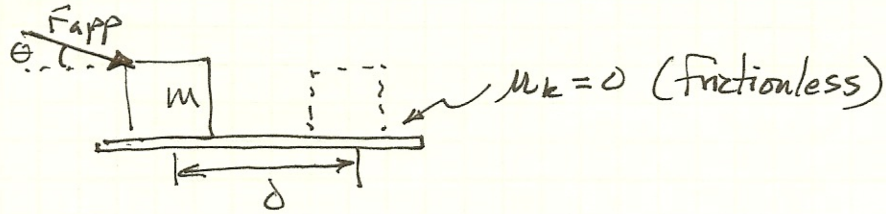
7.1



a) Work done by applied force

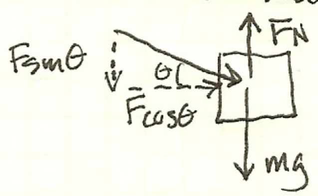
$$W = Fx\cos\theta = \vec{F}\cdot\vec{x}$$

$$= (16\,N\cos 25.0°)(2.20\,m) = \boxed{31.9\,J}$$

b) Work done by Normal force

There is no Work done by the normal force because there is no displacement in the direction of the normal force.

Formally: $W_{Normal} = F_N x\cos\theta$



$$\Sigma F_y = ma_y$$

$$F_N - F_{app}\sin\theta - F_g = 0$$

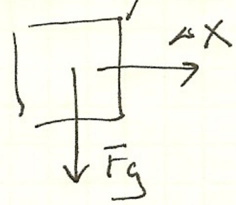
$$F_N = F_g + F_{app}\sin\theta = (2.5\,kg)(9.8) + 16\sin 25$$

$$F_N = 31.3\,N$$

$$W = F_N\,x\cos\theta$$

$$= (31.3\,N)(2.2\,m)\cos 90° = \boxed{0\,J}$$

c) For similar reasons, there is no Work done by the gravitational force.



$$W_g = F_g\,x\cos\theta$$

$$mg\,x\cos(90)$$

$$= \boxed{0\,J}$$

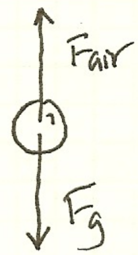
d) F_{net} on block is just $F\cos\theta = 16\cos 25$

$$= \boxed{14.5\,N}$$

There is no net Force in the y direction, because the Forces along that axis cancel out.

$\Sigma F_y = ma_y = 0$

7.2



raindrop, $m = 3.35e{-5}$ kg

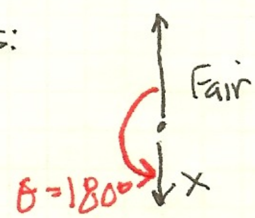
Falling at constant speed for a distance of 100 m.

a) Work done by gravity as it falls:

$$W_g = \vec{F_g} \cdot \vec{x} = F_g\, x \cos\theta$$
$$= mg\, x \cos\theta$$
$$= (3.35e{-5})(9.8)(100)(\cos 0^\circ)$$
$$= \boxed{0.0328\,\text{J}}$$

0° because angle between Fg & x is 0°.

b) Work done by air resistance as it falls:



$$\Sigma F_y = m a_y = 0 \quad \text{(constant speed)}$$

so

$$F_{air} = F_g = mg$$

$$W_{air} = F_{air}\, x \cos\theta$$
$$= (3.35e{-5})(9.8)(100)(\cos 180^\circ)$$
$$= \boxed{-0.0328\,\text{J}}$$

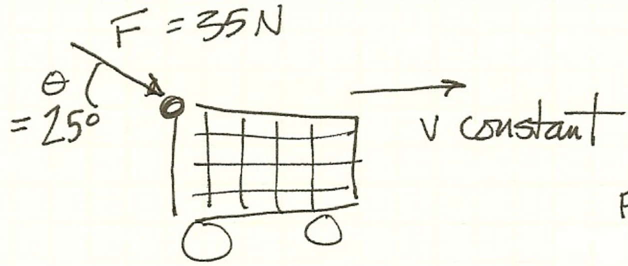
Note that we can simplify our analysis of each solution by just using + & − values here, rather than $\cos\theta$.

