}{2}(0.35)(0.02^2 + 0.03^2)$
$= 2.275e^{-4}$

Energy initial

$$U_1 + K_1 + K_2 + K_{rot} - \Delta E_{int} = \ldots$$

$$m_1gh_1 + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}I\omega^2 - F_{f2}x = \ldots$$

$\omega = \frac{v}{r}$

$$(0.420)(g)(0.700) + \frac{1}{2}(0.420)(0.820)^2 + \frac{1}{2}(0.850)(0.820)^2 + \frac{1}{2}(2.275e^{-4})\left(\frac{0.820}{0.030}\right)^2 -$$

$(\mu m_2 g)(x)$

$(0.250)(0.850)(g)(0.700)$

$$\Sigma E_i = 1.935J$$

Energy final

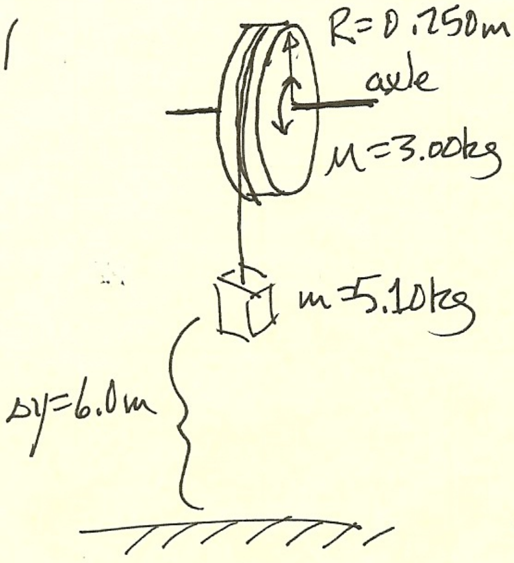
$$\ldots = U_1' + K_1' + K_2' + K_{rot}$$
$$= mgh' + \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 + \frac{1}{2}I\omega'^2$$
$$= 0 + \frac{1}{2}(0.420)v'^2 + \frac{1}{2}(0.850)v'^2 + \frac{1}{2}(2.275e^{-4})\left(\frac{v}{0.030}\right)^2$$

$$\Sigma E_f = 0.7614v^2$$

$$1.935J = 0.7614v^2$$
$$v = \boxed{1.59 m/s}$$

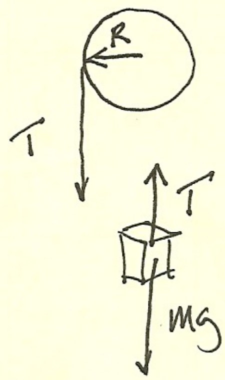
b) $\omega = \frac{v}{r} = \frac{1.59}{0.030} = \boxed{53.1 \text{ rad/s}}$

10.51



a) Find tension T in string.

Force analysis, so we need free body diagrams, & $F_{net} = ma$ for both bodies.



$\Sigma \tau = I\alpha$

$r \times T = \frac{1}{2}(MR^2)\frac{a}{R}$

$T = \frac{Ma}{2}$

$\Sigma F = ma$

$mg - T = ma$

$a = g - \frac{T}{m}$

Sub & solve for T

$T = \frac{M}{2}\left(g - \frac{T}{m}\right)$

$T = \frac{Mg}{2} - \frac{TM}{m2}$

$T + \frac{TM}{2m} = \frac{Mg}{2}$

$T\left(1 + \frac{M}{2m}\right) = \frac{Mg}{2}$

$T\left(1 + \frac{3}{2(5.1)}\right) = \frac{3 \cdot 9.8}{2}$

$T = \boxed{11.4N}$

b) acceleration of object?
Sub back in T = 11.4N

$a = g - \frac{T}{m} = 9.8 - \frac{11.4}{5.1} = \boxed{7.56\,m/s^2}$

c) $V_f^2 = V_i^2 + 2ax$

$V_f = \sqrt{0^2 + (2)(7.56)(6m)}$

$= \boxed{9.53\,m/s}$

d) Confirmation of this speed, using energy:

$U_i + K_i + K_{rot} = U_f + K_f + K_{rot}'$

$mgh + 0 + 0 = 0 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ $\omega = \frac{v}{r}$

$(5.1)(9.8)(6) = \frac{1}{2}(5.1)v^2 + \frac{1}{2}(0.09375)\left(\frac{v}{0.25}\right)^2$

$I_{disk} = \frac{1}{2}MR^2$

$= \frac{1}{2}(3)(0.25)^2$

$= 0.09375$

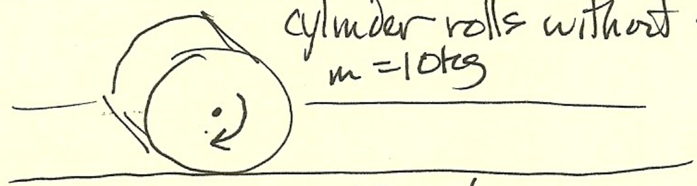
$300 = 2.55v^2 + 0.75v^2$

$v = \boxed{9.53\,m/s}$ ✓

10.55

cylinder rolls without slipping

$m = 10\,kg$



$v_{cm} = 10.0\,m/s$

a) $K_{translational} = \frac{1}{2} m v_{cm}^2 = \frac{1}{2}(10\,kg)(10\,m/s)^2 = \boxed{500\,J}$

b) $K_{rotational} = \frac{1}{2} I \omega^2$

$v_{cm} = R\omega = V_{tangential}$ for a rotating object

$K_{rot} = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{cm}}{R}\right)^2$

