Ch 7 – Energy & Work

Work

Work is a quantity that is useful in describing how objects interact with other objects.

"Work done by an agent exerting a constant force on an object is the product of the component of the force in the direction of displacement, and the magnitude of the displacement of the object."

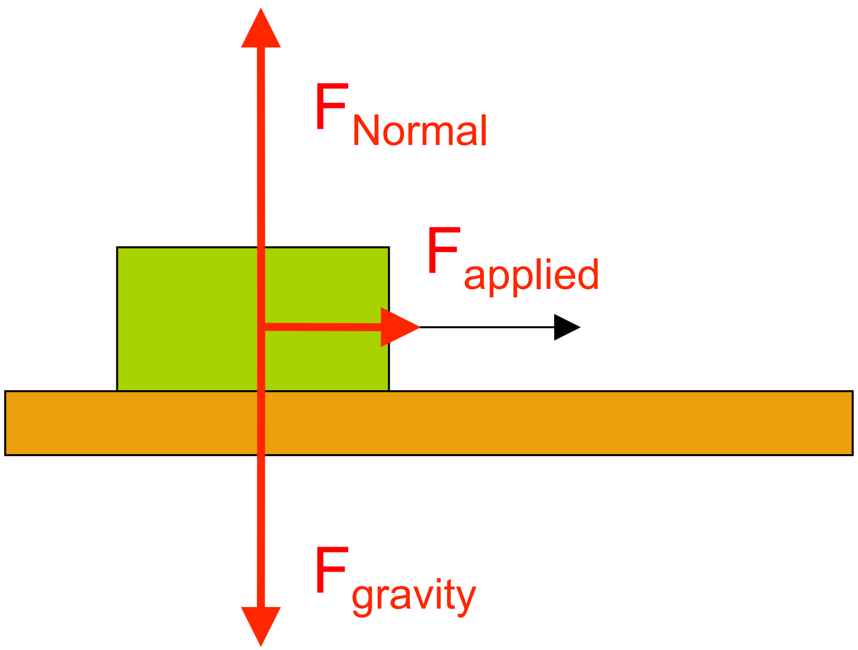
$$\boxed{W = \vec{\mathbf{F}} \bullet \vec{\mathbf{x}}}$$

$$W = Fx\cos\theta$$

$$W = Fx$$

Example 1

A 100-N horizontal force is used to drag a 20kg box 2.0-m across a frictionless table. How much Work is done on the box...



a) By the horizontal Force?

$$W = Fx\cos\theta$$

$$W = (100)(2.0)\cos(0°) = 200N \bullet m$$

$$W = 200 \quad Joules$$

b) By the table?

$$W = F_{Normal}x\cos\theta$$

$$W = (200N)(2.0m)\cos(90°) = 0J$$

c) By gravity?

$$W = F_{gravity}x\cos\theta$$

$$W = (200N)(2.0m)\cos(-90°) = 0J$$

Example 2

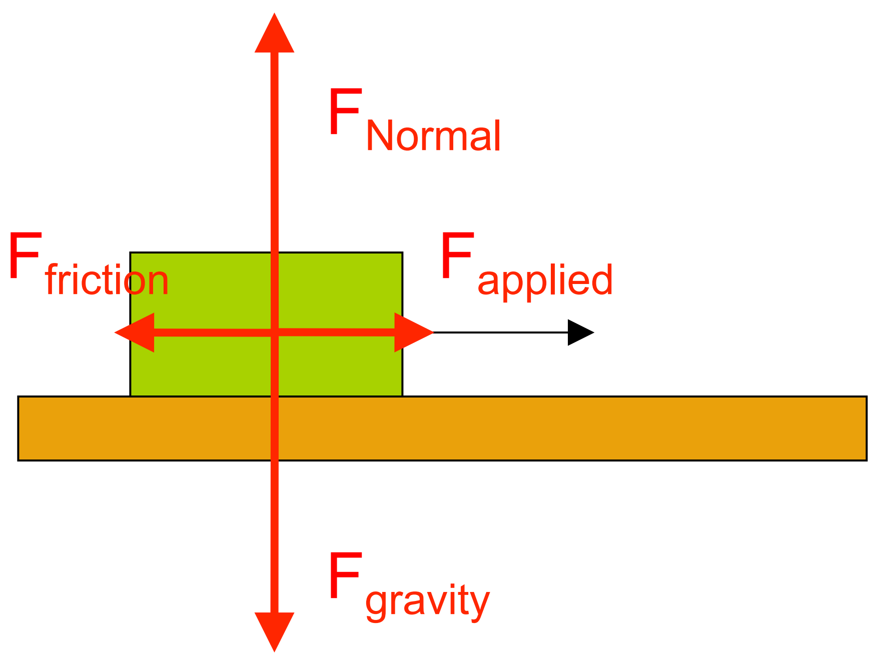
The same 100-N horizontal force is used to drag a 20kg box 2.0-m across a rough surface at constant velocity.

How much Work is done by Friction?

$$W = Fx\cos\theta$$

$$W = (100N)(2.0m)\cos(180°)$$

$$W = -200J$$

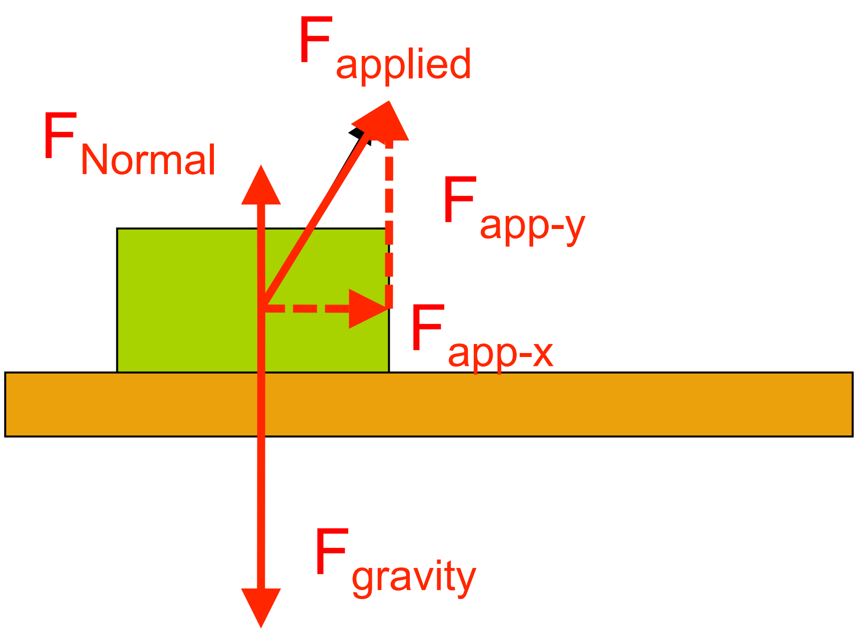


Example 3

A100-N is applied at 60° above the horizontal, and used to drag the 20-kg box 2.0-m across a frictionless surface.

How much Work is done by the applied Force?

