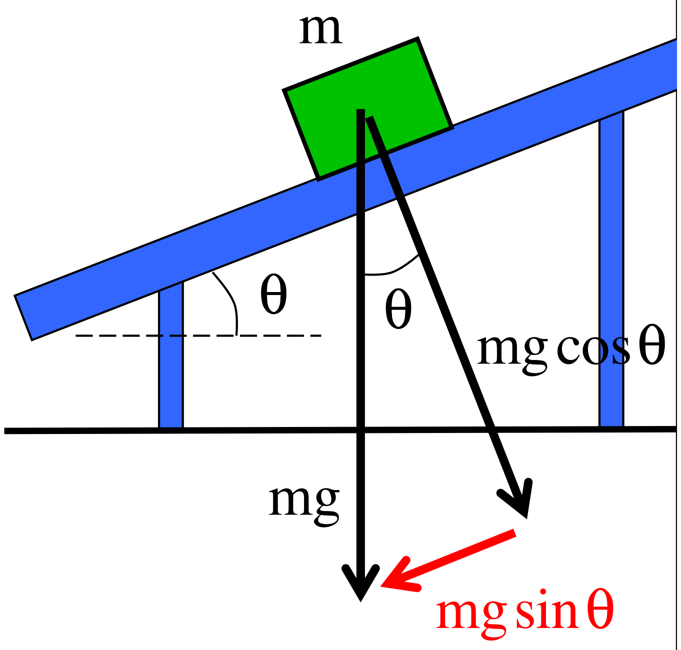
The Island Series:

You have been kidnapped by a crazed physics nerd and left on an island with twenty-four hours to solve the following problem. Solve the problem and you get to leave. Don't solve the problem and you don't.

The problem: You are given an incline plane, a protractor, a calculator and a cart with frictionless wheels on it. Determine the acceleration of the cart as it rolls freely down the incline.

Solution to Island Problem:



According to Newton, a net force in a particular direction will generate an acceleration in that direction with the net force equaling the *mass times that acceleration*. The only force acting on the cart along the line of the motion is the component of gravity along that line, which means

$$\cancel{m}g\sin\theta = \cancel{m}a$$

In other words, all you need to determine the acceleration is a *protractor* to determine the angle of the incline.

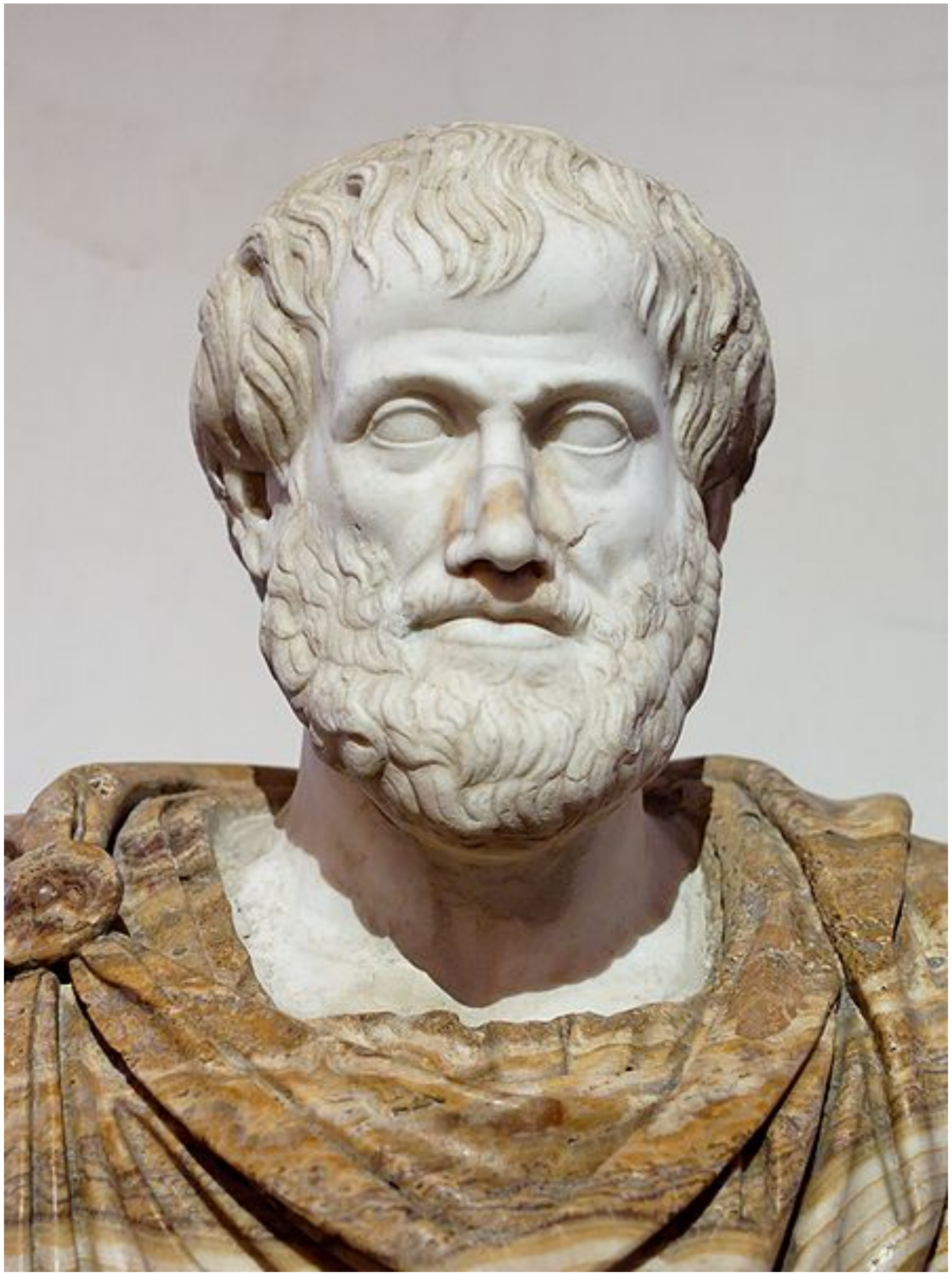
Newton's Second Law is a very powerful tool in analyzing systems. It is the approach with which this section is concerned.