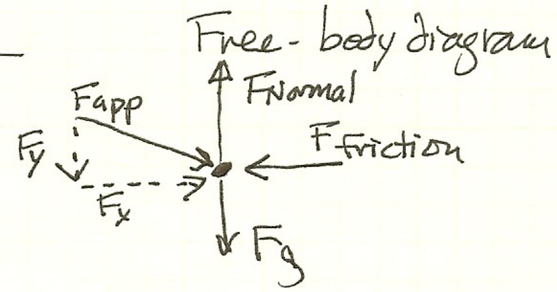
a) $W_g = \vec{F_g} \cdot \vec{x} = (mg)(x) = 0.0328\,\text{J}$

b) $W_{air} = \vec{F_{air}} \cdot \vec{x} = (-mg)(x) = -0.0328\,\text{J}$

Negative because Fair is in the opposite direction of x.

7.5





a) Work done by shopper along 50.0 m aisle.

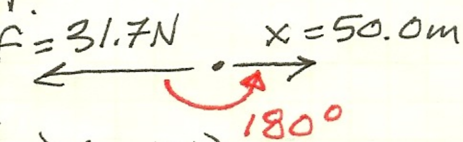
$W_{app} = \vec{F}_{app} \cdot \vec{x}$
$= Fx\cos\theta$
$= (35N)(50.0m)(\cos 25)$
$= \boxed{1590\ J}$

$\Sigma F_x = ma_x = 0$

$F_{app-x} + {}^{-}F_{friction} = 0$

$35\cos 25 = F_F = \underline{31.7N}$

b) Net work done on cart by all forces is $\boxed{0J}$, because $F_{friction}$ ($= F_{app-x}$) does negative 1590 J of Work on the cart.



$W_{friction} = Fx\cos\theta$
$(31.7N)(50m)(\cos 180)$
$= \boxed{-1590J}$

c) If the shopper pushes horizontally against same friction force, s/he has to apply less overall force, because there is no vertical component of force to be applied.
(31.7N < 35.0 N from first situation.)

d) Net work done on the cart will still be 0J.

7.7 Show that $\vec{A} \cdot \vec{B} = \underbrace{A_x B_x + A_y B_y + A_z B_z}_{\text{3 dimensions!}}$

$$\vec{A} = \vec{A}_x + \cancel{B}\vec{A}_y + \vec{A}_z$$

$$= A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x i + B_y \hat{j} + B_z \hat{k})$$

$$= A_x \hat{i} \cdot B_x i + A_x i \cdot B_y \hat{j} + A_x i \cdot B_z \hat{k}$$

$$+ A_y \hat{j} \cdot B_x \hat{i} + A_y \hat{j} \cdot B_y \hat{j} + A_y \hat{j} \cdot B_z \hat{k}$$

$$+ A_z \hat{k} \cdot B_x \hat{i} + A_z \hat{k} \cdot B_y \hat{j} + A_z \hat{k} \cdot B_z \hat{k}$$

Circled dot products – $i \cdot i$, $j \cdot j$, & $k \cdot k$ – have a value of 1, because unit vectors are parallel.

Other dot products – $i \cdot j$, $j \cdot k$, etc. are unit vectors that are perpendicular, & thus have a value of 0.

Thus, $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$ ✓

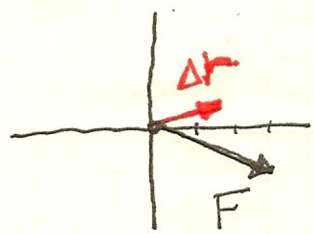
7.9 $\vec{F} = (6i - 2j)\,N$, while $\vec{\Delta r} = (3i + j)\,m$.

a) Work done by the force

$$W = \vec{F} \cdot \vec{\Delta r}$$
$$= (6i - 2j) \cdot (3i + j)$$
$$= (18 - 2)\,N\,m = \boxed{16\,J}$$

b) The angle between F & Δr.