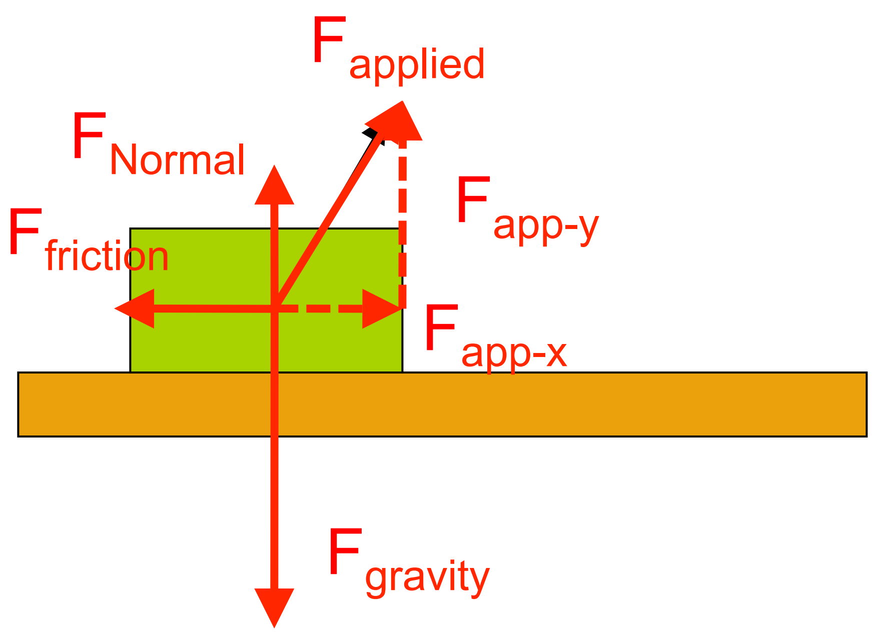
$$W = Fx\cos\theta$$
$$W = (100N)(2.0m)\cos 60°$$
$$W = 100J$$



$$W = F_x x$$
$$W = (100N\cos 60°)(2.0m)$$
$$W = 100J$$

Example 4

A 100-N is applied at 60° above the horizontal, and used to drag the 20-kg box 2.0-m across a rough surface (μ = 0.15).



How much Work is done by Friction?

$$W = F_{friction} x \cos\theta$$

$$F_{friction} = \mu F_{Normal}$$

$$\sum F_y = ma_y = 0$$

$$F_{Normal} + F_{applied-y} - F_g = 0$$

$$F_{Normal} = mg - F_{applied} \sin 60$$

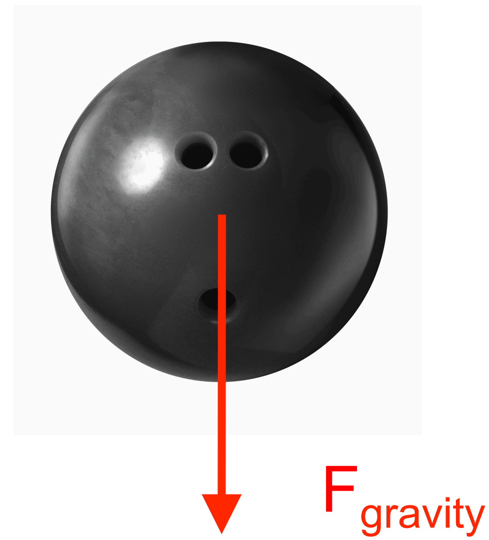
$$F_{Normal} = 109N$$

$$F_{friction} = (0.15)(109N) = 16N$$

$$W = (16N)(2m)\cos 180 = -32J$$

More Examples

1. I lift a 8-kg bowling ball up 50 cm into the air at constant velocity-- how much Work did I do?
2. How much Work did Earth's gravity do in the preceding problem?
3. How much Work do I do lowering an 8-kg bowling ball 50 cm down?
4. How much Work do I do holding an 8-kg bowling ball motionless in the air?
5. How much Work do I do carrying an 8-kg bowling ball sideways 50cm at constant velocity?



Dot products

$$W = \vec{\mathbf{F}} \bullet \vec{\mathbf{x}}$$

If F, x, and θ are known, *W* = Fx cos θ.

The dot product is especially easy to calculate in these cases:

- Parallel: Fx cos(0) = Fx, and Fx(cos(180)=-Fx
- Perpendicular: Fx cos(90) = 0

If F and x have been given to you in **i** , **j** notation, use the dot-product to find *W*.

Example 5

A=3**i** + 5**j**, and
B = -**i** + 2**j**.

a) What is **A** • **B**?

$$\mathbf{A} \bullet \mathbf{B} = ?$$

$$\mathbf{A} \bullet \mathbf{B} = (3\mathbf{i} + 5\mathbf{j}) \bullet (-1\mathbf{i} + 2\mathbf{j})$$

$$\mathbf{A} \bullet \mathbf{B} = -3 + 10 = 7$$

b) What is the angle between **A** and **B**?

$$\mathbf{A} \bullet \mathbf{B} = AB\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{A} \bullet \mathbf{B}}{AB}\right)$$

$$A = \sqrt{3^2 + 5^2} = 5.83$$

$$B = \sqrt{-1^2 + 2^2} = 1.73$$

$$\theta = \cos^{-1}\left(\frac{7}{5.83 \bullet 1.73}\right) = 46°$$

Example 6

A force of 4**i** + -3**j** Newtons acts on object, and displaces it 10**i**+3**j** meters.

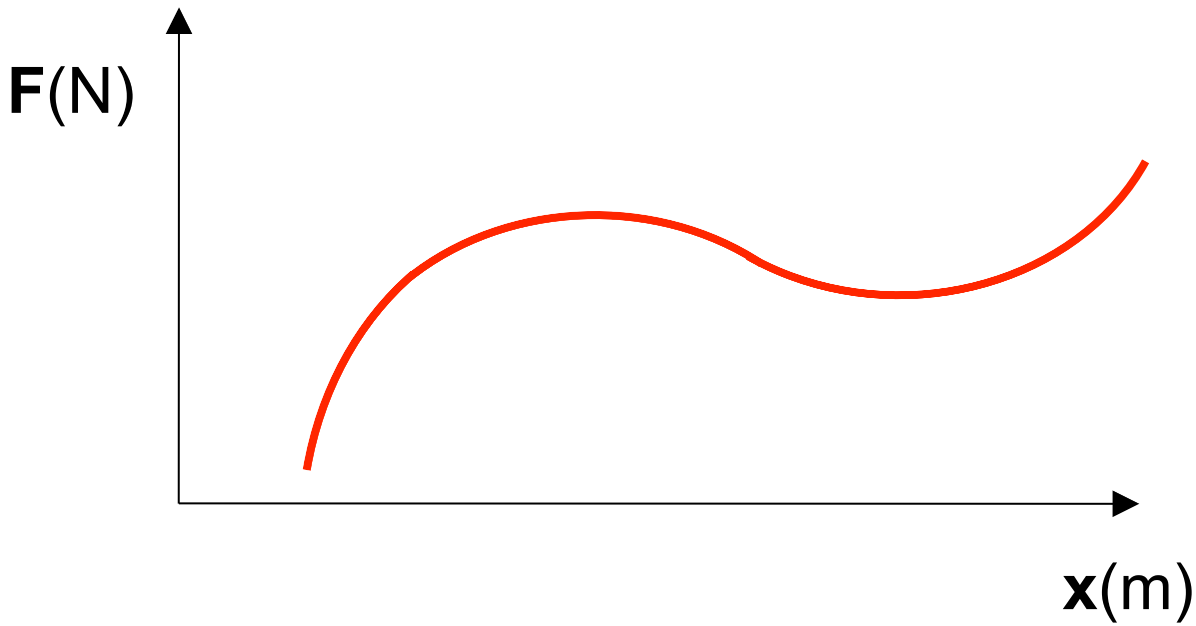
How much Work was done on the object by the force?

