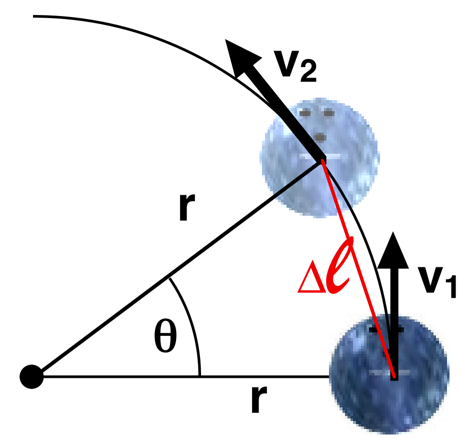
Ch 6 – Circular Motion, & more forces

Centripetal Acceleration

An object moving in a circle of radius *r* with constant speed *v* has an acceleration directed toward the middle of the circle, with a magnitude

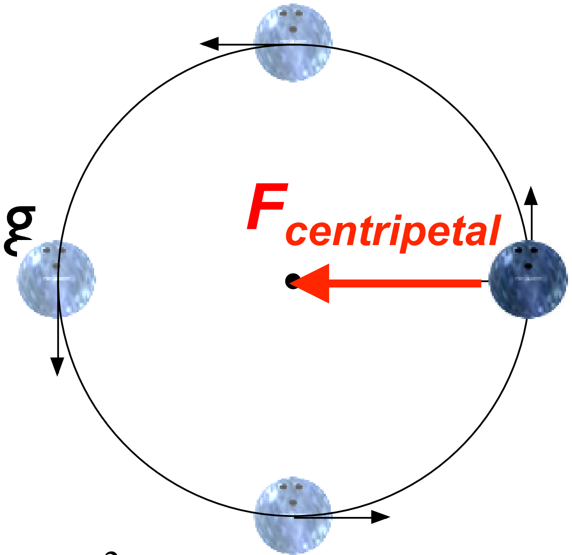
$$a_c = \frac{v^2}{r}$$



But what *gives* an object a centripetal acceleration?

Centripetal Force

Centripetal Force = a "center-seeking" force necessary to keep an object moving in a circle.



$$\sum F = ma$$

$$a_c = \frac{v^2}{r}$$

$$\sum F_c = ma_c = \frac{mv^2}{r}$$

Note that the phrase *centripetal force* is used generally to describe any force(s) that keep an object moving in a circle, just like we use F_{net} generally to describe the forces that accelerate an object linearly. Think of F_c as just "F_{net} in circular situations."

The *actual* forces that make an object move centripetally are due to things that we've already discussed: $F_{Tension}$ for ropes, cords, strings, etc., $F_{gravity}$ for gravitational force, $F_{friction}$ for... friction, and (in some cases) F_{normal}, etc.

Example 1

A 1000 kg car on a flat road is traveling at 14 m/s on a curve of radius 50m.

a. Draw a free-body diagram for the car.

b. How much centripetal force will be necessary to keep the car on the road?

c. If the μ static for this road is 0.60, will the car make the turn?

d. If the μ static for the road is 0.20, will the car make the turn?

e. What is the maximum speed the car can have and still make the turn with μ=0.6?

f. How high should we bank the turn to allow the car traveling at 14 m/s to stay on the track with no lateral friction from tires?

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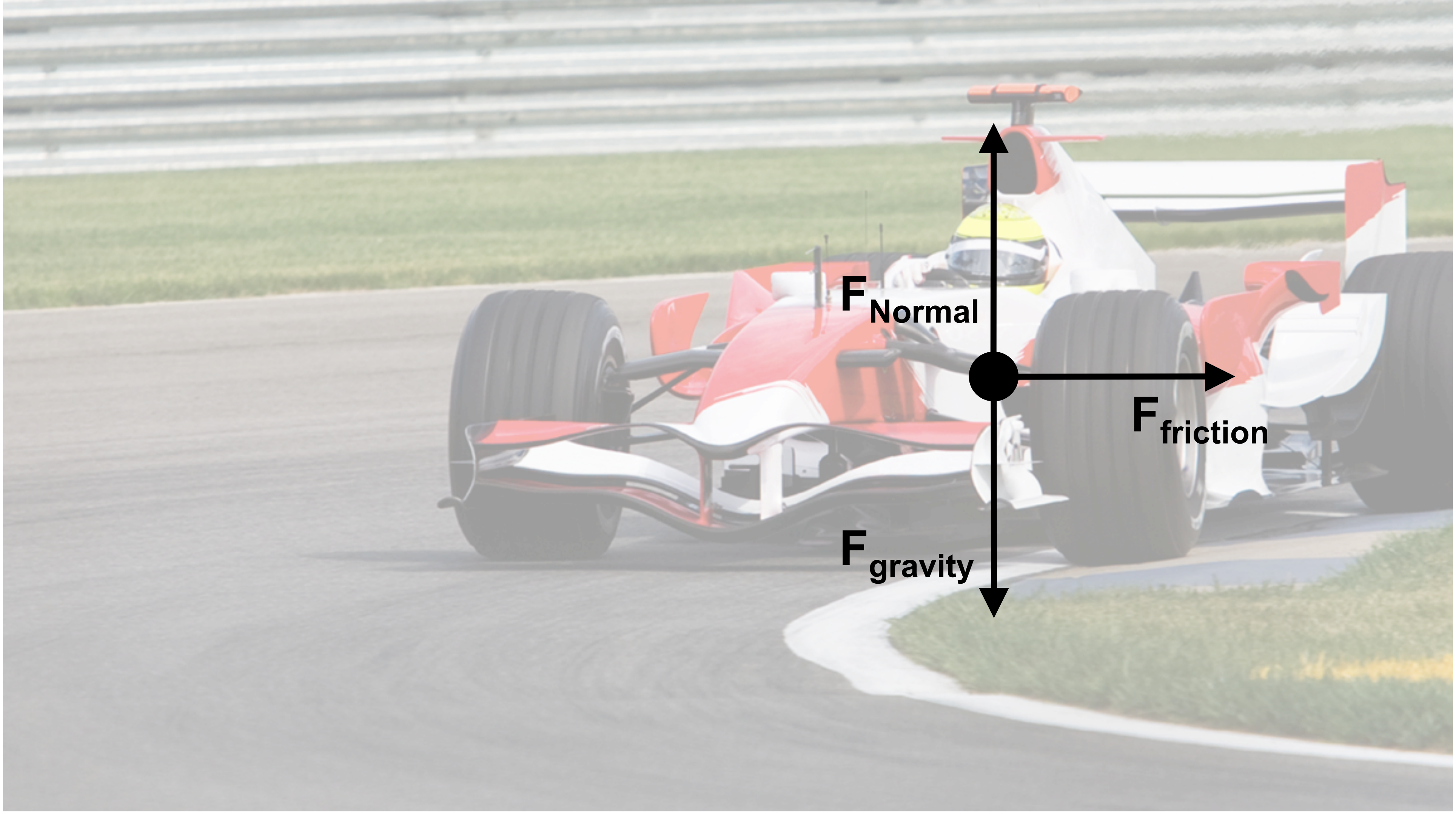


