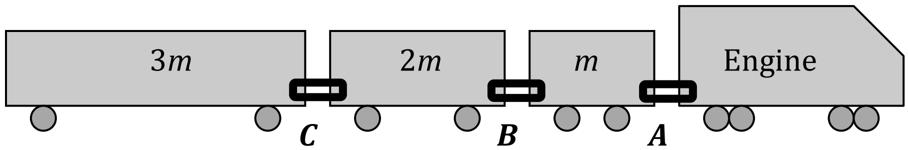
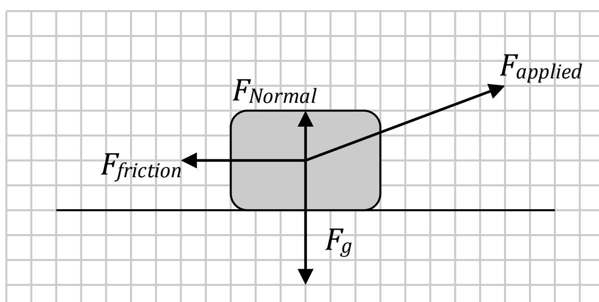
This test covers Newton's Laws of Motion, forces, coefficients of friction, free-body diagrams, and centripetal force.

Part I. Multiple Choice

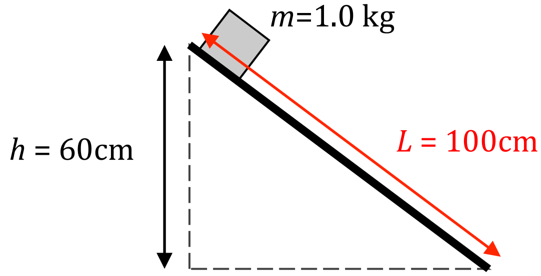


1. A locomotive engine of unknown mass pulls a series of railroad cars of varying mass: the first car has mass *m*, the second car has mass 2*m*, and the last car has mass 3*m*. The cars are connected by links *A*, *B*, and *C*, as shown. Which link experiences the greatest force as the train accelerates to the right?
   a. *A*
   b. *B*
   c. *C*
   d. Which link depends on the mass of the engine.
   e. *A*, *B*, and *C* all experience the same force.



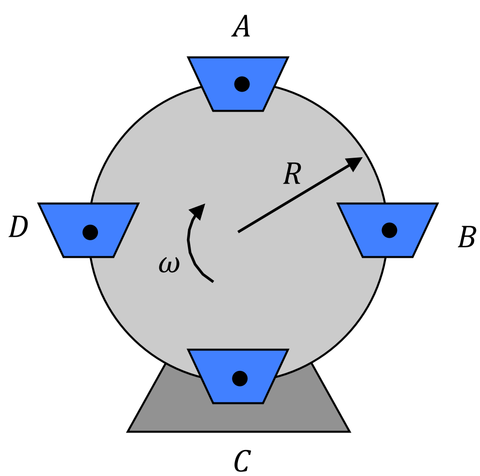
2. The free-body diagram shows all forces acting on a box supported by a horizontal surface, where the length of each force vector is proportional to its magnitude. Which statement below is correct?
   a. The box is accelerating downwards because the force of gravity is greater than the normal force.
   b. The box is accelerating to the right, but not upwards.
   c. The box is accelerating upwards, but not to the right.
   d. The box is accelerating upwards *and* to the right.
   e. None of the statements above is correct.

3. A 0.50-kg object moves along the *x*-axis according to the function $x = 4t^3 + 2t - 1$, where *x* is in meters and *t* is in seconds. What is the magnitude of the net force acting on the object at time *t* = 2.0s?
   a. 50 N
   b. 25 N
   c. 46 N
   d. 48 N
   e. 24 N



4. To determine the coefficient of friction between a block of mass 1.0kg and a 100cm long surface, an experimenter places the block on the surface and begins lifting one end. The block just begins to slip when the end of the surface has been lifted 60cm above the horizontal. The static coefficient of friction between the block and the surface is most nearly