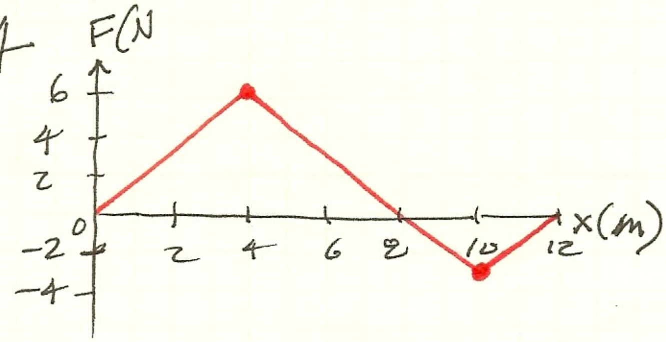
$$W = \vec{F} \cdot \vec{\Delta r} = F r \cos\theta$$



$$F = \sqrt{F_x^2 + F_y^2}$$
$$= \sqrt{6^2 + (-2)^2}$$
$$= \underline{6.33\,N}$$
$$\Delta r = \sqrt{3^2 + 1^2}$$
$$= \underline{3.16\,m}$$

$$16\,J = (6.33)(3.16)\cos\theta$$
$$16 = 20\cos\theta$$
$$\theta = \cos^{-1}\left(\frac{16}{20}\right) = \boxed{36.9^\circ}$$

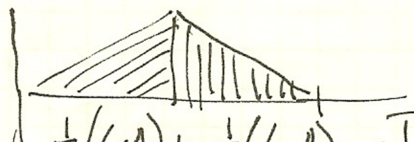
7.14



Force varies as a function of displacement as shown.

$W = \int F \cdot dx$ = area under F·x curve.

a) For ~~x~~ = 0-8 m, area under that curve is:



$\frac{1}{2}(6\cdot 4) + \frac{1}{2}(6\cdot 4) = \boxed{24\ J}$

b) For ~~x~~ = 8-10 m Force is negative while dx is positive, so Work done will be negative ("area over the curve").

$\frac{1}{2}(2\cdot 3) + \cancel{\frac{1}{2}(2\cdot 3)} = \boxed{\cancel{-6\ J}}$

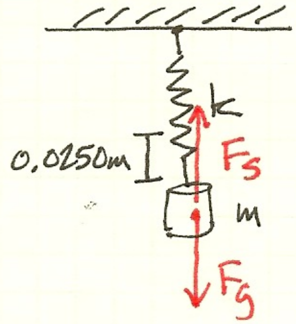
$-3\ J$

c) Net Work done from x = 0-10 m

$= 24\ J - 3\ J = \boxed{\cancel{18\ J}}$

$21\ J$

7.17



when first mass is hung,
$\Delta x = 0.0250\,m$.

$F_{spring} = -kx$

At equilibrium, $F_s = -F_g$, so

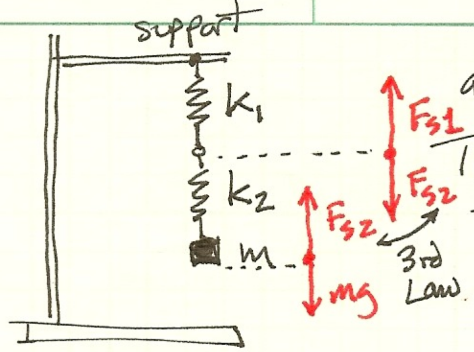
$$-F_g = -kx$$
$$mg = kx$$
$$k = \frac{mg}{x} = \frac{(4kg)(9.8\,m/s^2)}{(0.0250m)}$$
$$= \underline{1570\,N/m}$$

a) If 1.50kg mass is now hung on the spring:

$$\Delta x = \frac{mg}{k} = \frac{(1.5)(9.8)}{1570\,N/m} = \boxed{0.00938\,m}$$

b)
$$W = \int F_{app} \cdot dx$$
$$= \int_{x=0}^{0.04m} kx \cdot dx$$
$$= \frac{1}{2}kx^2 \Big|_0^{0.04}$$
$$= \frac{1}{2}k(0.04)^2 - \frac{1}{2}k(0)$$
$$= \frac{1}{2}(1570)(0.04)^2 = \boxed{1.26\,J}$$

7.21



a) Find total extension.