Example 1

A 1000 kg car on a flat road is traveling at 14 m/s on a curve of radius 50m.

b. How much centripetal force will be necessary to keep the car on the road?

$$\sum F_c = ma_c$$

$$\sum F_c = \frac{mv^2}{r}$$

$$F_{centripetal} = \frac{(1000kg)(14m/s)^2}{50m} = 3920N$$

Example 1

c. If the μ static for this road is 0.60, will the car make the turn?

$$\sum F_c = ma_c$$

$$F_{friction} = \frac{mv^2}{r}; \; F_{friction} = \mu F_{Normal}$$

$$\sum F_y = ma_y$$

$$F_{Normal} - F_{gravity} = 0$$

$$F_{Normal} = F_{gravity} = mg$$

$$\mu mg = \frac{mv^2}{r}$$

$$\mu = \frac{v^2}{rg} = \frac{(14m/s)^2}{50m \bullet 9.8m/s^2} = 0.40;$$

Yes, the car will make the turn.

Example 1

d. If the μ static for the road is 0.20, will the car make the turn?

$0.20 < 0.40$; No, the car will not make the turn.

There is not sufficient friction from the road to keep the car on the circular road.

Example 1

e. What is the maximum speed the car can have and still make the turn with μ=0.6?

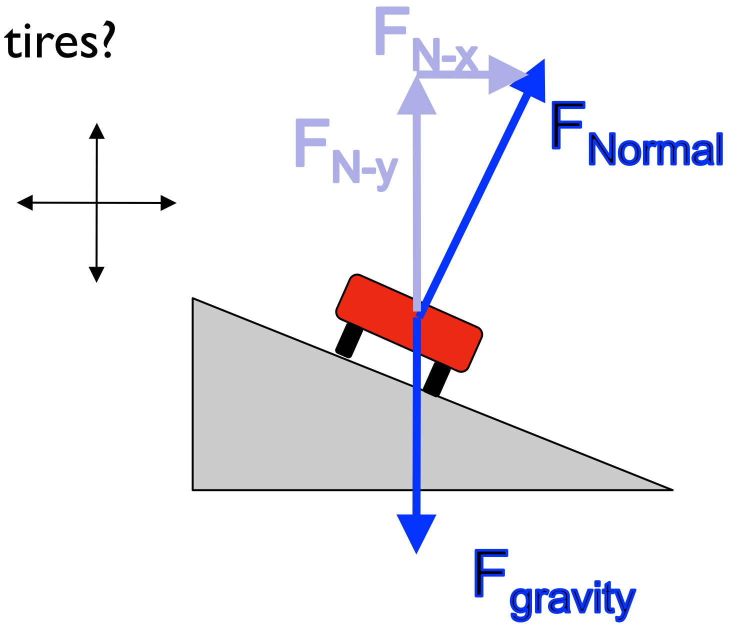
$$\mu mg = \frac{mv^2}{r}$$

$$v = \sqrt{r\mu g}$$

$$v = \sqrt{(50m)(0.60)(9.8m/s^2} = 17.1m/s$$

Example 1

f. How high should we bank the turn to allow the car traveling at 14 m/s to stay on the track with no lateral friction from tires?



