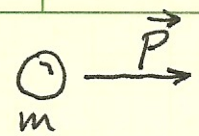
9.1




a) Show that $K = \frac{p^2}{2m}$.

$\vec{p} = m\vec{v}$, so $v = \frac{p}{m}$

$K = \frac{1}{2}mv^2$, so $K = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$ ✓

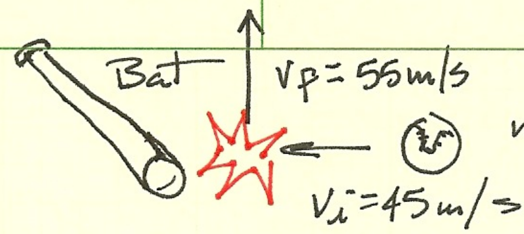
b) Magnitude of particles momentum in terms of K & m.

$p = mv$

$K = \frac{1}{2}mv^2$, so $v = \sqrt{\frac{2k}{m}}$

$p = (m)\left(\sqrt{\frac{2k}{m}}\right) = m\sqrt{\frac{2k}{m}} = \boxed{\sqrt{2km}}$

9.4




$m = 0.145\,kg \quad t = 2e{-}3\,s$

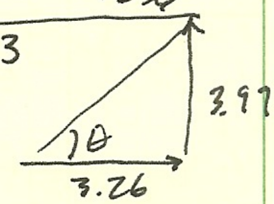
$F_{avg} = ?$

$$Ft = m\Delta V$$

$$F = m\frac{\Delta V}{t} = \frac{m(V_f - V_i)}{t} = \frac{(0.145)(55j - {-45i})}{2e{-}3}$$

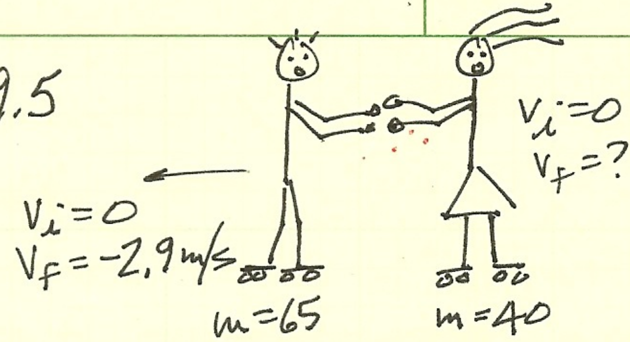
$$F = (3.26e3\,i + 3.99e3\,j)\,N$$

$$= \boxed{5.15e3\,N \;@\; 50.7^\circ} \text{ above horizontal}$$




$$\theta = \tan\left(\frac{3.99}{3.26}\right)$$

9.5




a) When girl pushes boy, reaction force from the push causes her to move away (in the +x direction, to the east).

Her velocity is...

$$\Sigma p_i = \Sigma p_f$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$0 + 0 = (65)(-2.9) + (40)(v_2')$$

$$v_2' = \boxed{4.71\,m/s} \rightarrow$$

b. Mechanical E of the system is based on the K of the boy & girl.

$$E = K_{♂} + K_{♀} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2}65(2.9)^2 + \tfrac{1}{2}40(4.71)^2$$

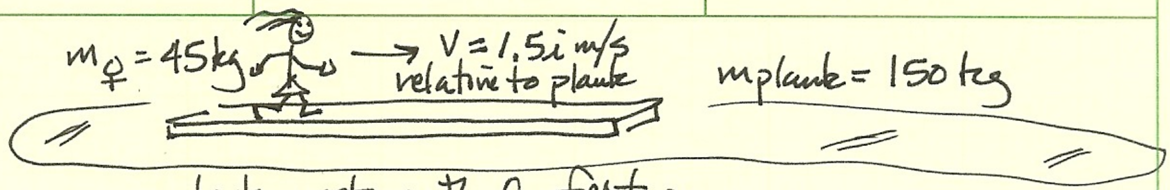
$$= \boxed{717\,J}$$

c. Yes, momentum is conserved - there is no external force acting on the boy-girl system.

d. There are large forces acting, but they are internal to the system.

e. Although there is "plenty of motion", some of the momentum is in one direction, & some in the opposite direction. The <u>net</u> momentum after the push remains <u>zero</u>.