$$W = \mathbf{F} \bullet \mathbf{x}$$

$$W = (4\mathbf{i} - 3\mathbf{j}) \bullet (10\mathbf{i} + 3\mathbf{j})$$

$$W = (40 - 9) = 31J$$

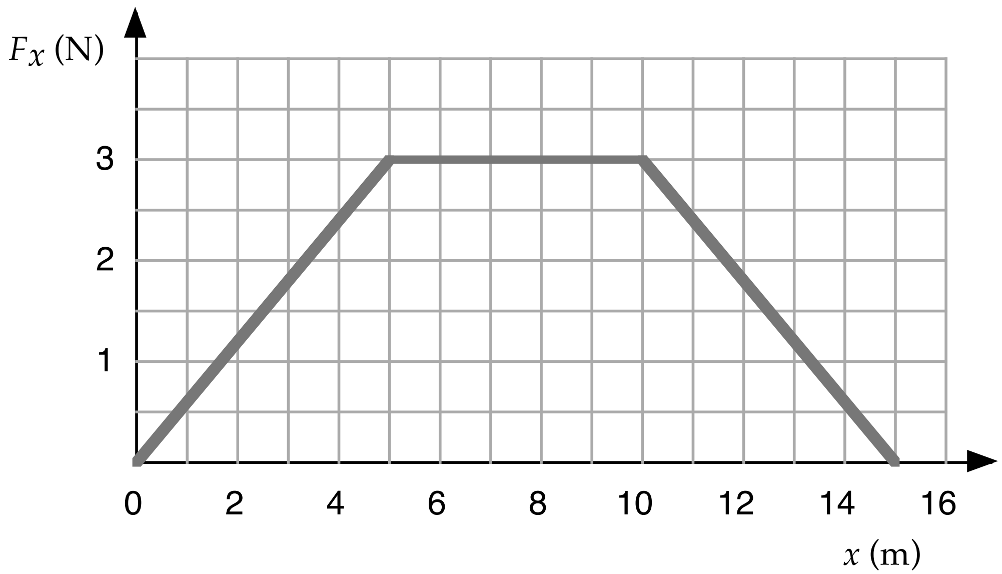
Work done by varying Force



$$W = \int_{x_i}^{x_f} F_x \bullet dx$$

Example 7

How much Work is done by the Force indicated in the graph here?



Hooke's Law-Springs

$$\boxed{F_{spring} = -kx}$$

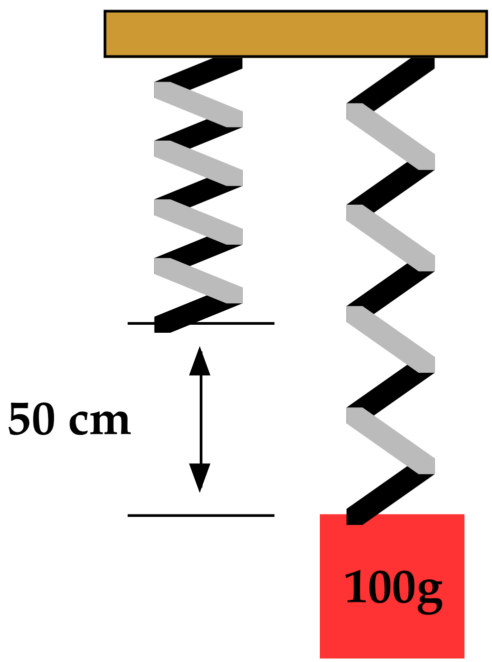
Example 8

A mass is suspended from a spring and allowed to come to rest. Calculate the spring constant of the spring shown here.

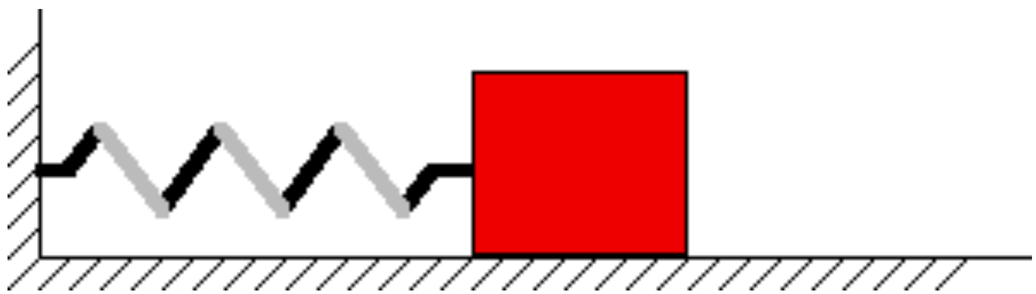
$$F_{spring} = -kx$$

$$k = \frac{F_{spring}}{x} = \frac{F_{gravity}}{x} \frac{mg}{0.50m}$$

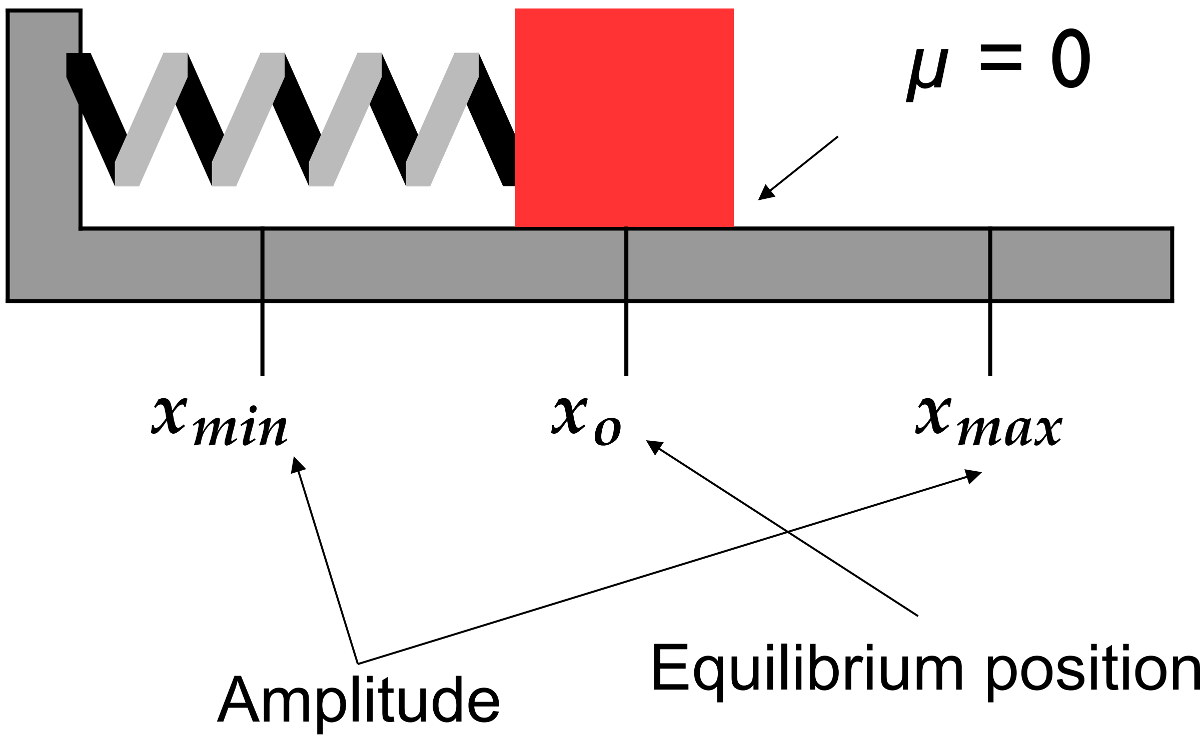
$$k = \frac{(0.100kg)(9.8m/s^2)}{0.50m} = 2.0N/m$$