$$\sum F_y = ma_y$$

$$F_{Normal-y} - F_{gravity} = 0$$

$$F_{Normal-y} = F_{Normal}\cos\theta = mg$$

$$\sum F_c = ma_c$$

$$F_{Normal-x} = F_{Normal}\sin\theta = \frac{mv^2}{r}$$

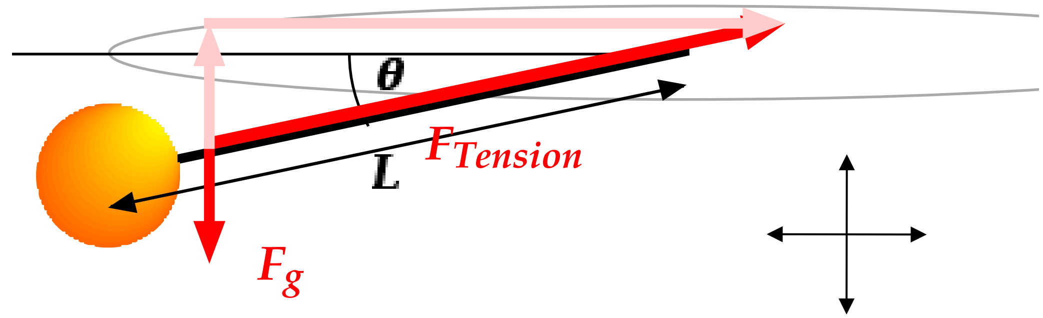
$$\frac{F_{Normal}\sin\theta = mv^2/r}{F_{Normal}\cos\theta = mg}$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = 21.8°$$

Example 2

A small body of mass *m* is suspended from a string of length *L* which makes an angle θ with the horizontal. The body revolves in a horizontal circle. Find the speed of the body and the period (time) of one revolution in terms of *L*, θ, and fundamental constants.



$$\sum F_y = ma_y$$

$$T_y - F_g = 0, \ so \ \ T\sin\theta = mg$$

$$\sum F_x = ma_x$$

$$T_x = \frac{mv^2}{r}, so \ \ T\cos\theta = \frac{mv^2}{r}$$

$$\tan\theta = \frac{rg}{v^2}, so \ \ v = \sqrt{\frac{rg}{\tan\theta}}$$

$$v = \sqrt{\frac{gL\cos\theta}{\tan\theta}}$$

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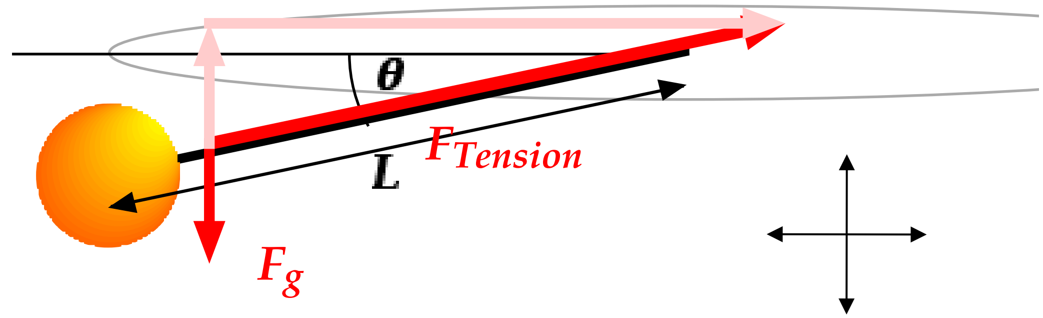
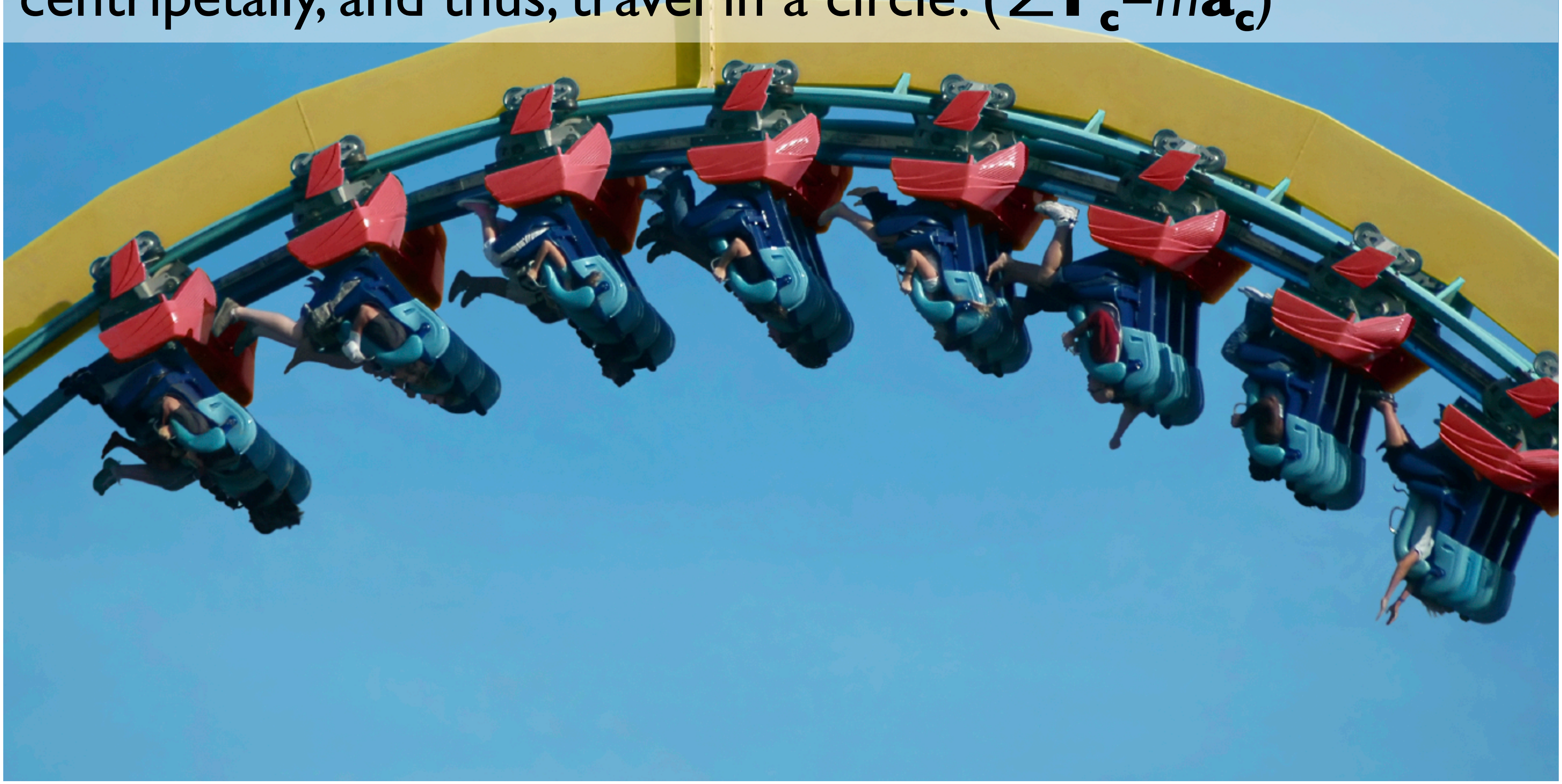


