
Ch 8–Conservation of Energy

# Cons. of Energy

It has been determined, through experimentation, that the total mechanical energy of a system remains constant in any isolated system of objects that interact only through conservative forces.

James Prescott Joule, 1818-1889

# Conservation of Energy

*"Energy is neither created nor destroyed -- energy is always conserved."*

$$\sum \Delta E = 0$$

$$\sum E_i = \sum E_f$$

$$U_i + K_i = U_f + K_f$$

$$\Delta K + \Delta U + \Delta E_{\text{int}} = W + Q + T_{MW} + T_{MT} + T_{ET} + T_{ER}$$

# Cons. of Energy - Non-isolated system

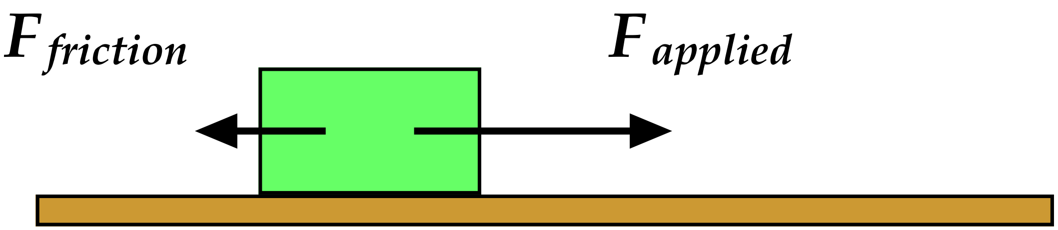
*Isolated system* = unaffected by outside influence

*Non-isolated system* = Work added to the system, or $\Delta E_{internal}$ “lost” as heat.

# Energy & Friction?

Friction forces convert K into *internal energy*.

$$\Delta E_{internal} = f_k d$$



# Conservation of Energy

*"Energy is neither created nor destroyed -- energy is always conserved."*

$$\sum E_i = \sum E_f$$

$$U_i + K_i = U_f + K_f$$

$$E_{system-initial} - \Delta E_{internal} + \Sigma W_{otherforces} = E_{system-final}$$

# Strategy

1. Identify problem as using an Energy analysis.
2. Define your system: Identify initial and final positions for all objects.
3. Select zero reference points for all potential energies.
4. If any type of friction is present, mechanical energy will not be constant--account for friction using $\Delta E_{\text{int}} = F_{friction}x$.
5. Write equation with all energies present, and solve for unknown.

# Example 1

A ball of mass *m* is dropped from a height *h* above the ground.

a) What is the speed of the ball at a height *y* above the ground? (Assume no air friction.)

$$U_{g-i} + K_i = U_{g-f} + K_f$$

$$mgh + 0 = mgy + \frac{1}{2}mv^2$$

$$v = \sqrt{2g(h-y)}$$

b) What is the speed of the ball if it is given an initial speed $v_i$ at the initial height *h*?