An Oscillating Spring



Work w/ Varying Force



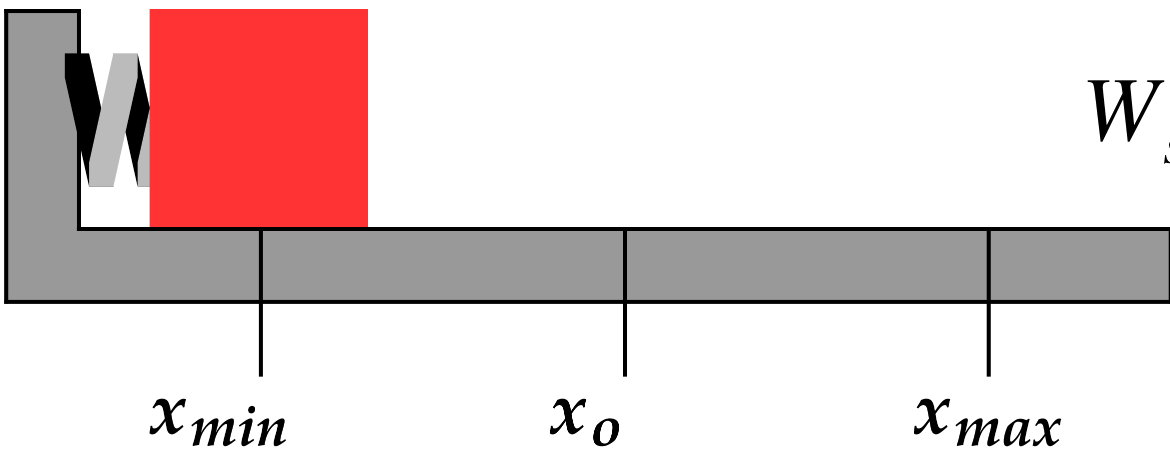
Example 9

How much Work is done by the spring to return the block to its equilibrium position?

$$W_{spring} = \int_{x_i}^{x_f} F_{spring} \bullet dx$$

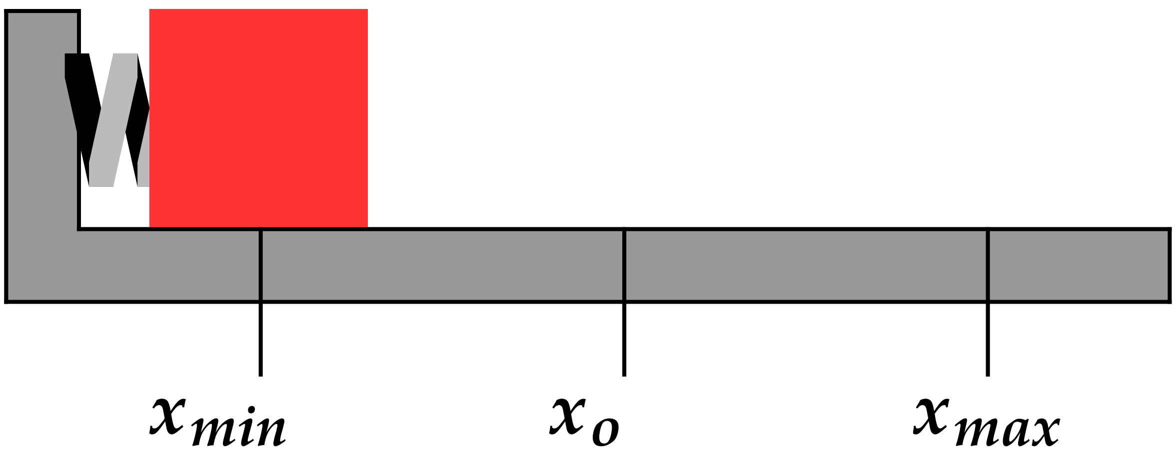
$$W_{spring} = \int_{-x}^{x_o} (-kx) \bullet dx$$

$$W_{spring} = \frac{1}{2} kx^2$$



Example 9a

If the mass is displaced twice the distance x before being released, how much more Work will the spring do on the mass?



$$W_{spring} = \frac{1}{2}kx^2$$

$$W_{spring} = \frac{1}{2}k(2x)^2$$

$$W'_{spring} = 4\left(\frac{1}{2}kx^2\right)$$

$$W'_{spring} = 4W_{spring}$$

Extra Credit

10/16/16	10/17/16	10/18/16
	CLASS: 7.5 Kinetic Energy and the Work-Kinetic Energy Theorem; 7.6: Potential Energy of a System. Extra Credit available.	
	HW: 7.31, 7.32, 7.33, 7.35, 7.42	
10/23/16	**10/24/16**	**10/25/16**
	CLASS: Lab-Conservation of Energy: Pendulum	**CLASS:** 8.1-8.2: Analysis Models for non-Isolated and Isolated Systems; 8.3: Situations Involving Kinetic Friction
	HW: Complete writeup if not finished in class	**HW:** 8.2, 8.3, 8.5, 8.7, 8.12, 8.14, 8.16
10/30/16	**10/31/16**	**11/01/16**
	CLASS: 9.1: Linear Momentum; 9.2: Analysis Model: Isolated System; 9.3 Analysis Model: Non-isolated System	**CLASS:** Review for Test.
	HW: 9.1, 9.4, 9.5, 9.6, 9.11, 9.15; Download Practice Test	**HW:** Study for test

2010-2011 / GP1 / AP PHYSICS

	B (87.4%)		
TEST: 79%			
ASSIGNMENT	DUE	SCORE	POSSIBLE
Test-Chapter 2	Sep 03	76	100
TEST-Ch 3, 4	Sep 20	82	100
LAB: 97.5%			
ASSIGNMENT	DUE	SCORE	POSSIBLE
LAB-Kinematics	Sep 15	19	20
Lab-Catch the Ball	Oct 06	20	20
OTHER: 101.9%			
ASSIGNMENT	DUE	SCORE	POSSIBLE
Activity-Velocity of Car	Sep 02	5	5
Homework-Ch 2	Sep 02	11	10
HW: 3:1	Sep 03	2	2
HW: 3: 3, 9, 17	Sep 07	2	2
HW: 3: 21, 33, 40	Sep 08	2	2
HW: 4: 1, 2, 5, 7	Sep 10	2	2
HW-4: 12, 17, 19, 21, 23	Sep 13	2	2
Activity-Bus Jump Scene from S	Sep 14	5	5
HW-4: 27, 30	Sep 14	2	2
HW-4: 33, 36, 41, 42, 45	Sep 15	2	2
HW-Ch 5: 3, 7, 8, 11	Sep 21	2	2
HW-Ch5: 17, 18, 19	Sep 22	2	2
HW-Ch5: 22, 23, 31, 33	Sep 24	2	2
HW-Ch 5: 41, 36, 44	Oct 05	2	2
Pop Quiz-Dragging a Student	Oct 07	5	5
HW-Ch 5: 45, 58, 65	Oct 07	2	2
HW-Ch 6: 1, 7, 6	Oct 10	2	2
HW-Ch 6: 17, 13, 15	Oct 13	2	2
EXTRA CREDIT: 100%			
ASSIGNMENT	DUE	SCORE	POSSIBLE
Student Info Form	Sep 01	5	Extra Credit

Work-Kinetic Energy Theorem

If **F** varies in a problem, we can't use our constant-a kinematics to analyze the motion…

... but we *can* use Work.

How is Work related to the acceleration of an object that has a Force being applied to it?