$$v = \sqrt{\frac{gL\cos\theta}{\tan\theta}}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi L\cos\theta}{\sqrt{\frac{gL\cos\theta}{\tan\theta}}}$$

Vertical Circles

Objects traveling in *vertical* circles are treated exactly the same as objects traveling in horizontal circles: the sum of the centripetal forces adds up to allow the object to accelerate centripetally, and thus, travel in a circle. ($\Sigma \mathbf{F_c} = m\mathbf{a_c}$)

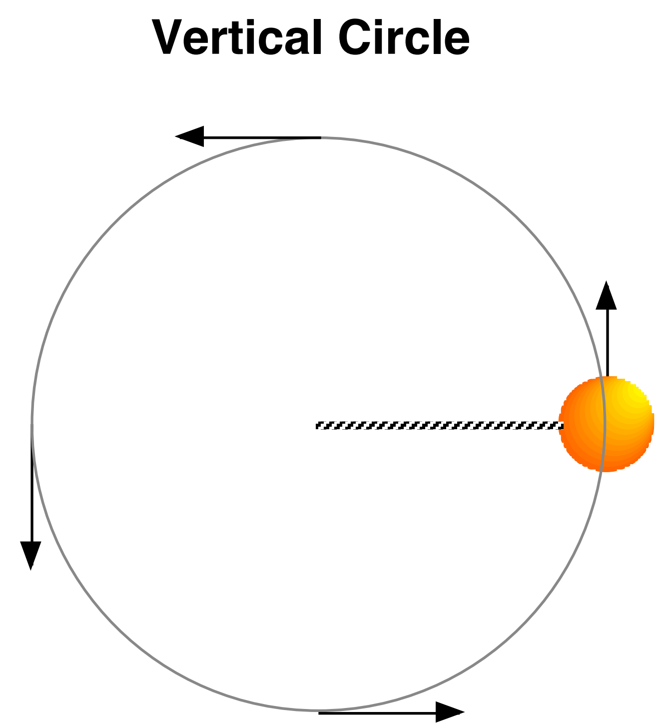


Example 3

A 1.8-kg ball is being swung in a vertical circle, at the end of a 1.2-m long rope.

a. What is the Tension in the rope as the ball swings past the bottom of the path at 5.0 m/s?

b. What is the *minimum* velocity the ball can have at the top of its circular path?



Example 3

a. What is the Tension in the rope as the ball swings past the bottom of the path at 5.0 m/s?