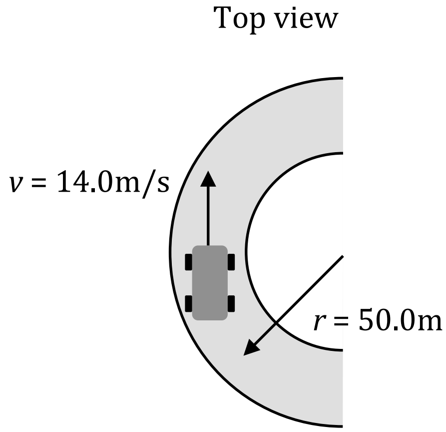
a. 0.60
b. 0.75
c. 0.90
d. 1.05
e. 1.20

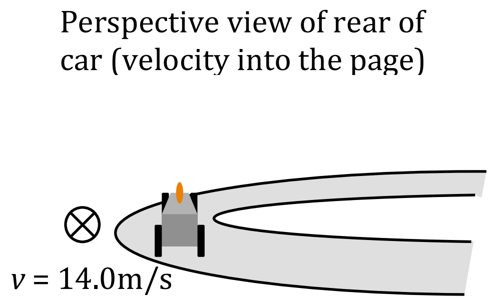


5. A large Ferris wheel at an amusement park has four seats, located 90° from each other and at a distance *R* from the axis. Each seat is attached to the wheel by a strong axle. As the Ferris wheel rotates with a constant angular velocity ω, the seats move past positions *A, B, C,* and *D* as shown.
At which position does a seat's axle apply the greatest force to the seat?

a. A
b. B
c. C
d. D
e. The axles applies the same force to the seat at all four positions.

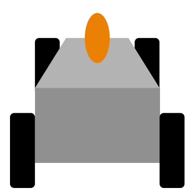
Part II. Free Response





6. A 500-kg race car is traveling at a constant speed of 14.0 m/s as it travels along a flat road that turns with a radius of 50.0m.

a. Draw a free-body diagram for the car as it negotiates the right-turning curve.

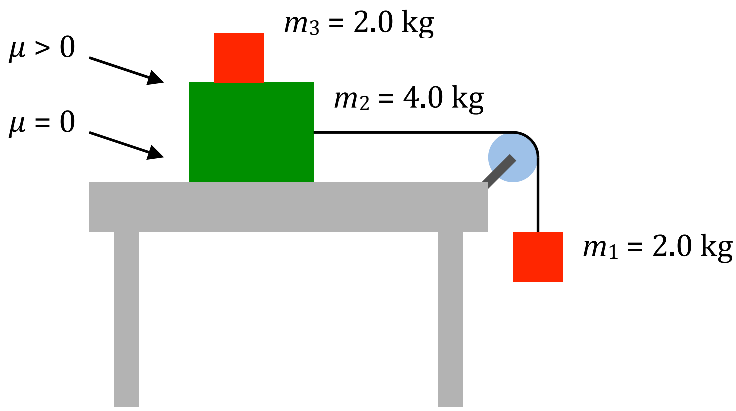


b. What is the magnitude of the centripetal force required for the car to travel through the turn?

c. The coefficient of static friction between the tires and the road is 0.78. Show that the car will be able to make this turn.

d. What is the maximum velocity that the car can have, and still make the turn without slipping off the road?

e. Now engineers want to redesign the curve so that no friction at all is required to stay on the road. How high should they bank the 50.0-meter radius turn so that the car will be able to travel through it at 14.0 m/s with no lateral friction required for the car to make the turn.



7. Blocks m_1 = 2.0 kg and m_2 = 4.0 kg are connected by a thin, light cord which is draped over a light pulley so that mass m_1 is hanging over the edge of the pulley as shown. The surface between m_2 and the table is essentially frictionless, but there is friction between m_2 and m_3, which has a mass of 2.0 kg and is resting on top of m_2.

a. Block m_2 is initially held so that it doesn't move. What is the Tension in the cord attached to m_1?

b. Block m_2 is now released, and it accelerates so that m_3 does not slip, and remains in place atop m_2.

i. What is the acceleration of mass m_2?