9.6




plank rests with 0 friction on icy lake.

a) $\Sigma p_i = \Sigma p_f$ (isolated system, no external forces)

$$m_g v_g + m_p v_p = m_g v_g' + m_p v_p'$$

$$0 + 0 = (45)\underbrace{(1.5 - v_p)}_{\text{Girl's velocity relative to external reference point.}} + (150)(v_p)$$

$v_{plank}$ is backwards, relative to external reference.

$$67.5 - 45 v_p = 150 v_p$$

$$67.5 = 195 v_p$$

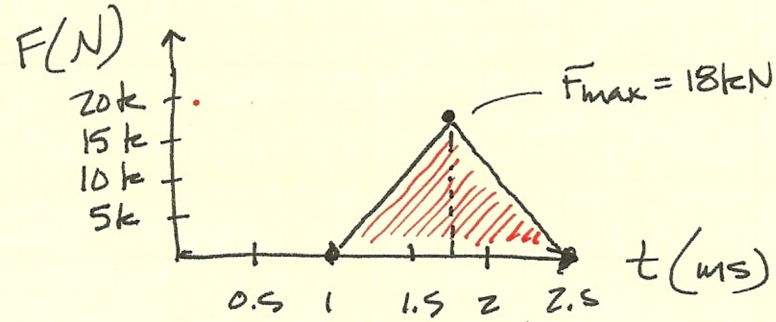
$$v_p = \boxed{0.346 \text{ m/s}}$$

b) $V_{girl-shore} = V_{girl-plank} + V_{plank-shore}$

$= 1.5$ m/s $+ {}^{-}0.346$ m/s

$= \boxed{1.16 \text{ m/s}}$

9.11 Force of bat on ball as a function of time.




a) Magnitude of impulse

$$J = \int F \cdot dt = \text{area under the curve.}$$

$$= \frac{1}{2}(18000 \cdot 0.75e{-3}) \times 2$$

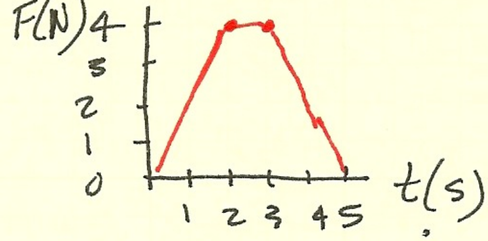
$$= \boxed{13.5 N \cdot s}$$

b) Average Force exerted on ball

$$J = F_{avg} t$$

$$F_{avg} = \frac{J}{t} = \frac{13.5 Ns}{1.5e{-3} s} = \boxed{9000 N}$$

9.15




$F_{net}$ on particle w/ mass $m = 2.5$ kg.

a) Impulse over 5-second interval?