The weight of mass m applies a force mg to the second spring, causing it to extend a distance x.

$$F_s = -kx$$
$$F_g = -F_s, \text{ so}$$
$$F_g = kx = mg$$
$$x_2 = \frac{mg}{k_2}$$

The spring k_2 itself is "light", & has no weight of its own so now the F of the spring pulling up on the mass ($F_2 = mg$) is transmitted to k_1 as well:

$$mg = k_1 x, \text{ or } x_1 = \frac{mg}{k_1}$$

Total displacement $x = x_1 + x_2$

$$= \frac{mg}{k_1} + \frac{mg}{k_2}$$
$$= \boxed{mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$$

b) $F_{\text{spring system}} = -k_{\text{eff}} x_{\text{total}}$

what drives system, = to all F_{springs} → $\frac{mg}{k_{\text{effective}}} = x_{\text{total}} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$

$$\frac{1}{k_{\text{effective}}} = \frac{1}{k_1} + \frac{1}{k_2}$$

or

$$k_{\text{effective}} = \boxed{\frac{k_1 k_2}{k_1 + k_2}}$$

7.31 $m = 0.600\,kg \quad v_A = 2.00\,m/s$

$K_B = 7.50\,J$

a) K energy at A:

$$K_A = \frac{1}{2} m v_A^2$$
$$= \frac{1}{2}(0.600\,kg)(2.00\,m/s)^2$$
$$= 1.20\,kg\,m^2/s^2 = \boxed{1.20\,J}$$

b) speed at B:

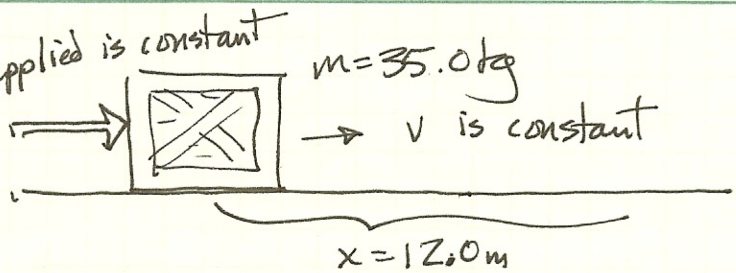
$$K_B = \frac{1}{2} m v_B^2$$
$$v_B = \sqrt{2K_B/m}$$
$$= \sqrt{\frac{(2)(7.50\,J)}{0.6}}$$
$$= \boxed{5.00\,m/s}$$

c) Net work done on particle from A to B?

$W = \Delta K$, so $W = K_f - K_i$

$$W = 7.5\,J - 1.2\,J$$
$$= \boxed{6.30\,J}$$ of Work done <u>on</u> particle.

7.32 $F_{applied}$ is constant



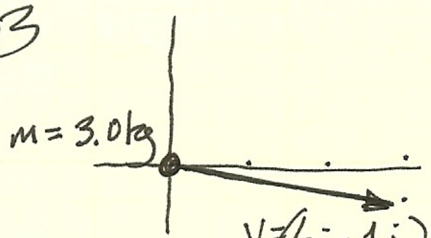
a) $W_{done} = F_{app}\, x$

$$F_{app} = \frac{W_{done}}{x} = \frac{350\,J\ (\text{given})}{12\,m} = \boxed{29.2\,N}$$

b) If worker applies a greater Force, net Work will be done on the box (because $W_{worker} > W_{friction}$), so the box will have a greater velocity (more K energy).

c) If worker applies a force < F, the Work done by the friction force will be greater than W_{worker}. Net work on the box will be negative ($\Delta W = K_f - K_i$), & the box will lose K (ie. slow down).