Derivation

$$W_{ext} = \int_{x_i}^{x_f} F_x \, dx$$

$$W_{ext} = \int_{x_i}^{x_f} m a_x \, dx$$

$$W_{ext} = m \int_{x_i}^{x_f} a_x \, dx$$

$$W_{ext} = m \int_{x_i}^{x_f} \frac{dv}{dt} dx$$

$$W_{ext} = m \int_{x_i}^{x_f} \frac{dv}{dx} \frac{dx}{dt} dx$$

$$W_{ext} = m \int_{v_i}^{v_f} v \, dv$$

$$W_{ext} = m \frac{1}{2} v^2 \Big]_{v_i}^{v_f}$$

$$\boxed{W_{ext} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2}$$

Kinetic energy

$$W_{ext} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$K \equiv \frac{1}{2}mv^2$$

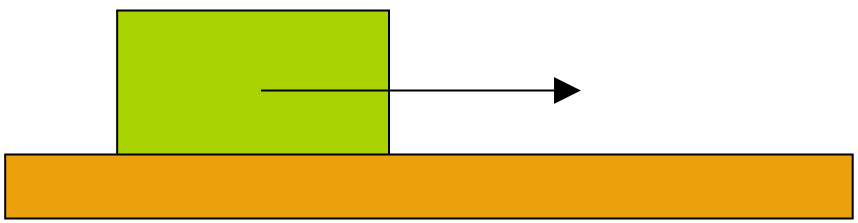
$$\sum W_{ext} = K_f - K_i = \Delta K$$

Work done *on* an object causes its *K* to *increase*.

Negative work done on an object causes its *K* to *decrease*.

Example 10

A 6.0-kg mass is pulled with a constant horizontal Force of 12.0-N for a distance of 3.0-m on a frictionless surface.



a) Find the final speed of the block using kinematics.

$$F_{net} = ma; \; a = \frac{F_{net}}{m} = \frac{12N}{6} = 2m/s^2$$

$$v_f^{\,2} = v_i^{\,2} + 2a\Delta x; \; v_f = \sqrt{2 \bullet 2 \bullet 3} = 3.46m/s$$

b) Find the final speed of the block using work-energy.

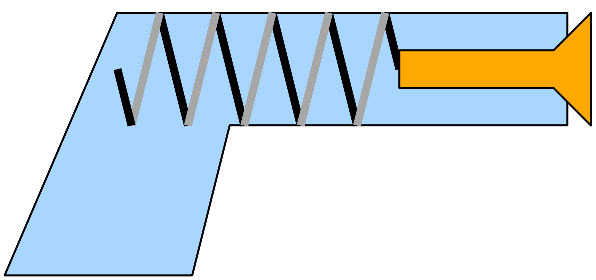
$$W = Fx = (12N)(3m) = 36J$$

$$W = K_f - K_i = \tfrac{1}{2}mv_f^{\,2} - 0$$

$$v_f = \sqrt{\frac{2 \bullet W}{m}} = \sqrt{\frac{2 \bullet 36}{6}} = 3.46m/s$$

Example 11

A dart is loaded into a spring-driven Nerf gun by pushing the spring in for a distance *d*. For the next loading, the spring is compressed a distance 2*d*.



How much faster does the second dart leave the gun, compared to the first?

$$W = \Delta K$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2}$$

$$v_f = \sqrt{\frac{kx^2}{m}} = \sqrt{\frac{k}{m}}x$$

Compressing the spring twice the distance produces a dart with twice the velocity.

Potential Energy

= "*stored*" *energy*, later converted to other forms.

Gravitational Potential Energy (U_g) = energy stored in a gravity field

Elastic Potential Energy (U_s) = energy stored in a spring system

Rethink

With this new idea of "stored" potential energy, we need to adjust how we think about problems that involve energy.

What happens when you lift a box?

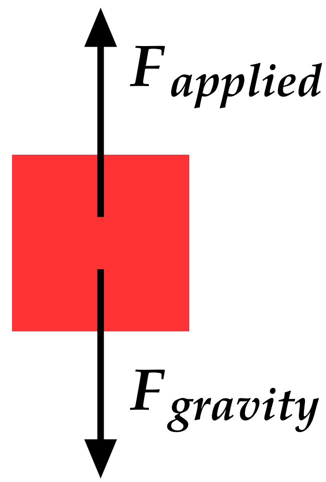
Old Analysis

You do positive work on the box, and gravity does negative work.

The box in the air has W_{net}=0.

Nothing will happen to box until we let it go. Then, we'll look at additional Work gravity does on box to pull it down.

$$W_{gravity} = \vec{\mathbf{F}} \bullet \vec{\mathbf{x}} = mgh$$



New Analysis

You do work on the box and move it through a gravity field, which is considered part of the system. The Work done on the box "gives it" a gravitational potential energy *U*, based on its position in the field.

$$U_g \equiv mgh$$

If you drop the box, where does its *K* come from?

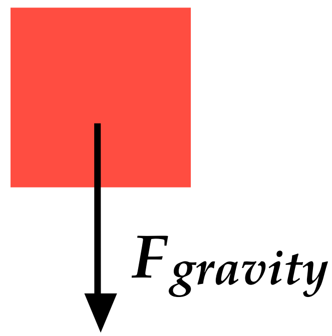
Old Analysis

Work done by gravity

$$W_{gravity} = K_f - K_i$$

$$F_g x = \frac{1}{2} m v_f^{\ 2} - 0$$

$$mgh = \frac{1}{2} m v_f^{\ 2}$$



New Analysis

U_g from when it was in the air.

$$U_g = K$$

$$mgh = \frac{1}{2} m v^2$$

Calculating *U*

For a spring:

$$W_{spring} = \int_{x_i}^{x_f} F \bullet dx$$

$$W_{spring} = \int_{x_i}^{x_f} -kx \bullet dx$$

$$W_{spring} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$W_{spring} = U_i - U_f \quad \text{where} \quad \boxed{U_s \equiv \frac{1}{2}kx^2}$$

Calculating *U*

For gravity:

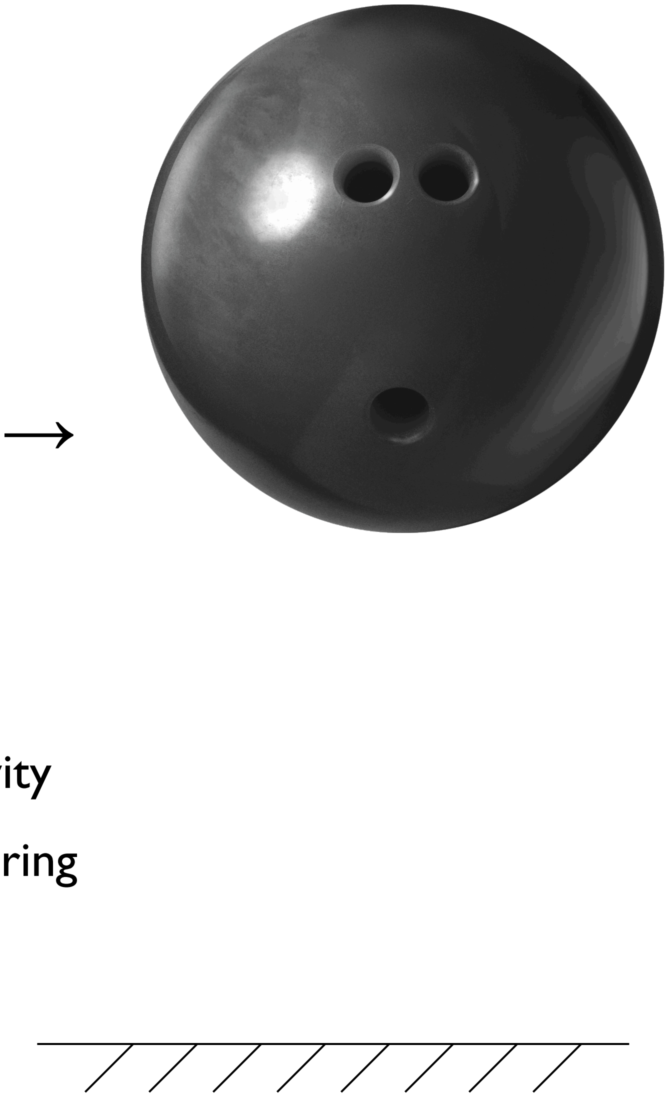
$$W_g = \int_{x_i}^{x_f} F_g \bullet \, dx$$