$$J = \int F \cdot dt = \text{area under the curve.}$$

$$= \tfrac{1}{2}(2 \cdot 4) + (1 \cdot 4) + \tfrac{1}{2}(2 \cdot 4)$$

$$= \boxed{12\ N \cdot s}$$

b) $$Ft = m\Delta v = m(v_f - v_i)$$

$$12\ N \cdot s = (2.5\ kg)(v_f - 0)$$

$$v_f = \boxed{4.8\ m/s}$$

c) $$12\ N \cdot s = (2.5)(v_f - -2i)$$

$$12 = 2.5 v_f + 5$$

$$7 = 2.5 v_f$$

$$v_f = \boxed{2.8\ m/s}$$

d) Average Force exerted

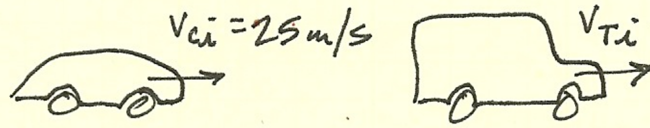
$$J = F_{avg} t$$

$$12\ N \cdot s = F_{avg}(5s)$$

$$F_{avg} = \frac{12}{5} = \boxed{2.4\ N}$$

9.18

$v_{ci} = 25\,m/s$ $\quad$ $v_{Ti} = 20\,m/s$



East $\rightarrow$

$m = 1200\,kg$ $\quad$ $m = 9000\,kg$

$v_{cf} = 18\,m/s$, East

a) $v_{Tf} = ?$ $\qquad m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$

$$(1200\,kg)(25\,m/s) + (9000)(20) = (1200)(18) + (9000)\,v_T'$$

$$v_T' = \boxed{20.9\,m/s}\ \boxed{\text{east}}$$

b) $\Delta E$ during collision?

$$\Sigma K_i = \Sigma K_f + \Delta E_{int}$$

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}mv_f^2 + \Delta E_{int}$$

$$\tfrac{1}{2}(1200)(25)^2 + \tfrac{1}{2}(9000)(20)^2 = \tfrac{1}{2}(1200)(18)^2 + \tfrac{1}{2}(9000)(20.9)^2 + \Delta E_{int}$$

$$\Delta E_{int} = \boxed{8.69 \times 10^3\,J}$$

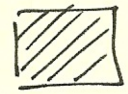
c) The change in mechanical energy is due to energy converted to heat during the collision: deformation of the cars, sound produced by the crash, etc.

Note that you'll get $1.50 \times 10^4\,J$ if you used 20.9 m/s rather than the 20.9333 m/s from the previous problem. Keep all available digits & only round for sig figs at the end of the problem!

9.19

bullet, $m = 10g = 0.0100 kg$    wood, $m = 5.00 kg$

(Before)



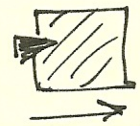
$v_i = 0$

$v_f = 0.600 m/s$

$v_i = ?$

$v_f = 0.600 m/s$ (w/ wood)

(After)



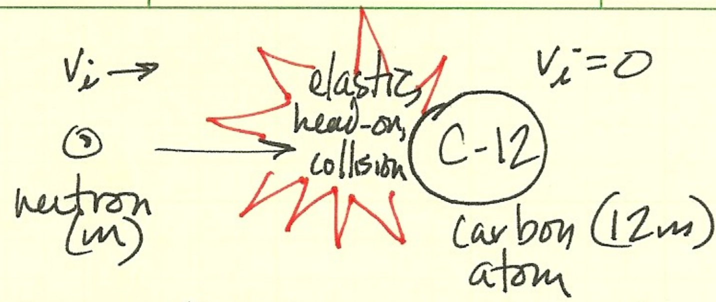
$v_f = 0.600 m/s$

$$m_1 v_1 + m_2 v_2 = m_1 v_f + m_2 v_f = (m_1 + m_2) v_f$$

$$(0.01)(v_1) + 0 = (5.0 + 0.01)(0.600)$$

$$v_1 = \boxed{301 m/s}$$

9.21




a) What fraction of neutron's K is transferred to nucleus?

Need to get velocities of 2 particles after the collision.
Using eqns 9.23 & 9.24 from book:

$$v_{1f} = \frac{m-12m}{m+12m} v_{1i} \rightarrow v_{1f} = \frac{-11}{13} v_1$$

$$v_{2f} = \frac{2m}{m+12m} v_{1i} \rightarrow \frac{2}{13} v_1$$

Use this to get fraction of initial energy transferred

$$\frac{\text{Transferred K}}{\text{Original K}} = \frac{\frac{1}{2} m_2 {v_2'}^2}{\frac{1}{2} m_1 v_1^2} = \frac{\frac{1}{2}(12m)\left(\frac{2}{13}v_1\right)^2}{\frac{1}{2}(m)(v_1)^2} = \frac{12 \cdot \frac{4}{169}}{1}$$