7.33



$V = (6i - 1j)\ m/s$

$= \sqrt{6^2 + (-1)^2}$

$= 6.08\ m/s$

a) $K = \frac{1}{2} m v^2$

$= \frac{1}{2}(3)(6.08\ m/s)^2$

$= \boxed{55.5\ J}$

b) If $K_f = \frac{1}{2} m v^2$, & $v = 8i + 4j$

$K_f = \frac{1}{2}(3)(64 + 16)$ ← $v^2 = \vec{v} \cdot \vec{v}$!

$K_f = 120 J$

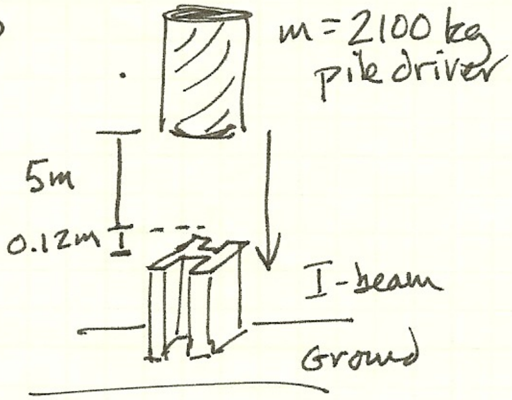
Work done $= \Delta K$

$= K_f - K_i$

$= 120 - 55.5$

$= \boxed{64.5\ J}$

7.35



Find average Force exerted on pile driver when it hits I-beam using energy analysis.

The total Work W_{net} done on the pile driver will cause its K energy to change, from 0 at top of fall, to... 0 at bottom of fall (it comes to rest after hitting I-beam).

$$W_{net} = \Delta K = K_f - K_i = 0 - 0 = 0$$

What does Work on pile driver? What applies Forces to pile driver?

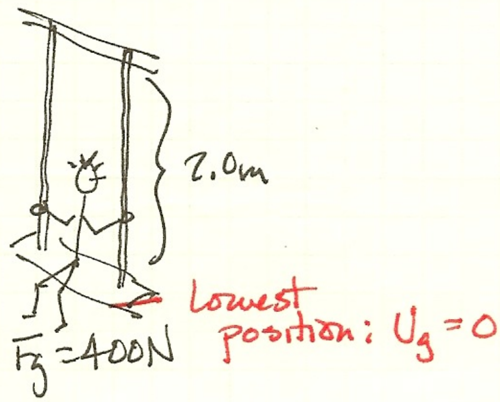
$$W_{gravity} + W_{I\text{-beam}} = 0$$

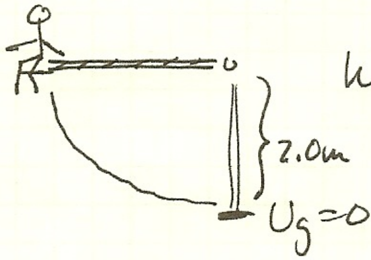
$$F_g \Delta y_g + F_{beam} \Delta y_b = 0$$

$$mg(5 + 0.12) + -F_{beam}(0.12) = 0$$

$$\frac{(2100)(9.8)(5.12)}{0.12} = F_{beam} = \boxed{8.78e5\ N}$$

7.42

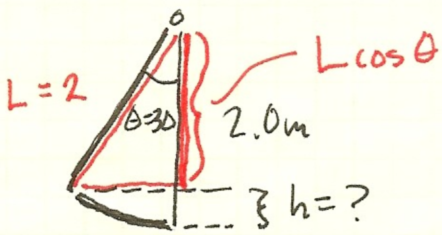


a) 