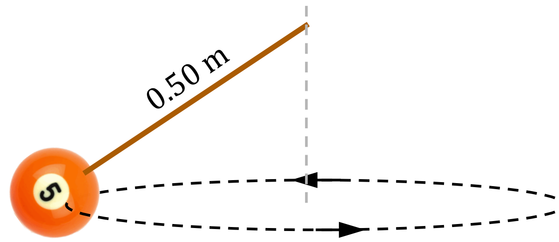
ii. Draw a free-body diagram of mass m_2, with vector arrows originating at the location where the force is applied.

iii. What is the Tension in the cord attached to m_1 now as the system accelerates?

iv. What is the minimum static coefficient of friction that can exist between m_2 and m_3 based on this situation? Explain your reasoning.

v. If the coefficient of static friction between m_2 and m_3 is 0.50, what is the maximum mass that m_1 can have so that m_3 will accelerate without sliding?

8. A billiard ball (mass $m = 0.150$ kg) is attached to a light string that is 0.50 meters long and swung so that it travels in a horizontal, circular path of radius 0.40 m, as shown.

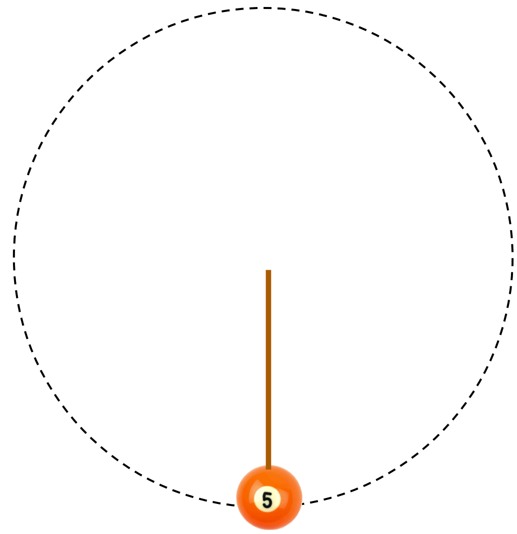


a. On the diagram, draw a free-body diagram of the forces acting on the billiard ball.

b. Calculate the force of tension in the string as the ball swings in a horizontal circle.

c. Determine the magnitude of the centripetal acceleration of the ball as it travels in the horizontal circle.

d. Calculate the period T (time for one revolution) of the ball's motion.

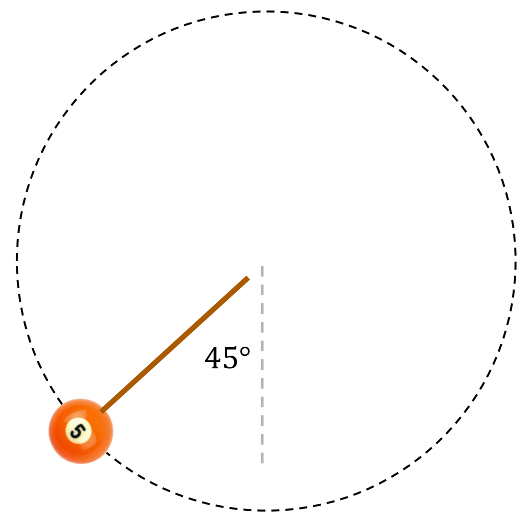


In a different experiment, the same ball with the same length of string is now swung in a vertical circle of radius 0.50 m.

e. If the ball is travelling at 3.00 m/s at the bottom of the vertical, circular path, what is the tension in the string at that moment? Include a free-body diagram as part of your solution.

f. If the ball is travelling at 3.00 m/s at the top of the vertical, circular path, what is the tension in the string at that moment? Include a free-body diagram as part of your solution.

g. If the ball is travelling at 3.00 m/s at the moment when the string makes an angle of 45° from the vertical as shown, calculate the tension in the string.



h. The string can handle a maximum force of 10.0 Newtons before it breaks. What is the maximum speed the ball can have at the bottom of its path before the string breaks?