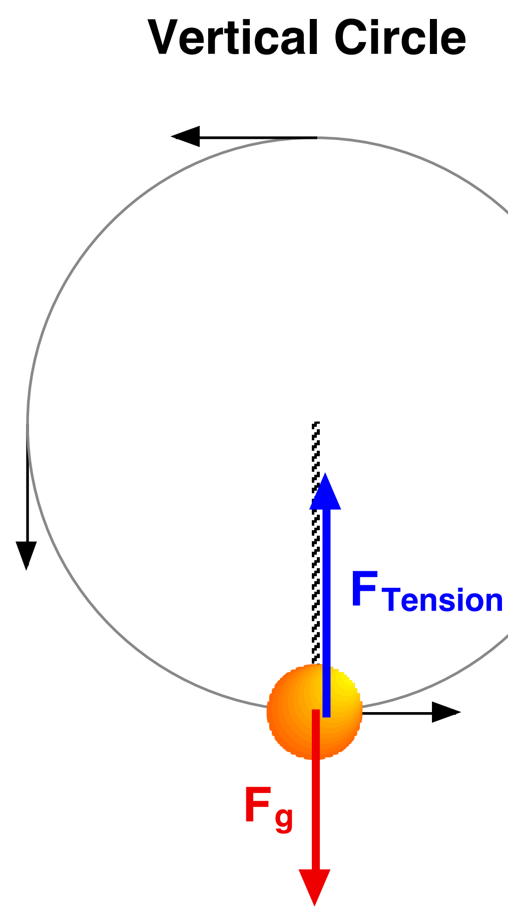
$$\sum F_c = ma_c$$

$$-F_g + F_{Tension} = m(\frac{v^2}{r})$$

$$-mg + F_{Tension} = m(\frac{v^2}{r})$$

$$F_{Tension} = m(\frac{v^2}{r}) + mg$$

$$F_{Tension} = (1.8kg)(\frac{(5m/s)^2}{1.2m}) + (1.8kg)(9.8m/s^2) = 55.1N$$



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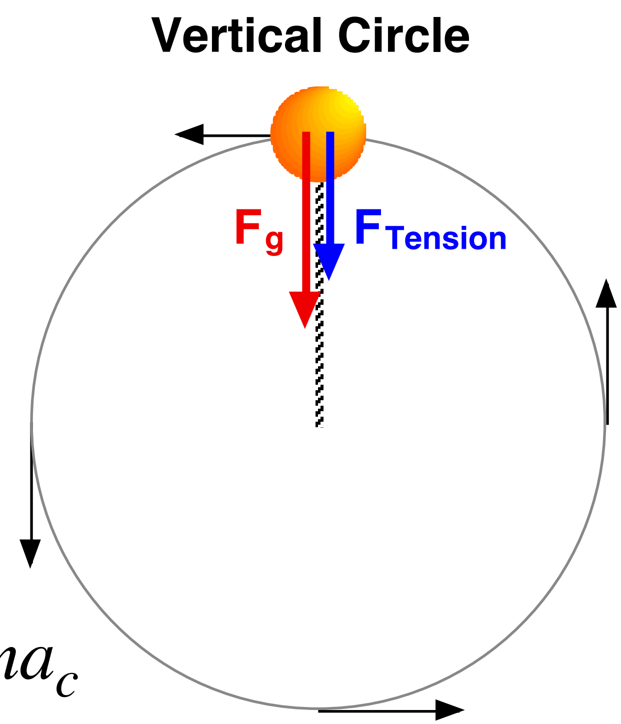

$$\sum F_c = ma_c$$

$$F_g + F_{Tension} = m(\frac{v^2}{r})$$

$$mg + 0 = m(\frac{v^2}{r})$$

$$v = \sqrt{rg} = \sqrt{(1.2m)(9.8m/s^2)}$$

$$v = 3.43m/s$$

Example 4

A 100 kg rollercoaster car travels inverted through a vertically-oriented circular loop of radius 20 meters.

a. At what velocity should the car travel through the top of the loop if the track is not to supply any force on the car?

b. Describe the force supplied by the track if the car travels 5 m/s *faster* than this velocity.

c. Describe the force supplied by the track if the car travels 5 m/s *slower* than this velocity.

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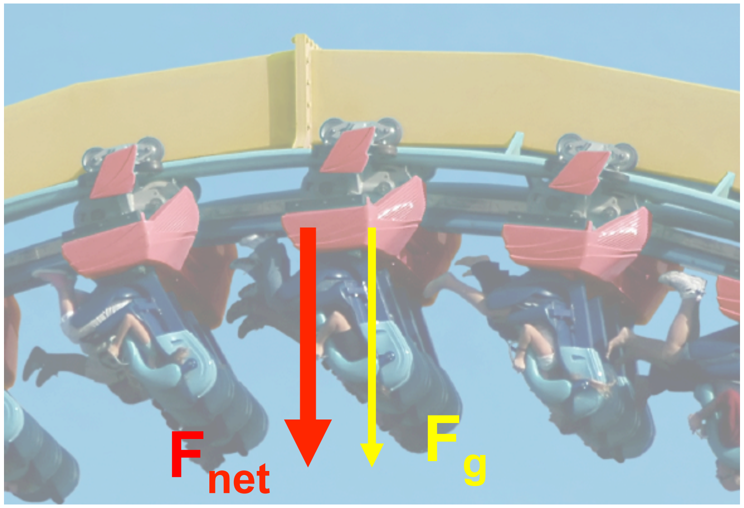