$= \frac{1}{2}\left(\frac{1}{2}M v_{cm}^2\right) = \boxed{250\,J}$

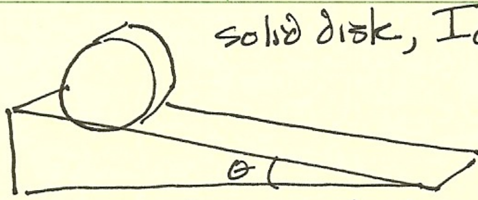
c) $K_{total} = K_{trans} + K_{rot}$

$= 500\,J + 250\,J$

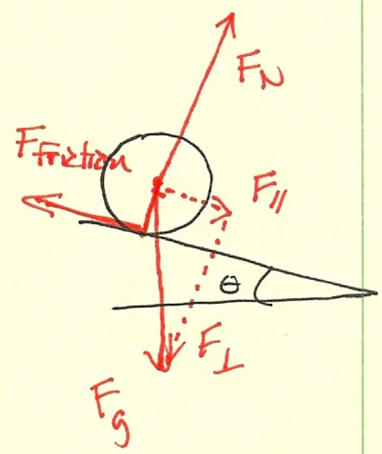
$= \boxed{750\,J}$

10.57

solid disk, $I_{cm} = \frac{1}{2}MR^2$



a) Find a_{cm} of solid disk rolling down an incline.



Because we're looking for acceleration, we'll used a <u>force</u> analysis, requiring a free-body diagram.

$$\Sigma \tau = I\alpha$$
$$r \times F_F = \left(\tfrac{1}{2}MR^2\right)\left(\tfrac{a}{R}\right)$$
$$F_f = \underbrace{\tfrac{1}{2}Ma} \quad \leftarrow \text{Don't know } a \text{ or } F_F\text{, so...}$$

$$\Sigma F_x = ma_x$$
$$F_{\parallel} - F_f = ma$$
$$mg\sin\theta - \left(\tfrac{1}{2}Ma\right) = Ma$$
$$\cancel{m}g\sin\theta = \tfrac{3}{2}\cancel{m}a$$
$$a = \boxed{\tfrac{2}{3}g\sin\theta}$$

b) For a hoop, do exact same analysis, w/ $I = MR^2$

$$\cancel{m}g\sin\theta - \cancel{M}a = \cancel{m}a \qquad r \times F_F = (MR^2)\left(\tfrac{a}{R}\right)$$
$$a = \boxed{\tfrac{1}{2}g\sin\theta} \qquad F_f = Ma$$

Hoop has a smaller acceleration than cylinder.

c) $$\mu_{minimum} = \frac{F_F}{F_N}$$

From cylinder analysis, sub a back in to get F_F required

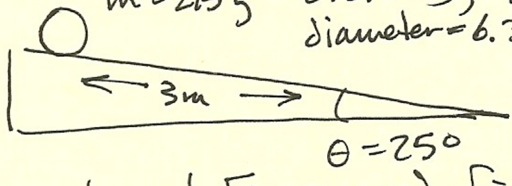
$$F_f = \tfrac{1}{2}Ma = \tfrac{1}{2}M\left(\tfrac{2}{3}g\sin\theta\right) = \tfrac{1}{3}Mg\sin\theta$$

$$\mu = \frac{\frac{1}{3}\cancel{Mg}\sin\theta}{\cancel{Mg}\cos\theta} = \boxed{\tfrac{1}{3}\tan\theta}$$

10.61

$m = 215\,g = 0.215\,kg$, height $= 10.8\,cm = 0.108\,m$

diameter $= 6.38\,cm \rightarrow$ radius $= 0.0319\,m$



$t = 1.50\,s$

a) If mechanical E conserved, find I.

$$U_i + K_{trans\,i} + K_{rot\,i} = U_f + K_{trans\,f} + K_{rot\,f}$$

$$mgh + 0 + 0 = 0 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\left(\frac{v_{cm}}{R}\right)^2$$

$$(0.215)(9.8)(1.27) = \frac{1}{2}(0.215)(4)^2 + \frac{1}{2}I\left(\frac{4}{.0319}\right)^2$$

$$I = \boxed{1.22e{-4}\ kg\cdot m^2}$$

$h = 3\sin 25 = 1.27\,m$

$\cancel{v_f = \sqrt{v_i^2 + 2a}}$

$v_{avg} = \frac{x}{t} = \frac{3}{1.5} = 2\,m/s$

$v_f = 4\,m/s$

b) height of can was not required.

c) The can is not of uniform density. The metal outside & the soup inside are different substances.