9. A ping-pong ball has a mass of 2.7 g and a diameter of 40mm so that its cross-sectional area is about $1.26 \times 10^{-3} m^2$. The ball is released from the top of a tall cliff at time $t = 0$, and as it falls through the air, experiences a drag force $R = \frac{1}{2} D\rho A v^2$, where D is the drag coefficient (0.5 for this ping-pong ball), ρ is the density of air (129 kg/m^3), and v is the velocity.

a. Draw a free-body diagram for the ping-pong ball:

i. Just after it has been released

ii. After it has fallen some distance but before it has reached terminal velocity

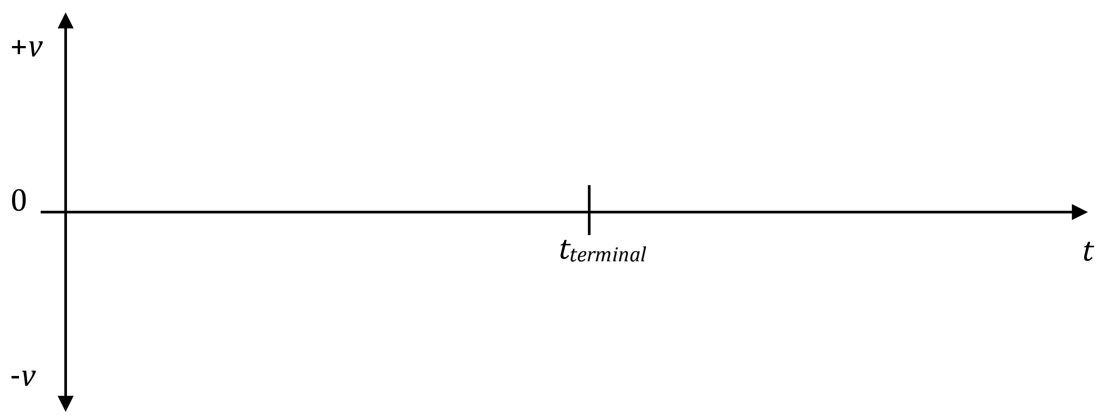
iii. After the ball has reached terminal velocity

b. Use Newton's Second Law to determine the ball's acceleration as a function of velocity.

c. Determine the terminal velocity of this ping-pong ball.

d. Develop, but do not solve, a differential equation that could be used to determine the velocity of the ball as a function of time.

e. Sketch a graph of the ball's velocity as a function of time, including the time at which the ball reaches terminal velocity.



1. The correct answer is *a*. Link *A* is responsible for pulling the entire mass of the train ($m + 2m + 3m = 6m$ total) to the right. Link *B* only needs to pull $5m$, and Link *C* only $3m$.

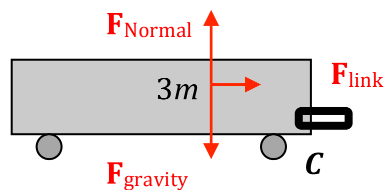
A more quantitative analysis, although not required for finding the answer here, might include determining the net acceleration of the train as a function of the Force of the engine and the total mass of the train:

$$F_{net} = ma$$

$$F_{engine} = (m_1 + m_2 + m_3)a$$

$$a = \frac{F_{engine}}{(m + 2m + 3m)} = \frac{F_{engine}}{6m}$$

A free-body analysis on the 3m car, then, would determine that the force acting on that car was:



$$F_{link} = ma = (3m)\left(\frac{F_{engine}}{6m}\right) = \frac{1}{2}F_{engine}$$

Similar analyses for car 2m and car 1m reveal that link *A* experiences a force of $\mathbf{F}_{\text{engine}}$ that is greater than the forces on the other links.

2. The correct answer is *b*. The box has a net force in the positive-*x* direction, but the forces in the *y*-direction are balanced.

3. The correct answer is *e*. The acceleration of the object is determined by using $a = \frac{d^2x}{dt^2}$, as follows:

$$v = \frac{dx}{dt}$$

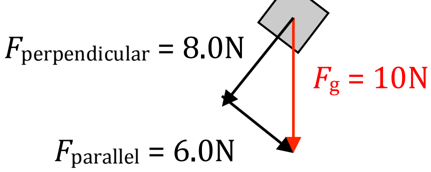
$$v = \frac{d}{dt}(4t^3 + 2t - 1) = 12t^2 + 2$$

$$a = \frac{dv}{dt}$$

$$a = \frac{d}{dt}(12t^2 + 2) = 24t$$

Substitute in $t = 2.0$s to get $\mathbf{a} = 48\text{m/s}^2$. Use $\mathbf{F}_{net} = m\mathbf{a}$ to get $\mathbf{F}_{net} = 24$ N.