$$\sum F_c = ma_c$$

$$F_g + F_{Normal} = m(\frac{v^2}{r})$$

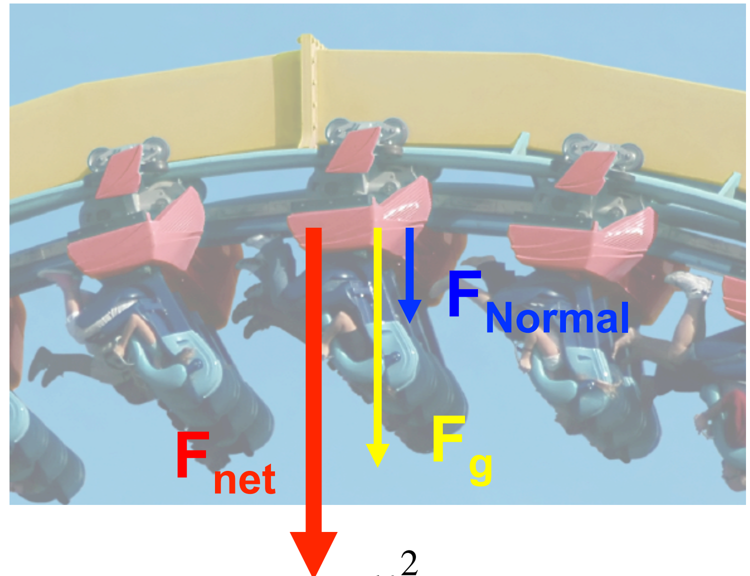
$$mg + 0 = m(\frac{v^2}{r})$$

$$v = \sqrt{rg} = \sqrt{(20m)(9.8m/s^2)} = 14m/s$$

Example 4

A 100 kg rollercoaster car travels inverted through a vertically-oriented circular loop of radius 20 meters.

b. Describe the force supplied by the track if the car travels 5 m/s *faster* than this velocity.



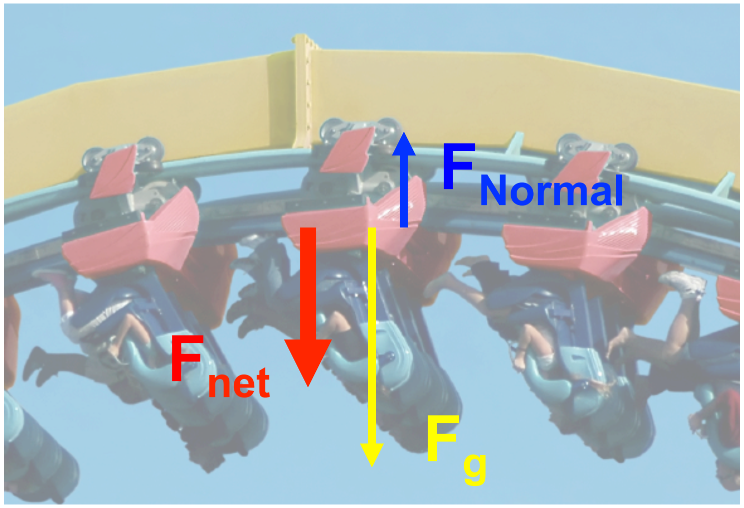
$$F_g + F_{Normal} = m(\frac{v^2}{r})$$

$$F_{Normal} = m(\frac{v^2}{r}) - mg = 825N,\ down$$

Example 4

A 100 kg rollercoaster car travels inverted through a vertically-oriented circular loop of radius 20 meters.

c. Describe the force supplied by the track if the car travels 5 m/s *slower* than this velocity.



$$F_g + F_{Normal} = m(\frac{v^2}{r})$$

$$F_{Normal} = m(\frac{v^2}{r}) - mg = -575N\ (up)$$

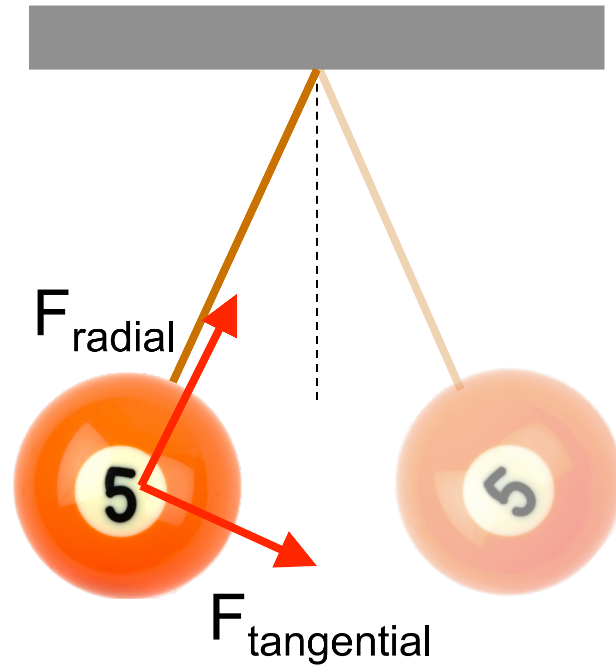
Non-Uniform Circular Motion

Earlier, we said that an object moving in a circle can have radial and tangential accelerations.