$$W_g = \int_{y_i}^{y_f} -mg \bullet \, dy \ \ (\text{where } y_i > y_f)$$

$$W_g = mgy_i - mgy_f$$

$$\boxed{U_g \equiv mgh}$$

3 Notes re: *U*



- Where does h=0?
- "The potential energy of the ball" → "... the ball *in the field*".
- In the new analysis,
  "Work done by gravity" → $-\Delta U_{gravity}$
  "Work done by a spring" → $-\Delta U_{spring}$

Example 17

A hiker carries a 15.0 kg backpack up a 20m long slope, inclined at 30° above the horizontal.

a) How much Work was done on the backpack by the hiker?

b) How much Work was done on the backpack by gravity?

c) What was the net Work done on the backpack?

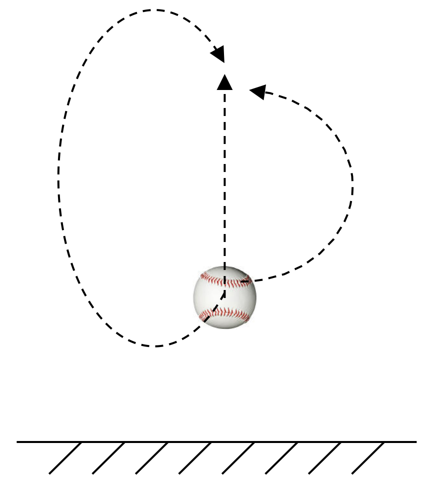
PhET demo

Simulations of the relationship between potential energy U and kinetic energy K.

- Energy Skate Park
- Masses & Spring

Conservative Forces

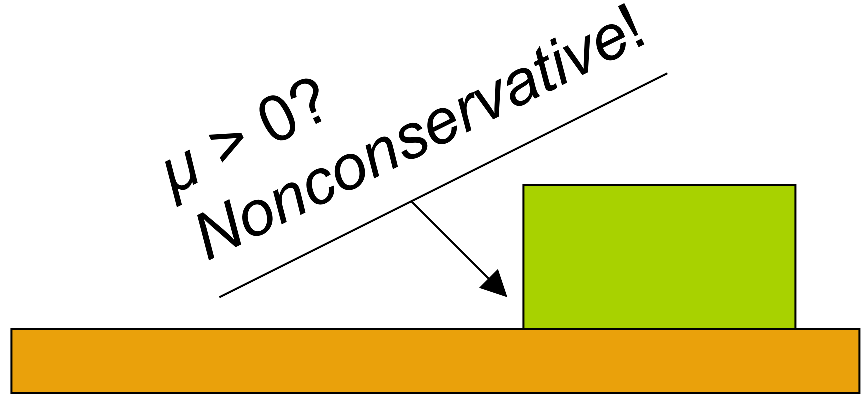
A force is *conservative* if work it does on a particle moving between any 2 points is independent of the path taken by the particle. Also, work done by a conservative force exerted on a particle moving through any *closed path* is 0.



Nonconservative Forces

A force is "nonconservative" if it causes a change in mechanical energy, ie. you have to do net work on an object to return it to its same location and state of motion.

Potential Energy functions can only be defined for Conservative Forces

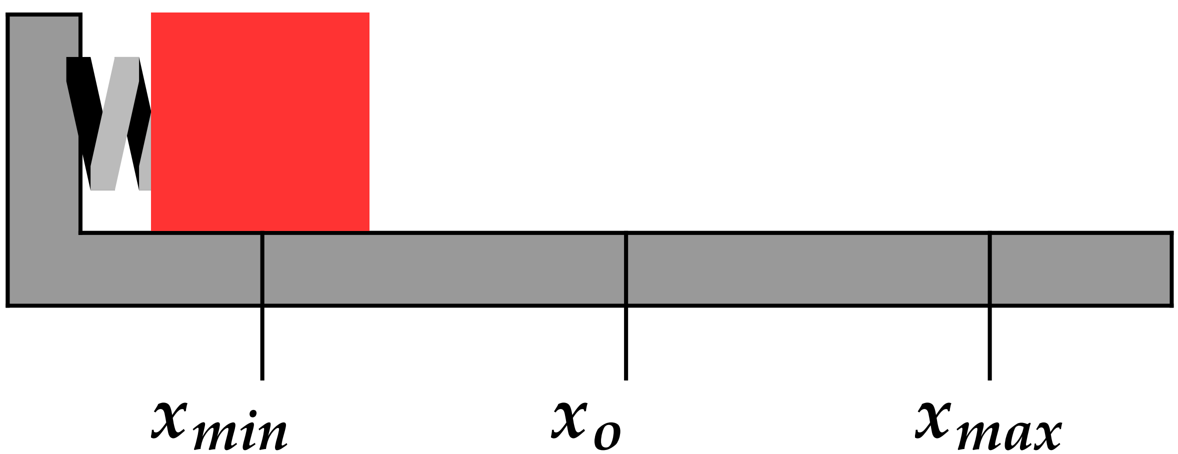


Conservative Forces

So how *do* we define Potential Energy for conservative forces?

$$W_{conservative\ force(\text{int})} = \int_{x_i}^{x_f} F_x\,dx = -\Delta U = U_i - U_f$$

$$\Delta U = U_f - U_i = -W_c = -\int_{x_i}^{x_f} F_x dx$$



Mechanical Energy

You will occasionally be asked to consider the "mechanical energy" of a system.

$$E_{mechanical} = K + U$$

In some systems, the total mechanical energy will remain constant, as long as energy is not "lost" (converted) via non-conservative forces.

Conservative Forces & *U*

$$Work_{donebyaconservativeforce} = \int \vec{F}_x \bullet d\vec{x} = -\Delta U$$

$$F_x \bullet dx = -dU$$

$$F_x = \frac{-dU}{dx}$$

Example 6

Show that $F_x = \dfrac{-dU}{dx}$

using elastic and gravitational potential energy expressions.