4. The correct answer is *b*. The ramp can be thought of as the hypotenuse of a 3-4-5 right triangle, with a corresponding 3-4-5 right triangle as part of the free-body diagram for the block.



The force of friction when the block just begins to slip equal the force $F_{parallel}$, and the normal force F_{Normal} equals the force $F_{perpendicular}$. The coefficient of friction, then, can be calculated:

$$\mu = \frac{F_{friction}}{F_{Normal}}$$

$$\mu = \frac{6.0N}{8.0N} = 0.75$$

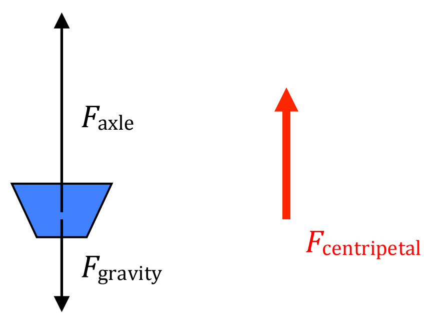
5. The correct answer is *c*. The axles need to support the seats against the force of gravity, and for a non-rotating Ferris wheel, the force would be the same at each position. For a rotating wheel, however, a centripetal force is required to keep the seats moving in a circle. At position *C*, the axle needs not only to support the seat, but also provide additional force to keep it accelerating centripetally (moving in a circle).

Quantitatively:

$$F_{centripetal} = \frac{mv^2}{r}$$

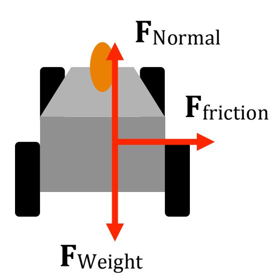
$$+F_{axle} - F_{gravity} = \frac{mv^2}{r}$$

$$F_{axle} = \frac{mv^2}{r} + mg$$



6.

a. The free-body diagram for the car needs to take into account all the forces acting on the car. All vectors should have labels. Note that the force of friction acts centripetally toward the center of the circular motion, and it should *not* be labeled $\mathbf{F}_{\mathbf{centripetal}}$.



b. For the car to be able to make it around the turn, it needs a centripetal force of

$$F_c = \frac{mv^2}{r} = \frac{(500kg)(14m/s)^2}{50m} = 1960N$$

This will obviously be supplied by the force of friction between the road and the tires.

c. If the static (non-slipping) coefficient of friction between the tires and the road is 0.78, we can determine the maximum amount of centripetal force that that friction will supply:

$$F_{friction} = \mu F_{Normal}$$

$$F_{friction} = \mu mg = (0.78)(500kg)(9.8m/s^2) = 3820N$$

Because this force of friction is greater than the centripetal force we need (3820N > 1960N), the car will easily make the turn.

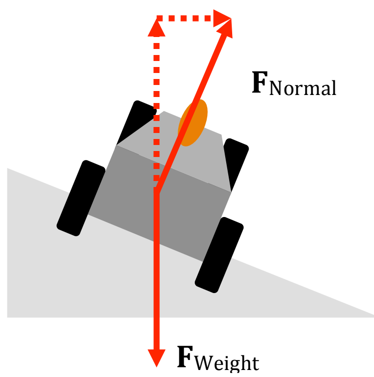
d. With that 3820 N maximum friction force available to us, the maximum speed the car can have to negotiate this turn would be:

$$F_c = \frac{mv^2}{r}$$

$$F_{friction} = 3820N = \frac{(500kg)v^2}{50m}$$

$$v = 19.5m/s$$

e. We need to bank the turn so that the horizontal component of the Normal force is what supplies the centripetal force that keeps the car moving in a horizontal circle. Notice that we have chosen *not* to tilt our *x-y* axes (as we sometimes do for inclined planes), because the centripetal force is directed horizontally toward the center of the circle. Therefore our calculations will be easier if we keep standard *x-y* orientations on our axes.