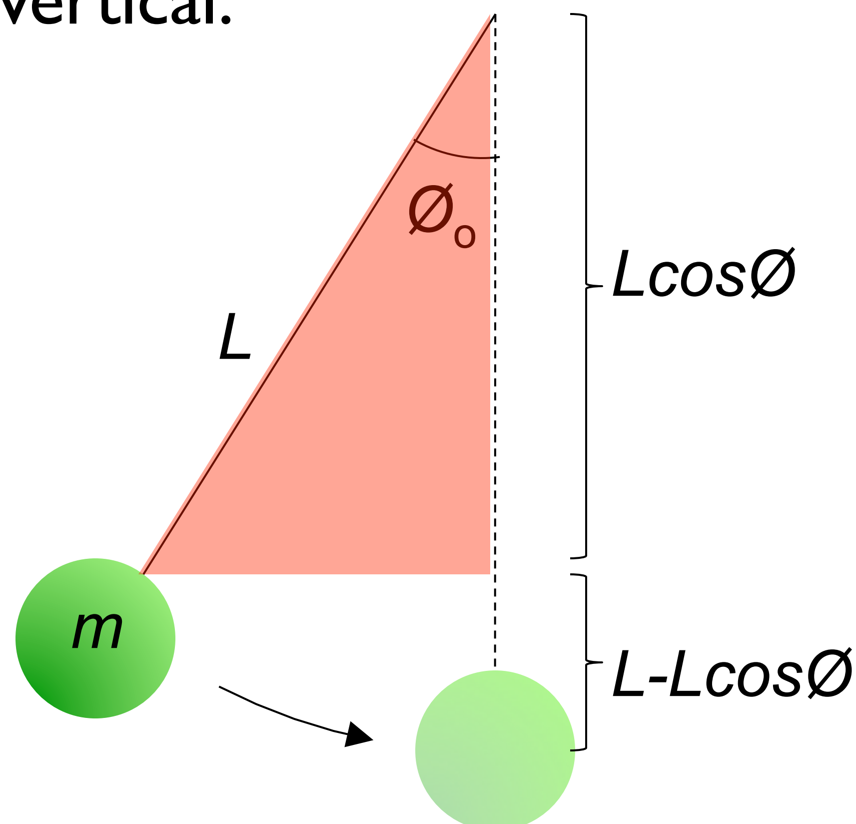
$$U_{g-i} + K_i = U_{g-f} + K_f$$

$$mgh + \frac{1}{2}mv_i^{\ 2} = mgy + \frac{1}{2}mv_f^{\ 2}$$

$$v_f = \sqrt{2g(h-y) + v_i^{\ 2}}$$

# Example 2

A pendulum of length L, with bob of mass *m*, is released from an angle $Ø_o$ relative to the vertical.




a) Find the speed of the pendulum at its lowest point.

$$U_{gi} + K_i = U_{gf} + K_f$$

$$mg(L - L\cos\phi + 0 = 0 + \frac{1}{2}mv^2$$

$$v = \sqrt{2gL(1 - \cos\phi)}$$

b) What is the tension T at this lowest point?

$$F_c = \frac{mv^2}{r}$$

$$T - mg = \frac{mv^2}{r}, \quad so \quad T = \frac{mv^2}{r} + mg$$

$$T = \frac{m\left(\sqrt{2gL(1 - \cos\phi)}\right)^2}{L} + mg$$

$$T = mg(3 - 2\cos\phi_o)$$

# Example 3

A child of mass *m* slides down a 6.00-m high slide, starting from rest.

a) Find $v_f$ assuming no friction.

$$U_{gi} + K_i = U_{gf} + K_f$$

$$mgh + 0 = 0 + \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = 10.8m/s$$

b) Find how much energy is lost to friction if $v_f$=8.00 m/s and *m* = 20.0 kg

$$U_{gi} + K_i - \Delta E_{\text{int}} = U_{gf} + K_f$$

$$\Delta E_{\text{int}} = mgh - \frac{1}{2}mv^2$$

$$\Delta E_{\text{int}} = 536J$$

c) Find the coefficient of kinetic friction in part (b) if the slide angle is 30° above the horizontal.

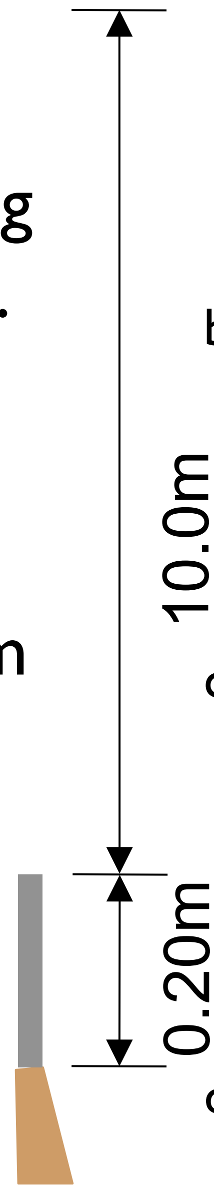
$$\Delta E_{\text{int}} = F_{friction}d$$

$$536J = F_{friction}\frac{6}{\sin 30}; F_{friction} = 44.7N$$

$$\mu = \frac{F_{friction}}{F_{Normal}} = \frac{44.7}{mg\cos 30} = 0.263$$

# Example 4

The launching mechanism for a toy rifle uses a spring with unknown constant *k*. A 35.0 g projectile placed on the spring compresses it a negligible amount. When compressed 0.20 m from the barrel end, the vertically-oriented gun launches the projectile to a height of 10.0 m above the end of barrel exit.




a) Neglecting friction, determine the spring constant.

b) Find the equilibrium position of the spring with the mass resting on it.

c) Find the speed of the projectile as it moves through the equilibrium position of the spring.

d) What is the speed of the projectile at 5.0 m above the end of the barrel exit?