$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_t + \vec{\mathbf{a}}_r$$

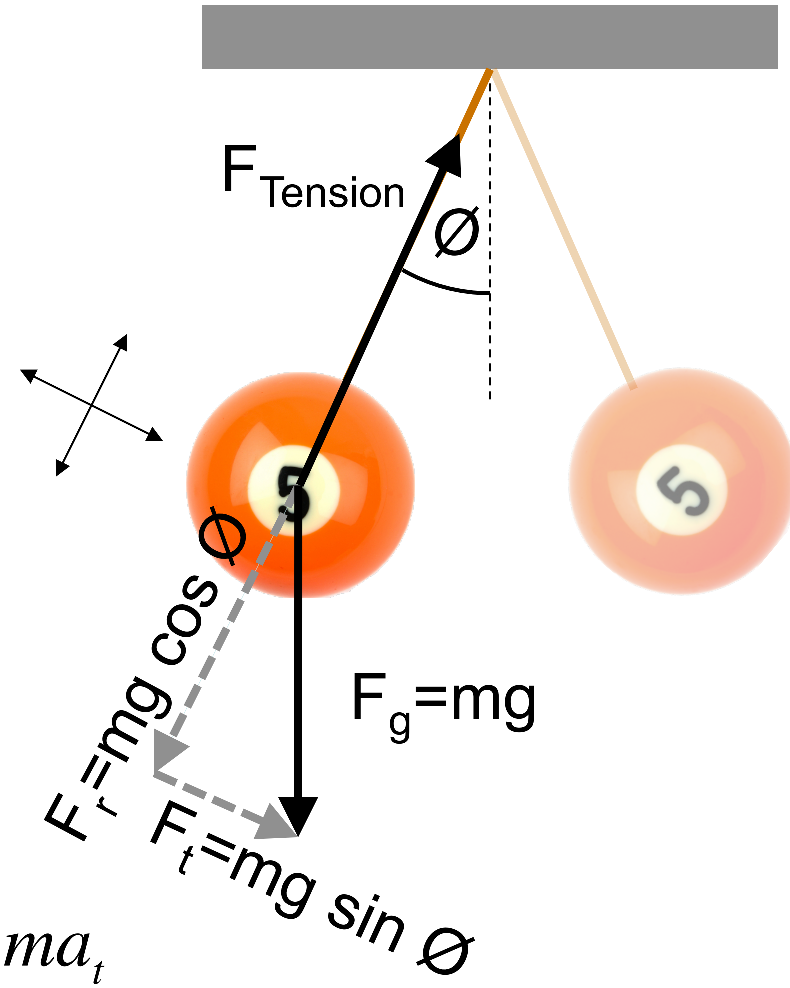
These obviously result from radial and tangential *forces*.

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_t + \vec{\mathbf{F}}_r$$



Example 5

Calculate the radial and tangential forces acting on the billiard ball, in terms of *m*, *v*, *r*, and Ø (angle from vertical), and radial and tangential accelerations.