The analysis:

$$y: \quad F_{net} = ma; \; F_{Normal-y} - F_{gravity} = 0$$

$$F_{Normal-y} = F_{Normal}\cos\theta = mg$$

$$x: \quad F_{net} = ma; \; F_{Normal-x} = \frac{mv^2}{r}$$

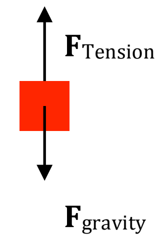
$$F_{Normal}\sin\theta = \frac{mv^2}{r}$$

Combine the *x* and *y* equations to get: $\dfrac{F_{Normal}\sin\theta = \dfrac{mv^2}{r}}{F_{Normal}\cos\theta = mg}$, *so* $\tan\theta = \dfrac{v^2}{rg}$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = 21.8°$$

7.

a. The Tension in the cord may be determined by drawing a free-body diagram and examining the forces acting on m_1:



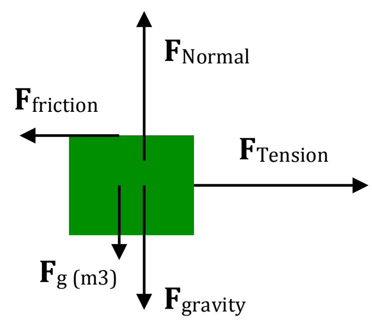
$$\sum F_y = ma = 0$$
$$F_{Tension} - F_{gravity} = 0$$
$$F_{Tension} = F_{gravity} = mg = (2kg)(9.8m/s^2) = 19.6N$$

b.

i. Once the block m_2 is released, the force of gravity acting on m_1 begins to accelerate the entire system. Although we could do an independent analysis of each individual mass, it's easier in this case simply to sum all the forces that are acting on the entire mass of the system:

$$F_{net-x} = ma$$
$$a_{system} = \frac{F_{net-x}}{m_{system}} = \frac{m_1 g}{(m_1 + m_2 + m_3)} = \frac{19.6}{(2+4+2)} = 2.45m/s^2$$

ii. The free-body diagram for m_2 has to include all forces acting on the block. Two important details to consider: the Weight of m_3 on top of m_2 applies a force down on that block, which results in a larger $\mathbf{F}_{\text{Normal}}$ pushing up on m_2. Also, note that there is a force of friction between m_2 and m_3—it is this force of friction acting to the right on m_2 that causes that block to accelerate off to the right with the system. The logical result of that, however, is that m_3 experiences that same force of friction in the opposite direction, to the left. That friction works opposite the force of Tension, and keeps m_2 from accelerating as quickly as it would otherwise.



iii. The Tension in m_1 is easily calculated using a new free-body diagram for that block:

$$\sum F_y = ma$$
$$F_g - F_{Tension} = ma$$
$$F_{Tension} = F_g - ma = mg - ma = m(g-a) = (2kg)(9.8 - 2.45) = 14.7N$$

where we've chosen to make the down direction *positive* in our equations.

iv. Because m_3 is accelerating to the right at 2.45 m/s^2, we can determine the force of friction acting on it:

$$\sum F_x = ma$$
$$F_{friction} = ma = (2kg)(2.45m/s^2) = 4.90N$$

We know the Normal force acting on m_3, so we can get the minimum coefficient of static friction as follows:

$$\mu = \frac{F_{friction}}{F_{Normal}} = \frac{4.90}{mg} = \frac{4.90}{19.6N} = 0.25$$

If there's a *greater* coefficient of friction between these two surfaces—ie. if the contact between them is *more* sticky—that's fine. We don't really have any more we can say about that. But we do know that because m_3 is still stuck under the current circumstances, the coefficient of friction has to be at least 0.25.

v. First let's figure out what the maximum acceleration for that block can be based on friction:

$$F_{friction} = \mu F_{Normal} = \mu mg = (0.5)(2)(9.8) = 9.8N$$

$$\sum F_{m3} = F_{friction} = ma$$

$$a = \frac{F_{friction}}{m} = \frac{9.8N}{2} = 4.9m/s^2$$

Now let's look at the system again to see what forces we can apply and NOT exceed an acceleration of 4.9 m/s^2:

$$\sum F_x = ma$$

$$F_g = ma; \ m_1 g = m_{total} a$$

$$m_1 g = (m_1 + m_2 + m_3)(4.9)$$

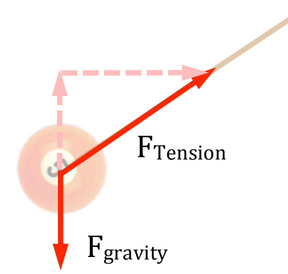
$$m_1 (g - 4.9) = (m_2 + m_3)(4.9)$$

$$m_1 = \frac{(2+4)4.9}{4.9} = 6kg$$

More than 6kg for m_1 and we'll accelerate the system too quickly, causing m_3 to break loose and start sliding.