# Example 4

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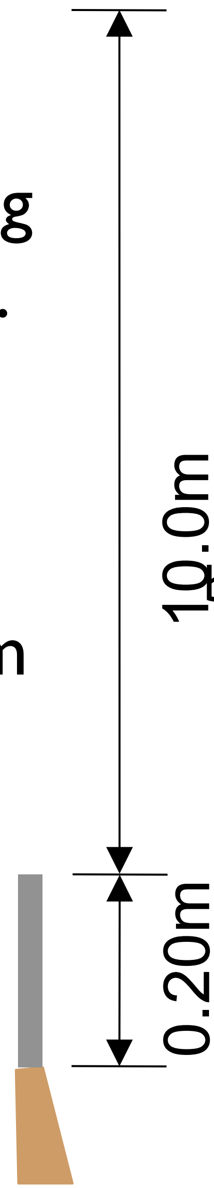



a) Neglecting friction, determine the spring constant.

$$U_{s-initial} + U_{g-initial} + K_i = U_{s-final} + U_{g-final} + K_f$$

$$\frac{1}{2}kx_i^2 + mgh + \frac{1}{2}mv_i^2 = \frac{1}{2}kx_f^2 + mgh_f + \frac{1}{2}mv_f^2$$

$$\frac{1}{2}kx_i^2 + 0 + 0 = 0 + mgh + 0$$

$$k = \frac{2mgh}{x^2} = \frac{2(0.035kg)(9.8)(10.2m)}{(0.2m)^2} = 175N/m$$

b) Find the equilibrium position of the spring with the mass resting on it.

$$F_{net} = 0$$

$$F_{spring} - F_g = 0$$

$$F_{spring} = -mg = -kx$$

$$x = \frac{mg}{k} = \frac{(0.035kg)(9.8m/s^2)}{175N/m} = 0.00196$$

# Example 4

c) Find the speed of the projectile as it moves through the equilibrium position of the spring.

$$U_{s-initial} + U_{g-initial} + K_i = U_{s-final} + U_{g-final} + K_f$$

$$\frac{1}{2}kx_i^2 + mgh_i + \frac{1}{2}mv_i^2 = \frac{1}{2}kx_f^2 + mgh_f + \frac{1}{2}mv_f^2$$

$$\frac{1}{2}kx_i^2 + 0 + 0 = \frac{1}{2}kx_f + mgh + \frac{1}{2}mv_f^2$$

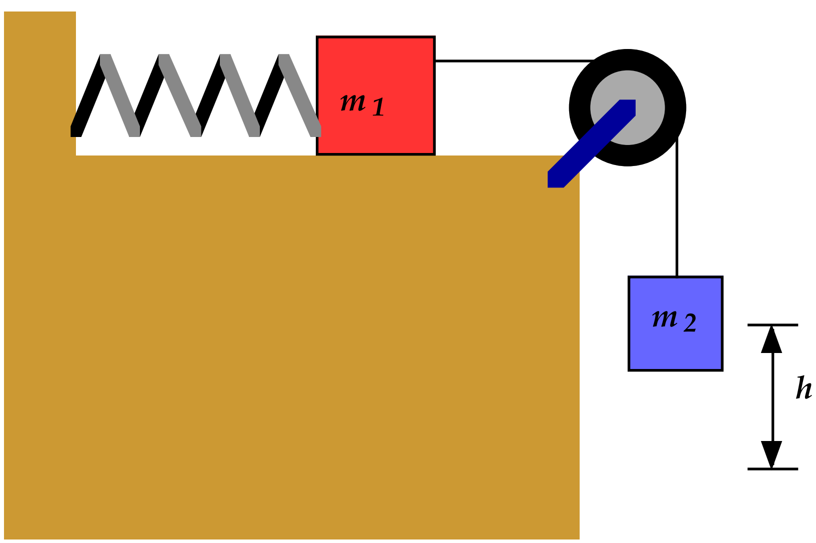
$$\frac{1}{2}(175)(-0.2)^2 + 0 + 0 = \frac{1}{2}(175)(-0.00196)^2 + (0.035)g(2 - 0.00196) + \frac{1}{2}(0.035)v_f^2$$

$$v_i = 14.0m/s$$

d) What is the speed of the projectile at 5.0 m above the end of the barrel exit?