sticky calculator key! $\rightarrow$ ~~= 71.6%~~

$0.284 \times 100$

$\boxed{= 28.4\%}$

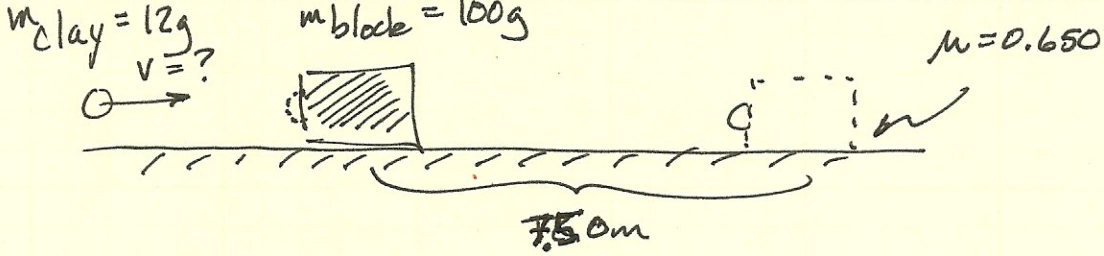
b.) $K_i = 1.60e{-13}\,J = \frac{1}{2} m v_1^2 \rightarrow v_1$

$K_f$ neutron = ? $\frac{1}{2} m \left(-\frac{11}{13} v_1\right)^2 = 71.6\% K_i =$

$\boxed{1.15e{-13}\,J}$

$K_{nucleus} = 28.4\% \times 1.60e{-13}\,J = \boxed{4.54e{-14}\,J}$

9.23




Find $v_{clay}$ just before impact.

Two part problem: Cons of momentum for impact
Cons of Energy for block sliding.

Working backwards (Energy analysis first):

$$K_i \text{ (just after impact)} = \Delta E_{int}$$

$$\tfrac{1}{2}mv^2 = fd, \quad \text{where } f = \mu N$$

$$\tfrac{1}{2}mv^2 = \mu m g d, \text{ so}$$

$$v \text{ (just after impact)} = \sqrt{2\mu g d}$$

$$= \sqrt{2(0.650)(9.8)(7.5)}$$

$$= \underline{9.77\,m/s}$$

Now, use this with cons of momentum to get clay velocity <u>before</u> impact:

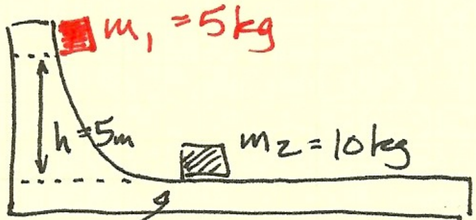
$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

$$m_{clay} v_{clay} + 0 = (m_{clay} + m_{block}) v'$$

$$0.012\, v_{clay} = (0.012 + 0.100)(9.77)$$

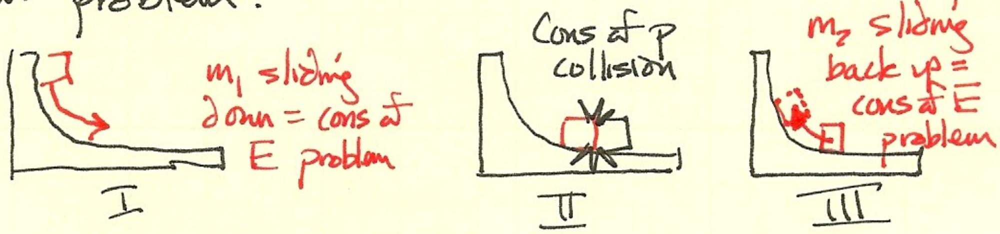
$$v_{clay} = \boxed{91.2\,m/s}$$

9.27




Magnets prevent physical collision — model as an elastic collision. Find height to which $m_1$ rises <u>after</u> collision.

3 part problem!