8.

a.



b. Using the free-body diagram, and knowing that the billiard ball is not accelerating vertically, we can write

$$\sum F_y = ma_y = 0$$

$$F_{Tension-y} - F_{gravity} = 0$$

$$F_{Tension} \sin\theta = mg$$

We can determine θ by using the dimensions of the string and the circle radius:

$$\cos\theta = \frac{0.40m}{0.50m}$$

$$\theta = \cos^{-1}\left(\frac{0.40m}{0.50m}\right) = 36.9°$$

Solving for tension:

$$F_{Tension} \sin 36.9° = (0.15kg)(9.8)$$

$$F_{Tension} = 2.45N$$

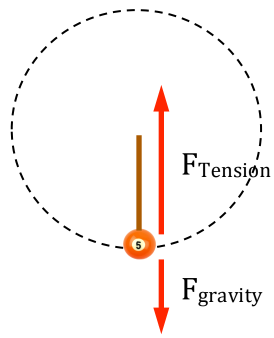
c. Centripetal acceleration can be determined using $a_c = \frac{v^2}{r}$, but we don't know the velocity of the ball. Perhaps we can determine acceleration using a force analysis. Considering the *x*-direction:

$$\sum F_x = ma_x \text{ , } or \ \sum F_c = ma_c$$
$$F_{Tension-x} = ma_c$$
$$a_c = \frac{F_{Tension-x}}{m} = \frac{F_{Tension}\cos\theta}{m}$$
$$a_c = \frac{(2.45N)\cos(36.9°)}{(0.15kg)} = 13.1m/s^2$$

d. The period T for one revolution is calculated using $T = \frac{distance}{speed} = \frac{2\pi r}{v}$. We haven't yet determined v, but now that we know centripetal acceleration, we can get it. Using that relationship:

$$a_c = \frac{v^2}{r}, \ so \ v = \sqrt{ra_c}$$
$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{ra_c}} = \frac{2\pi\sqrt{r}}{\sqrt{a}} = \frac{2\pi\sqrt{0.40m}}{\sqrt{13.1m/s^2}} = 1.10s$$

e. At the bottom of its path, the Force of tension and the Force of gravity are acting in opposite directions. Taking *up* to the positive direction:



$$\sum F_c = m\frac{v^2}{r}$$
$$F_{Tension} - F_g = m\frac{v^2}{r}$$
$$F_{Tension} = m\frac{v^2}{r} + F_g$$
$$F_{Tension} = \frac{(0.15kg)(3.00m/s)^2}{0.50m} + (0.15kg)(9.8m/s^2) = 4.17N$$

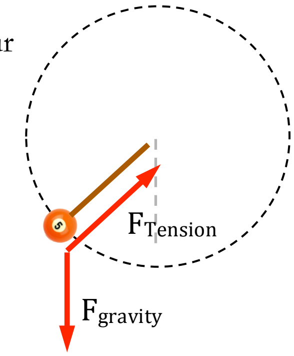
This is much greater than the weight of the billiard ball by itself, just 1.47 N. That's because the Tension in the string has to support the weight of the ball, *and* accelerate the ball toward the middle of the circle so that it will travel in that circular path.

f. At the top of its path, the Force of tension and the Force of gravity are both acting downwards toward the center of the circle. Taking *down* to be the positive direction:

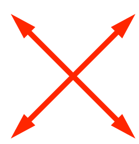
$$\sum F_c = m\frac{v^2}{r}$$
$$F_{Tension} + F_g = m\frac{v^2}{r}$$
$$F_{Tension} = m\frac{v^2}{r} - F_g$$
$$F_{Tension} = \frac{(0.15kg)(3.00m/s)^2}{0.50m} - (0.15kg)(9.8m/s^2) = 1.23N$$

It takes some amount of force directed downward to keep the ball moving in a circle at this speed, but gravity helps to supply some of that downward force, so the force required from the tension in the string is much less than it was before.