When ropes are horizontal,

h relative to bottom = 2.0m

$$U_g = mgh$$
$$= (400N)(2m)$$
$$= \boxed{800\,J}$$

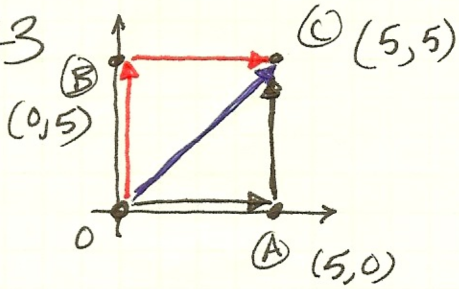
b) 

$$h = L - L\cos\theta$$
$$= 2 - 2\cos 30$$
$$= \underline{0.268\,m}$$

$$U_g = mgh$$
$$= (400)(0.268)$$
$$= \boxed{107\,J}$$

c) At bottom, h = 0 so $U_g = \boxed{0\,J}$

7.43



$m = 4.00\,kg$, &

$F_g = mg = 39.2\,N$ in $-y$ direction

Find Work done by gravity in moving partice along each path.

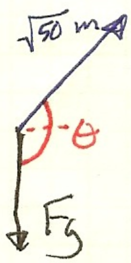
a) Along $\rightarrow\uparrow$ path.

$$W_{total} = W_{0-A} + W_{A-C}$$
$$= F_g x \cos\theta + F_g y \cos\theta$$
$$= (39.2)(5)(\cos 90) + (39.2)(5)(\cos 180)$$
$$= \boxed{-196N}$$

b) Along $\uparrow\rightarrow$ path.

$$W_{total} = W_{OB} + W_{BC}$$
$$= F_g y \cos\theta + F_g x \cos\theta$$
$$= (39.2)(5)(\cos 180) + (39.2)(5)(\cos 90)$$
$$= \boxed{-196N}$$

c) $$\text{Work total} = F_g x \cos\theta$$
$$= (39.2)(\sqrt{50})(\cos 135)$$
$$= \boxed{-196N}$$

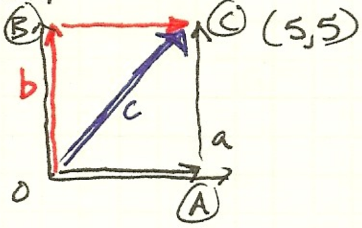


Note that the net Work done by gravity in each case is the same.

This is because gravity is a "conservative force," in which the Work done is independent of the path taken in going from one point to another.

Potential energies may only be defined for these path-independent conservative forces. In this case the force of gravity has an associated "gravitational potential energy" U_g.

7.45 Force acting on particle is $F = (2y\hat{i} + x^2\hat{j})$ N as particle moves through these three paths:



Find Work done by F as particle moves through these paths.

a) $W_{total} = W_{OA} + W_{AC}$

$= \int_0^5 F \cdot dx + \int_0^5 F \cdot dy$

$= \int_0^5 2y\hat{i} + x^2\hat{j} \cdot dx + \int_0^5 2y\hat{i} + x^2\hat{j}\, dy$

← 0 here, & $\hat{j}$ & x are ⊥ so 0 J.

← $\hat{i}$ & y are ⊥ so 0 J, but...

$\int_0^5 5^2 j \cdot dy$

$= 5^2 y\Big|_0^5 = \boxed{125\text{ J}}$

b) $W_{total} = W_{OB} + W_{BC}$

$= \int_0^5 2y\hat{i} + x^2\hat{j} \cdot dy + \int_0^5 2y\hat{i} + x^2\hat{j} \cdot dx$

← 0 here, & $\hat{i}$ & y are ⊥, so 0 J

← $\hat{j}$ & dx are ⊥ so 0 J, but...