I $\quad U_g = K_f$ (just before collision)

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 5} = \underline{9.90 \, m/s}$$

II $\quad \cancel{m_1 v_1 + m_2 v_2 = m_1 +}$

Elastic collision, cons of momentum $\rightarrow$ $v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i}$ (Eqn 9.23 in book)

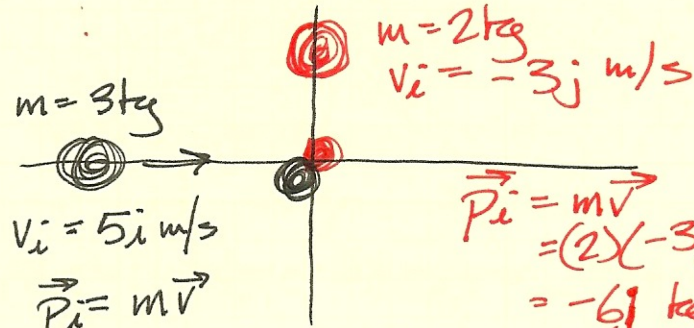
$$= \left(\frac{5-10}{5+10}\right) 9.9 = \underline{-3.3 \, m/s}$$

III $\quad K_i = U_{gf}$

$$\frac{1}{2}mv^2 = mgh$$

$$h = \frac{v^2}{2g} = \frac{(3.3)^2}{2 \cdot 9.8} = \boxed{10.556m}$$

9.29




$\vec{p}_i = m\vec{v}$
$= 3 \cdot 5$
$= 15i$ kg m/s

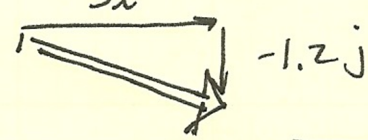
$\vec{p}_i = m\vec{v}$
$= (2)(-3j)$
$= -6j$ kg m/s

$p_{1i} + p_{2i} = p_{12f}$

$(15i + -6j)$ kg m/s $= (3+2)$ kg $(\vec{v}_f)$

$\frac{15i - 6j}{5} = v_f$

$v_f = \boxed{(3i - 1.2j) \text{ kg m/s}}$

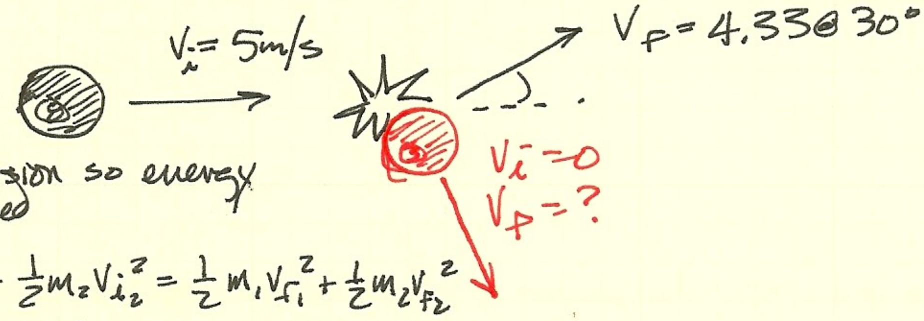



or

$v_f = \sqrt{3^2 + 1.2^2} = \boxed{3.23 \text{ m/s @}}$

$\theta = \tan^{-1}\left(\frac{-1.2}{3}\right) = \boxed{-21.8°}$

9.33




Elastic collision so energy conserved

$$\tfrac{1}{2}m_1 v_{i_1}^2 + \tfrac{1}{2}m_2 v_{i_2}^2 = \tfrac{1}{2}m_1 v_{f_1}^2 + \tfrac{1}{2}m_2 v_{f_2}^2$$

$$\rightarrow v_{1i}^2 + 0 = v_{1f}^2 + v_{2f}^2$$

$$5^2 = 4.33^2 + v_{2f}^2 \rightarrow v_{2f} = \boxed{2.50\text{ m/s}}$$

If I know the "elastic glancing collisions w/ an object of equal mass at rest produces velocities at 90°" trick, I can predict that $v_2$ moves at a direction of $\boxed{-60°}$.