$$F_x = \frac{-dU}{dx} = \frac{-d\left(\frac{1}{2}kx^2\right)}{dx} = -kx$$

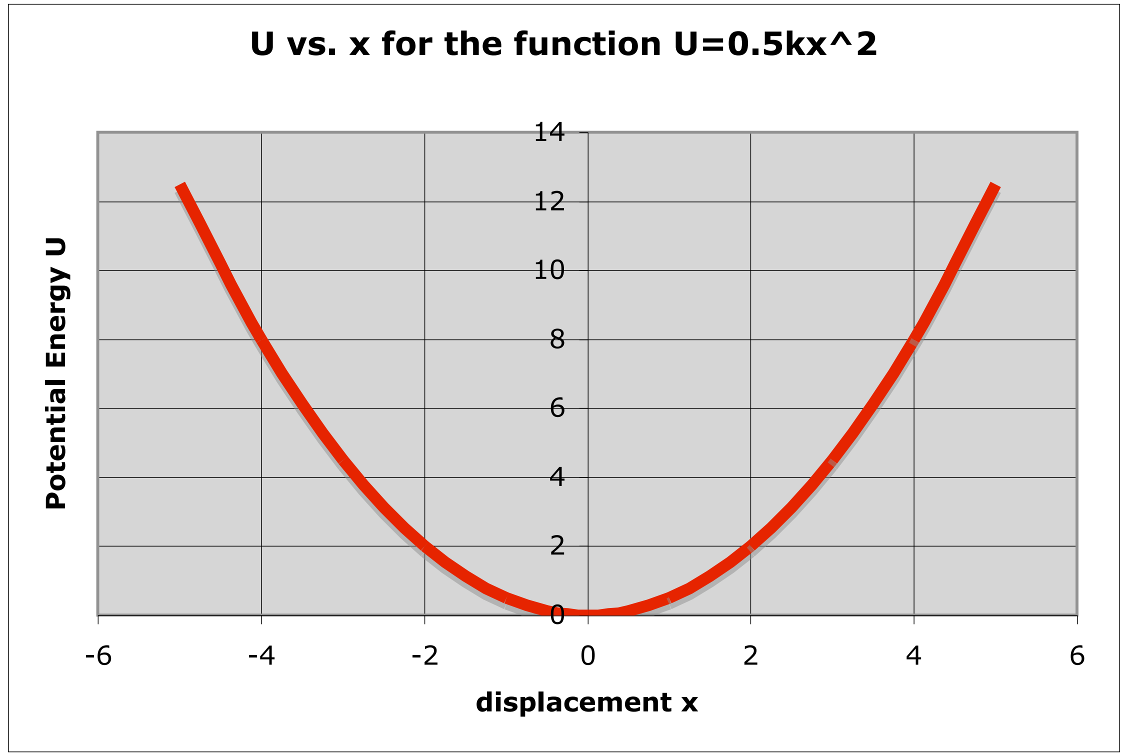
$$F_y = \frac{-dU}{dy} = \frac{-d(mgy)}{dy} = -mg$$

Energy Diagrams

Potential Energy Curve

a) What does the slope of this curve represent?

A conservative Force



b) What's happening at the bottom of the curve?

A point of "stable equilibrium"

c) What happens if the mass is given an initial displacement *x*?

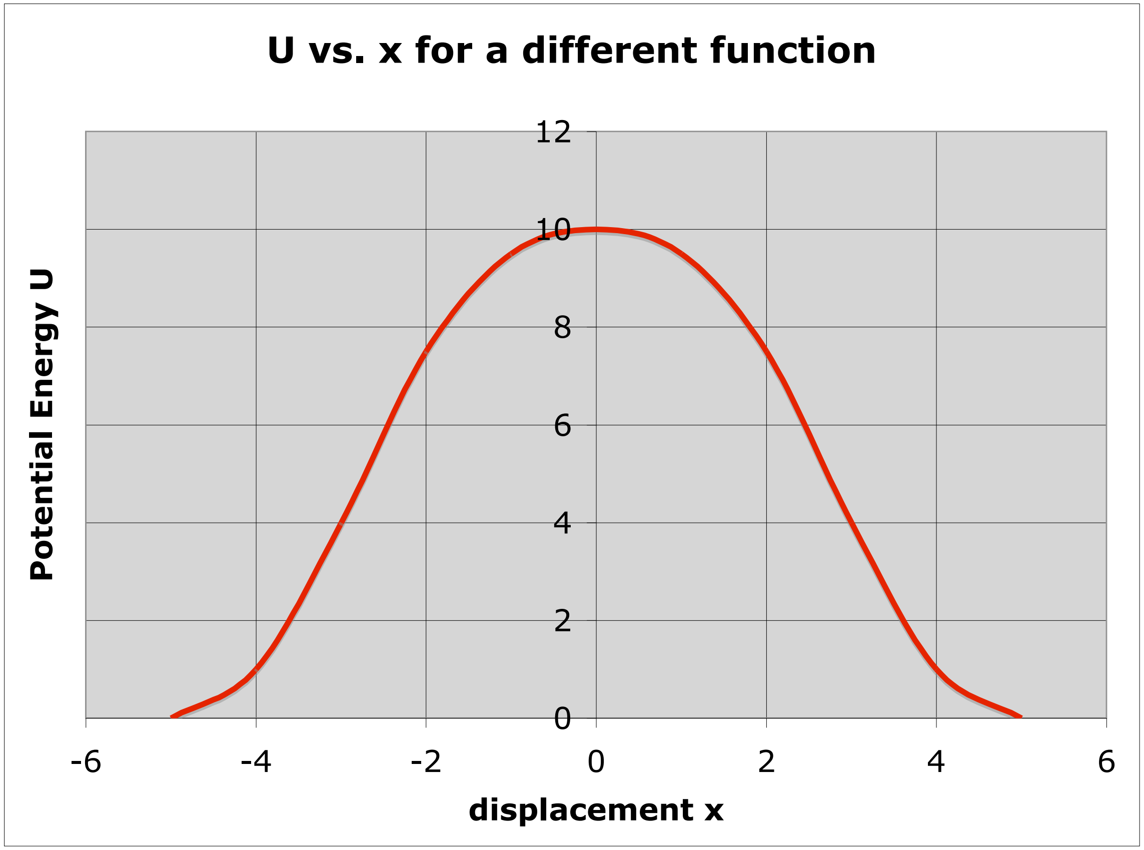
A displacement in the positive *x* direction results in a conservative "restoring force" in the –*x* direction. If the mass is released, it will oscillate back and forth, with U+K = k. Visualize this oscillating system as a "marble in a bowl."

Energy Diagrams

Potential Energy Curve

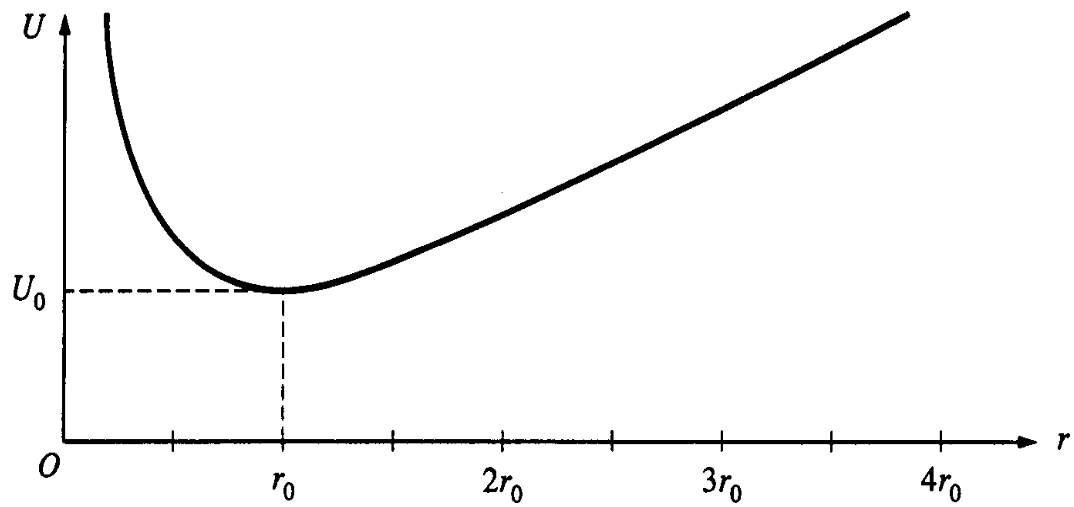
How is this diagram different?