$$\sum F_r = ma_r$$

$$F_{Tension} - F_{g-radial} = \frac{mv^2}{r}$$

$$F_{Tension} - mg\cos\phi = \frac{mv^2}{r}$$

$$F_{Tension} = \frac{mv^2}{r} + mg\cos\phi$$

$$\sum F_t = ma_t$$

$$F_{g-tang} = mg\sin\phi$$

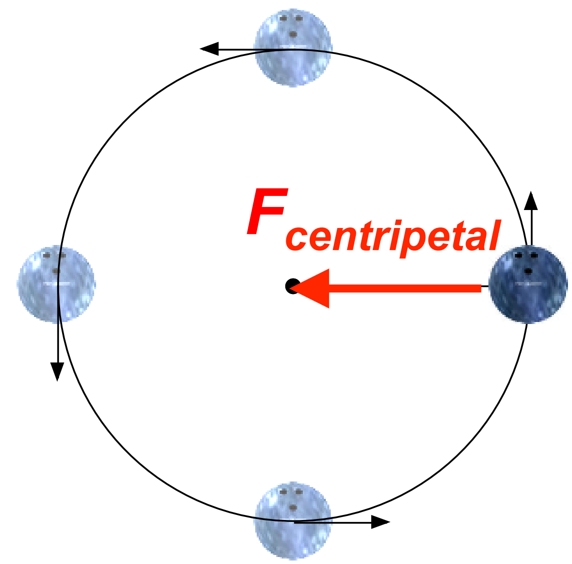
$$mg\sin\phi = ma_t$$

$$a_t = g\sin\phi$$

$$a = \frac{F_{radial}}{m} = \frac{v^2}{r}$$

Centrifugal Force

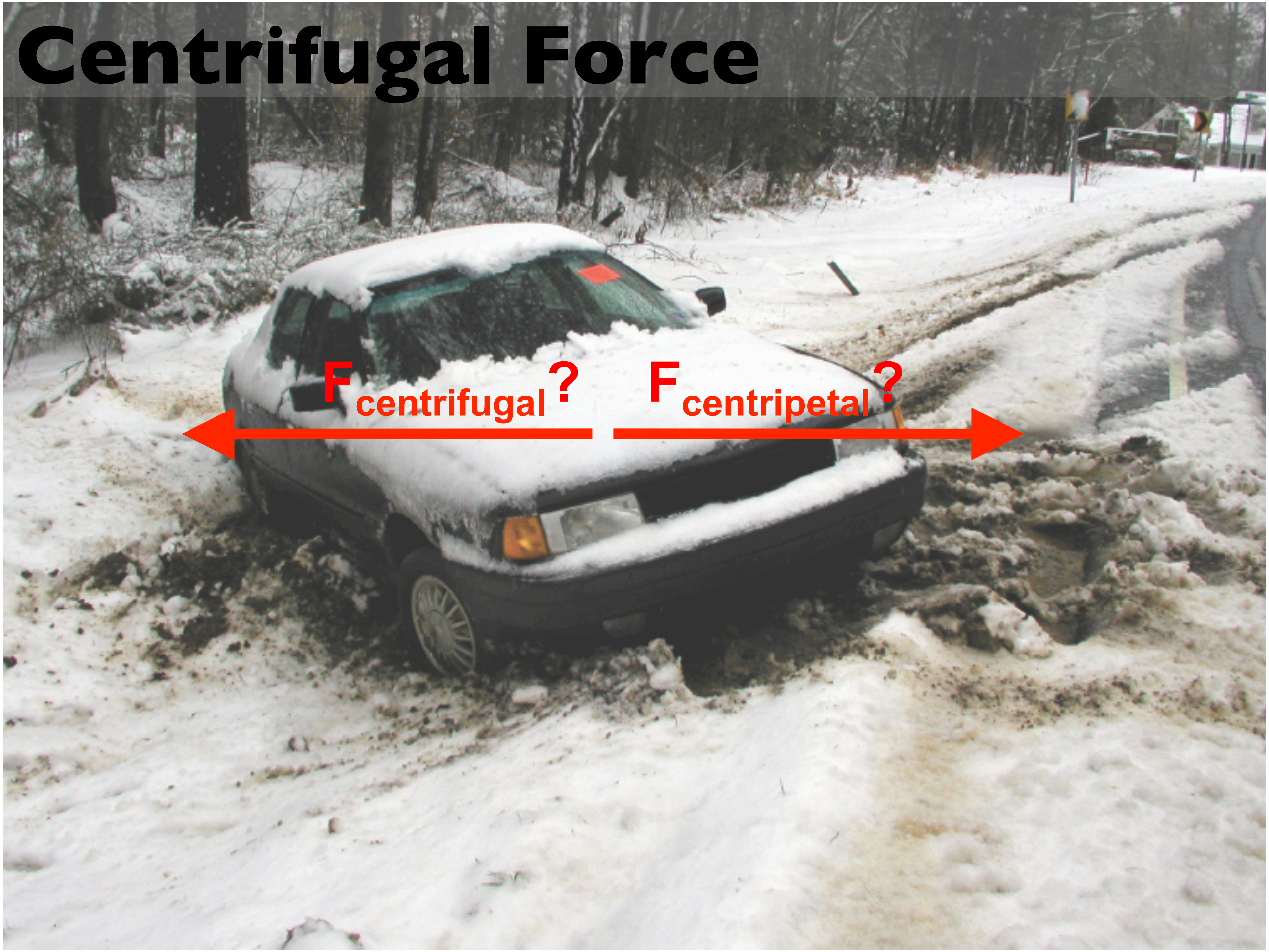
... *doesn't exist!*



Centrifugal force is an "apparent" force that we mistakenly think pulls an object *away* from the center of the circle. **There is no force pulling the ball outward!!!**

If you're holding on to the string attached to the ball while it goes in a circle, it's true that your hand feels an outward pull: *this is due to Newton's 3rd Law* (your hand pulls on the ball to keep it moving in a circle, the ball pulls back on your hand).

Centrifugal Force



Centrifugal Force

- the outward force on a body moving in a curved path around another body
  wordnetweb.princeton.edu/perl/webwn
- Centrifugal force (from Latin centrum "center" and fugere "to flee") represents the effects of inertia that arise in connection with rotation and which are experienced as an outward force away from the center of rotation. ...
  en.wikipedia.org/wiki/Centrifugal_force
- in everyday understanding, centrifugal force is the effect that tends to move an object away from the center of a circle it is rotating about (a consequence of inertia); : In a rotating reference frame, the apparent force that seems to push all bodies away from the centre of rotation of the ...
  en.wiktionary.org/wiki/centrifugal_force
- Attributive form of centrifugal force
  en.wiktionary.org/wiki/centrifugal-force
- The natural tendency of a language to branch into a set of regional dialects (cf. Centripetal Force). The various distinctive dialects of American English (Southern, Northern, Northeastern) all display centrifugal force.
  www.catchwordbranding.com/resources/glossary.php
- Also known as G-force. Describes the acceleration of gravity and, in racing terminology, the force that presses a car outwards in a corner. The unit of measurement is the G (1 G being the equivalent of 9.81 metres per second squared). ...
  en.lfsmanual.net/wiki/Glossary
- The reaction to a centripetal force, acting radially outwards from the centre of rotation.
  www.quarrying.org/dictionary/c.html
- Not an actual force but, rather, the result of an object's inertia trying to maintain motion along a straight line when the object is forced to travel along a curve.
  www.discoverhover.org/infoinstructors/vocab.htm
- The force away from the centre of a rapidly spinning impeller.
  www.diydoctor.org.uk/projects/buildingdictionaryc11.htm
- The action that causes something to move away from its center of rotation.
  www.lighttower.net/helpful-terms
- Centrifugal force is generally not believed to be a source of energy. It is simply a FORCE that acts internally on all rotating objects, and therefore cannot be harnessed for the production of energy. Recently, this problem has been overcome. ...
  www.freenergyword.com/energy-concepts.htm

Centrifugal Force - xkcd