If I don't know that trick, I'll have to use component analysis of momentum vectors.

$$x: p_{1x} + p_{2x} = p_{1x}' + p_{2x}'$$

$$m_1 v_x + 0 = m_1 v_x' + m_2 v_x'$$

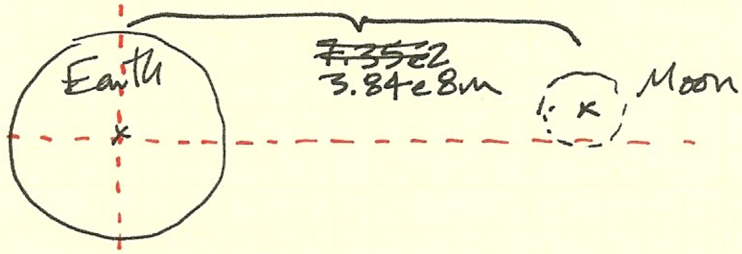
Because $m_1 = m_2$

$$v_{1x} = v_{1x}' + v_{2x}'$$

$$5 = 4.33\cos 30 + 2.50\cos\theta$$

$$\theta = \pm 60° = \boxed{-60°}$$

9.36




$m_e = 5.97e24\,kg$ $\qquad m_m = 7.35e22\,kg$

$x_{cm} = ?$

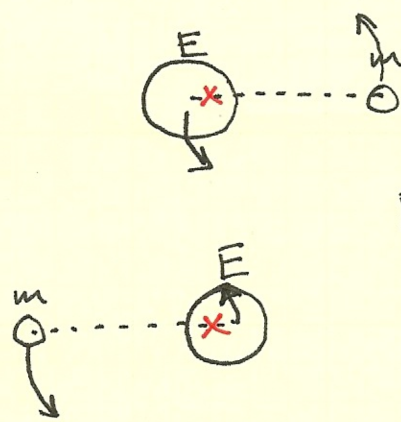
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Consider center of earth as 0

$$x_{cm} = \frac{(5.97e24)(0) + (7.35e22)(3.84e8)}{(5.97e24 + 7.35e22)}$$

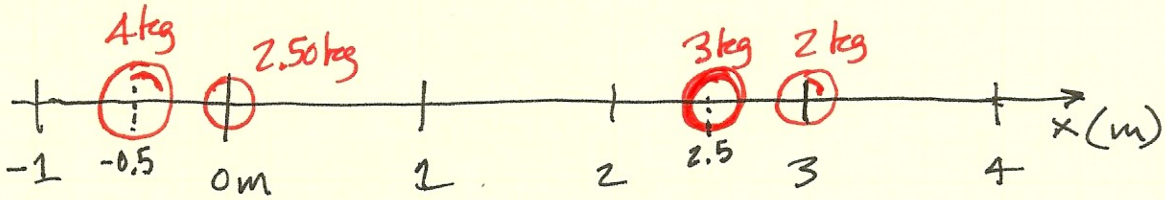
$= \underline{4.67e6\,m}$ from center of earth

Interestingly, this center of mass is located within the earth, which has a ~~center~~ radius of 6.38e6m.




Earth-moon system rotates about the center of mass.

9.37



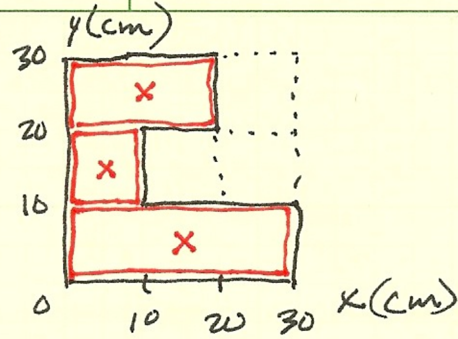

Find cm?

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4}{m_1 + m_2 + m_3 + m_4}$$

$$x_{cm} = \frac{(4)(-0.5) + (2.5)(0) + (3)(2.5) + (2)(3)}{4 + 2.5 + 3 + 2}$$

$$= \boxed{1.00\,m}$$

9.38




Find cm for this piece of metal. Sheet is uniform.

Strategy: simplify shape, & calculate $x_{cm}$ & $y_{cm}$ independently.

One simple symmetry is shown above, with cm of each component indicated.