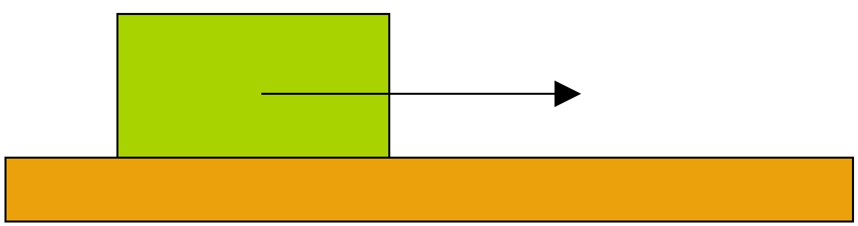
# Example 5

Two blocks are connected as shown. Spring starts uncompressed. When $m_2$ is released, it slowly descends a distance $h$ before coming to rest. Calculate $\mu$ between $m_1$ and surface.



# Example 6

A 6.0-kg mass is pulled with a constant horizontal Force of 12.0-N for a distance of 3.0-m on a rough surface with $\mu$=0.15.



Find the final speed of the block using work-energy.

$$W + K_i - \Delta E_{\text{int}} = K_f$$

$$Fx + 0 - fd = \frac{1}{2} m v_f^2$$

$$f = \mu N = \mu m g$$

$$v_f = \sqrt{\frac{2(Fx - \mu m g d)}{m}}$$

$$v_f = \sqrt{\frac{2(12 \bullet 3 - 0.15 \bullet 6 \bullet 9.8 \bullet 3)}{6}}$$

$$v_f = 1.78 m/s$$

# Example 7

A car traveling at a speed $v$ skids a distance $d$ after the brakes lock up.

a) How far will it skid if its initial velocity is 2$v$?

$$K_i - \Delta E_{\text{int}} = K_f$$

$$\frac{1}{2}mv^2 - fd = 0$$

$$d = \frac{mv^2}{2f} \rightarrow d \propto v^2$$

$$d' = \frac{m(2v)^2}{2f} = 4\left(\frac{mv^2}{2f}\right) = 4d$$

b) What happens to the car's $K$ as it skids to a stop?

*It's converted to random K of molecules in tire & road.*

# Example 8

A 1.6 kg block is attached to a horizontal spring with $k$ = 1.0x10$^3$N/m. The spring is compressed 2.0 cm and released.

a) What is the speed of the block as it passes through the equilibrium position? (Assume frictionless.)

$$W_{spring} + K_i = K_f$$

$$\frac{1}{2}kx^2 + 0 = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{1000}{1.6}}(0.02) = 0.5m/s$$

b) What is the speed of the block as it passes the equilibrium position if there is a constant friction force of 4.0 N retarding its motion?

$$W_{spring} + K_i - \Delta E_{\text{int}} = K_f$$

$$\frac{1}{2}kx^2 + 0 - fd = \frac{1}{2}mv^2$$

$$\frac{1}{2}1000(0.02)^2 + 0 - (4)(0.02) = \frac{1}{2}1.6v^2$$

$$v = 0.39m/s$$

# Power

*Power* = "the rate at which Work is done."

$$Power \quad P_{avg} = \frac{\Delta Work}{\Delta time}$$

$$P_{\text{instantaneous}} = \frac{dW}{dt}$$

$$P_{\text{instantaneous}} = \frac{F \bullet ds}{dt} = F \bullet \frac{ds}{dt} = \vec{F} \bullet \vec{v}$$

# Example 9

A student in class is lifted in a chair.

a) How much Work was required to lift the student?

b) How much Power was used to lift the student?

# Example 10

An elevator with a mass of 1000 kg carries a load of 800 kg. 4000 N of friction retards the elevator's upward motion.

a) Find minimum power necessary to lift the elevator at a speed of 3.00 m/s.

6.48e4 W

b) If the motor needs to have a 3:1 safety factor, what should the horsepower rating on the motor be? (746 W = 1 hp)

3*86.9=261 hp

c) What Power must the motor deliver at any instant (as a function of *v* ) if it's designed to provide an acceleration of 1.00 m/s$^2$?

P=F•v=2.34e4v W