$\int_0^5 2(5) \cdot dx = 10x\Big|_0^5 = \boxed{50\text{ J}}$

c) $W_{total} = W_{OC}$ ← We don't typically do these types of problems: x & y are both changing, & that requires multivariable calculus! But here, because $y = x$ along the path,

$x = y$

$F = 2x\hat{i} + x^2\hat{j}$

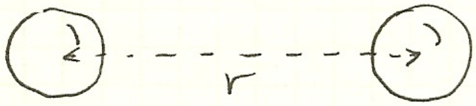
$W = \int F \cdot dx = \int_0^5 2x\hat{i} + x^2 i \cdot dx$

$= x^2 + \frac{x^3}{3}\Big|_0^5 = 5^2 + \frac{125}{3} = \boxed{66.7\text{ J}}$

d) This must be a non conservative force...

e) ... because different paths yield different amounts of Work required to move the particle.

7.47



2 particles

$U_r = \frac{A}{r}$; Find F_r.

$\Delta U = -\Delta W = -\int F \cdot dr$

This integral is related to the derivative

$F_r = -\frac{dU}{dr}$

In this case,

$$F_r = -\frac{d}{dr}\left(\frac{A}{r}\right)$$
$$= -A \frac{d}{dr} r^{-1}$$
$$= -A\left(-r^{-2}\right)$$
$$= Ar^{-2} = \boxed{\frac{A}{r^2}}$$

7.49 Particle moving along x-axis has mass $m = 5\,kg$
Conservative $F_x = 2x + 4$. For $x = 1m$ to $5m$,

a) Calculate Work done by force.

$$W = \int F \cdot dx = \int_1^5 2x + 4 \cdot dx = x^2 + 4x \Big|_1^5$$

$$= (5^2 + 20) - (1^2 + 4)$$

$$= \boxed{40J}$$

b) Because Work has been done by the force ($W > 0$), the U of the system has ~~+~~ decreased 40J.

$$\Delta U = -\Delta W$$

$$\boxed{\Delta U = -40J}$$

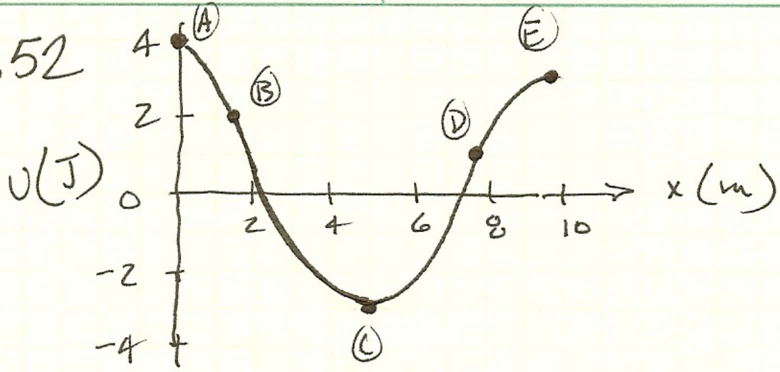
c) $\Delta U + \Delta K = 0$, or $U_i + K_i = U_f + K_f$, or

$$W = \Delta K = K_f - K_i$$

$$40J = \underbrace{\cancel{\tfrac{1}{2} m V_f^2}}_{K_f} - \tfrac{1}{2} m V_i^2$$

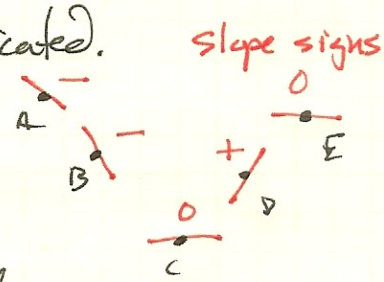
$$K_f = 40J + \tfrac{1}{2}(5kg)(3^2) = \boxed{\overset{62.5}{\cancel{58.0}}\,J}$$

7.52



a) Determine sign of F at each pt indicated.

$F = -\frac{\partial U}{\partial x} \longrightarrow$ opposite of slope



F_A = opposite of negative = positive

F_B = positive

F_C = zero

F_D = opposite of positive = negative

F_E = zero

b) E is unstable equilibrium

C is stable equilibrium

c) F_x vs. x for the same area?