Top piece $= \frac{2}{6}m$ ~~cm~~ cm top piece is located at 10, 25 cm
Middle piece $\frac{1}{6}m$ cm middle piece " 5, 15 cm
Bottom piece $= \frac{3}{6}m$ cm bottom piece " 15, 5 cm

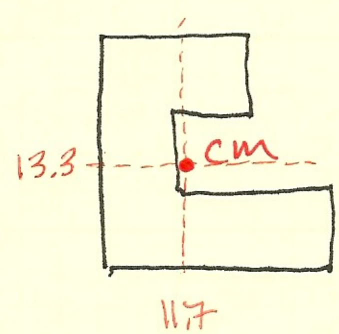
$$x_{cm} = \frac{m_{top}x_{top} + m_{middle}x_{middle} + m_{bottom}x_{bottom}}{m_{top} + m_{middle} + m_{bottom}}$$

$$= \frac{\left(\frac{2}{6}m\right)(10) + \left(\frac{1}{6}m\right)(5) + \left(\frac{3}{6}m\right)(15)}{\frac{2}{6}m + \frac{1}{6}m + \frac{3}{6}m}$$

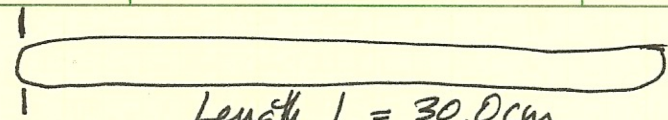
$$= \boxed{11.7\,\text{cm}}$$

$$y_{cm} = \frac{\left(\frac{2}{6}m\right)(25) + \left(\frac{1}{6}\right)(15) + \left(\frac{3}{6}\right)(5)}{m}$$

$$= \boxed{13.3\,\text{cm}}$$



9.40



$x=0$

Length $L = 30.0\,cm$

Linear density $\lambda = (50 + 20x)\ g/m$

a) What is the mass of the rod?

$$\lambda = \frac{M}{L} = \frac{dm}{dl}, \text{ or } \frac{dm}{dx} \text{ (along the x-axis)}$$

We can also write

$dm = \lambda\, dx$, which leads to

$$M_{total} = \int dm = \int_0^{.30} \lambda\, dx$$

$$M = \int_0^{.30} 50 + 20x$$

$$= 50x + 10x^2 \Big|_0^{.30}$$

$$= 50(.30) + (10)(.30)^2 - 0 = \boxed{15.9\ g}$$

b) How far from the end is the cm?

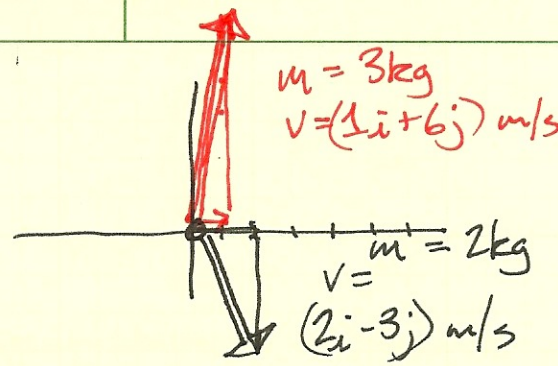
$$x_{cm} = \frac{1}{M}\int x\, dm, \text{ where } dm = \lambda\, dx. \text{ Therefore}$$

$$x_{cm} = \frac{1}{M}\int x \lambda\, dx = \frac{1}{M}\int x(50 + 20x)\, dx$$

$$x_{cm} = \frac{1}{M}\int 50x + 20x^2\, dx = \frac{1}{M}\left(25x^2 + \frac{20}{3}x^3\right)\Big|_0^{.30}$$

$$x_{cm} = \frac{1}{15.9}\left(25(.3)^2 + \frac{20}{3}(.3)^3 - 0\right) = \boxed{0.153\,m}$$

9.41




a) $v_{cm} = ?$

If there are no external forces acting on the system, to momentum remains constant, & the overall momentum $p_{total} = \sum p_i$.