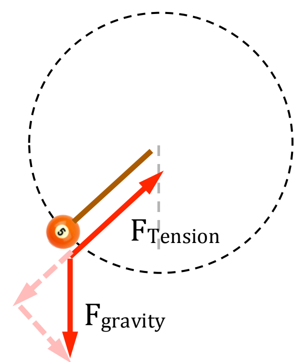
g. To solve this problem, we need to think about whether or not we should tilt our axes to solve the problem. We typically want to do that if the direction of acceleration in the problem has been tilted. Is that the case here? In which direction is the ball accelerating?



Given that the ball is accelerating centripetally (toward the center of the circle), we choose to tilt our axes as indicated.



With these axes in mind, we can see that we'll have to split the force of gravity up into components, and then use $F_{net} = ma$ to calculate the force of tension.



$$\sum F_c = m\frac{v^2}{r}$$

$$F_{Tension} - F_{gravity-radial} = m\frac{v^2}{r}$$

$$F_{Tension} = m\frac{v^2}{r} + F_{gravity-radial}$$

$$F_{Tension} = m\frac{v^2}{r} + mg\cos\theta$$

$$F_{Tension} = (0.15kg)\frac{(3.0m/s)^2}{(0.5m)} + (0.15kg)(9.8m/s^2)\cos 45° = 3.74N$$

h. We can use an analysis similar to what we did in part (e), although here we're solving for velocity instead of tension:

$$\sum F_c = m\frac{v^2}{r}$$

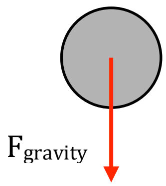
$$F_{Tension} - F_g = m\frac{v^2}{r}$$

$$v = \sqrt{\frac{r\left(F_{Tension} - F_g\right)}{m}} = \sqrt{\frac{(0.5m)(10N - (0.15kg)(9.8m/s^2))}{0.15kg}} = 5.33m/s$$

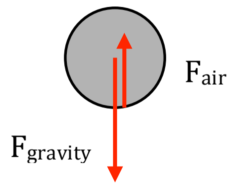
9.

a.

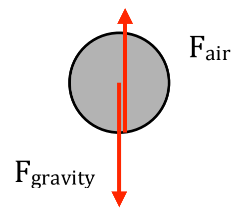
i. Just after it has been released



ii. After it has fallen some distance but before it has reached terminal velocity



iii. After the ball has reached terminal velocity



Some instructors prefer that students draw their vectors with the beginning of the vector located at the point of application for that Force. Thus, Force of gravity acts on the center of mass, and originates at the center of the ball, while the force of air friction is acting at the surface of the ball. The labels on the vectors are important, of course, and their relative lengths should be to scale.

b. The ball's acceleration as a function of velocity is determined using Newton's 2nd Law and the drag function given in the problem statement:

$$F_{net} = ma$$

$$F_g - R = ma$$

$$mg - \frac{1}{2}D\rho Av^2 = ma$$

$$a = g - \frac{D\rho Av^2}{2m}$$

$$a = 9.8 - \frac{(0.5)(129)(1.26e-3)v^2}{2(0.0027)} = 9.8 - 15v^2$$

c. The terminal velocity of the ball will occur when air resistance R is equal to the Weight of the ball:

$$F_{net} = ma$$

$$F_g - R = 0$$

$$mg = \frac{1}{2}D\rho Av^2$$

$$v = \sqrt{\frac{2mg}{D\rho A}} = \sqrt{\frac{2 \bullet 0.0027 \bullet 9.8}{0.5 \bullet 129 \bullet 1.26 \times 10^{-3} m^2}} = 0.807 m/s$$

d. We can develop this integral by using the function that's been given to us, along with Newton's Second Law. Note, also, that acceleration is the derivative of velocity, so:

$$F_{net} = ma$$

$$F_g - R = ma$$

$$mg - \frac{1}{2}D\rho Av^2 = ma = m\frac{dv}{dt}$$

$$\frac{dv}{dt} = g - \frac{D\rho A}{2m}v^2$$

e. The graph of the ping-pong ball's velocity should begin with 0 velocity, then increase in speed in the downward (negative velocity) direction with a negative acceleration (slope) that decreases over time to approach the constant (terminal) velocity.