A point of “unstable equilibrium”



AP Problem

1995 M2. A particle of mass m moves in a conservative force field described by the potential energy function
$U(r) = a(r/b + b/r)$,
where a and b are positive constants and r is the distance from the origin. The graph of $U(r)$ has the following shape.



a. In terms of the constants a and b, determine the following.

i. The position r_o at which the potential energy is a minimum

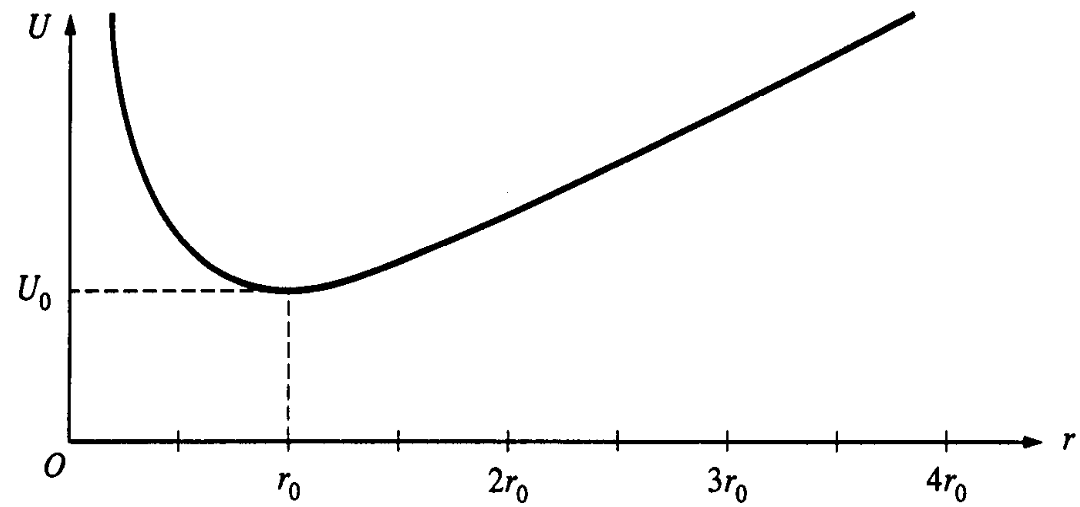
ii. The minimum potential energy U_o

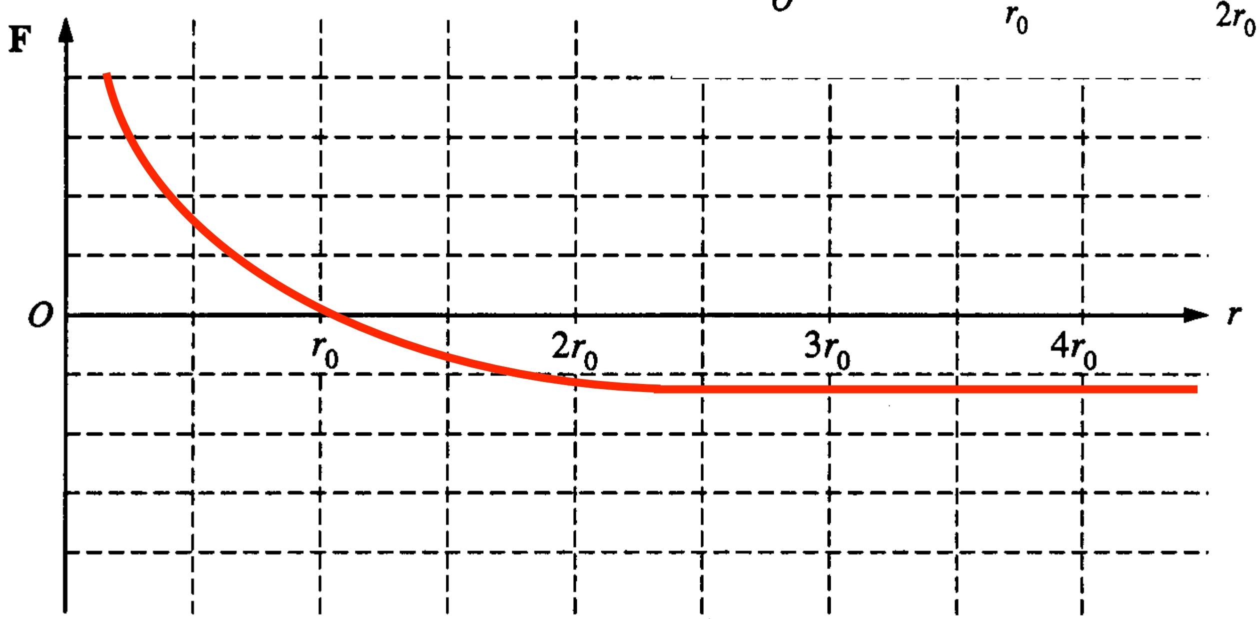
i. r_0=b

ii. U_0=2a

AP Problem

b. Sketch the net force on the particle as a function of *r* on the graph below, considering a force directed away from the origin to be positive, and a force directed toward the origin to be negative.





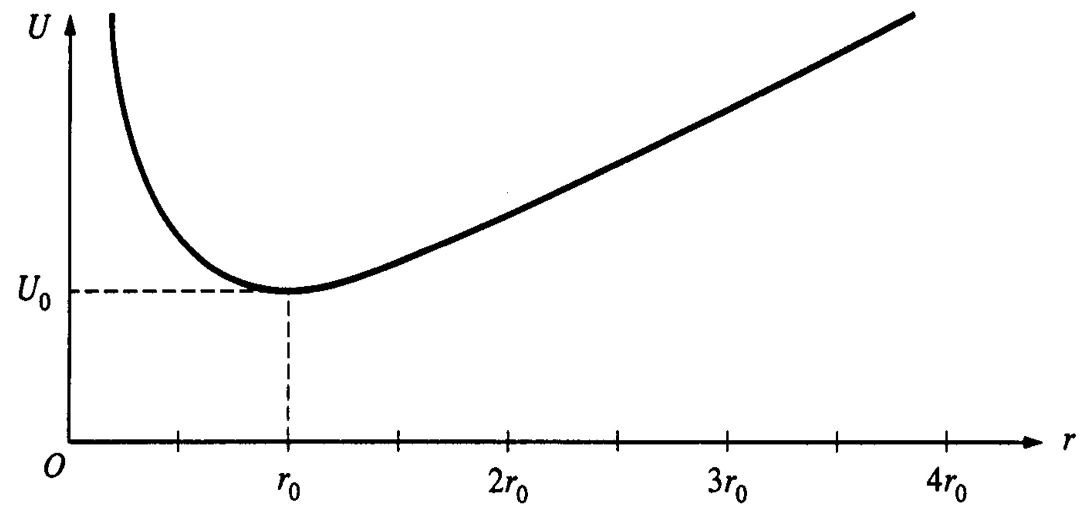
AP Problem

The particle is released from rest at $r = r_o/2$.

c. In terms of U_o and m, determine the speed of the particle when it is at $r = r_o$.

d. Write the equation or equations that could be used to determine where, if ever, the particle will again come to rest. It is not necessary to solve for this position.

e. Briefly and qualitatively describe the motion of the particle over a long period of time.



c. $v=\sqrt{(U_0/2m)}$

d. $U(r_1)=U(r_0/2)$

e. Particle will oscillate, energy is not lost