$$p_{total} = \sum p_i$$

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$v_{cm} = \frac{1}{M}(m_1\vec{v}_1 + m_2\vec{v}_2)$$

$$= \frac{1}{3+2}\left(3(1i+6j) + 2(2i-3j)\right)$$

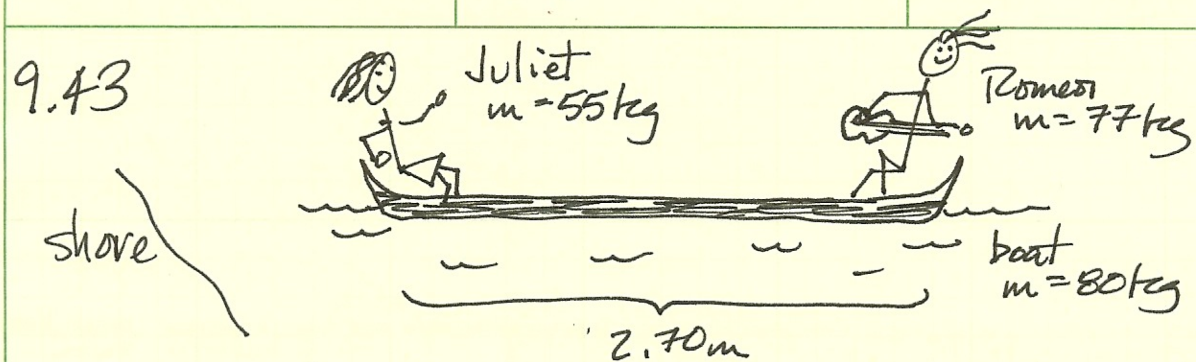
$$= \frac{1}{5}(7i + 12j) = \boxed{\left(\frac{7}{5}i + \frac{12}{5}j\right) m/s}$$

b) Total momentum of system p?

$$p_{total} = M\vec{v}_{cm} = (5\,kg)\left(\frac{7}{5}i + \frac{12}{5}j\right) m/s$$

$$= \boxed{7i + 12j \ kg \cdot m/s}$$

9.43

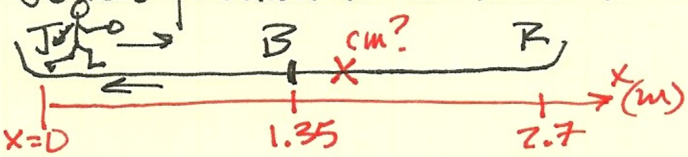



How far does boat move towards shore when Juliet stands up & walks to Romeo?

$F_{ext} = \frac{dp}{dt} = 0$; no external force, so momentum of system remains constant.

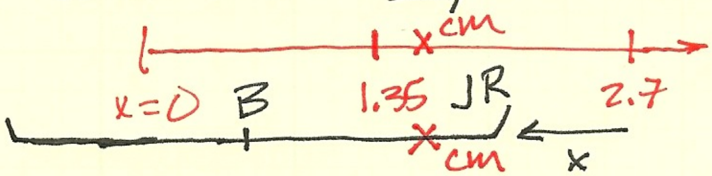
$V_{cm}$ before & after is 0, so no change in $x_{cm}$, before & after.

Consider Juliet's position to be $x = 0$.




$$x_{cm} = \frac{1}{M}\left(m_J x_J + m_R x_R + m_B x_B\right)$$
$$= \frac{1}{212}\left(55 \cdot 0 + 77 \cdot 2.7 + 80 \cdot 1.35\right) = \underline{1.49\,m}$$

That $x_{cm}$ doesn't change, even after Juliet, Romeo, & boat have redistributed their locations. Boat + Romeo move toward shore a distance $x$ while Juliet moves away from shore a distance $2.7 - x$.




After moving: $x_{cm} = \frac{1}{M}\left(m_J(2.7-x) + m_R(2.7-x) + m_B(1.35-x)\right)$

$$1.49 = \frac{1}{212}\left(55(2.7-x) + 77(2.7-x) + 80(1.35-x)\right)$$

Solve for $x$ to get $\boxed{x = 0.700\,m}$ that boat has moved

Note that other reference frames & solution strategies are possible!