CHAPTER 5:
Newton's Laws

You are horribly spoiled . . . a quick excursion into the history of the computer.

Life hasn't always been the bowl of cherries it is today, at least for scientists . . . A quick history of the evolution of science . . .

Some of this is lifted from Mr. White's lecture notes:



The Greek philosopher & metaphysicist Aristotle (384-322 B.C.), teacher of Alexander the Great and student of Plato (who was, in turn, the student of Socrates), was the first western scientist (at least that we know about) in the sense that he looked at the world and tried to find "laws" that governed its existence.

"Heavier objects fall faster in proportion to their weight;"

"The sun orbits the earth;"

"For an object to move with a constant velocity, a force is required;"

"Objects come to rest while in motion because it is their natural state to be at rest, and object tend to their natural state."

All incorrect, but all following logically from everyday observation.

The rub:

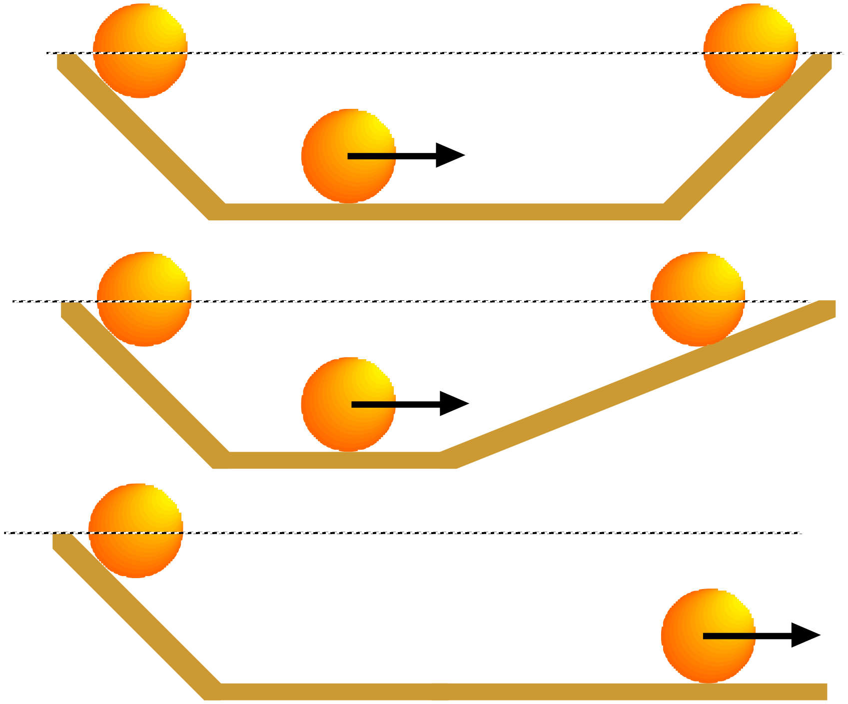
Religion in Italy in the 1500 and 1600's was dominated by the Catholic church, located at the Vatican, and church doctrine at the time was quite vehement about several points.

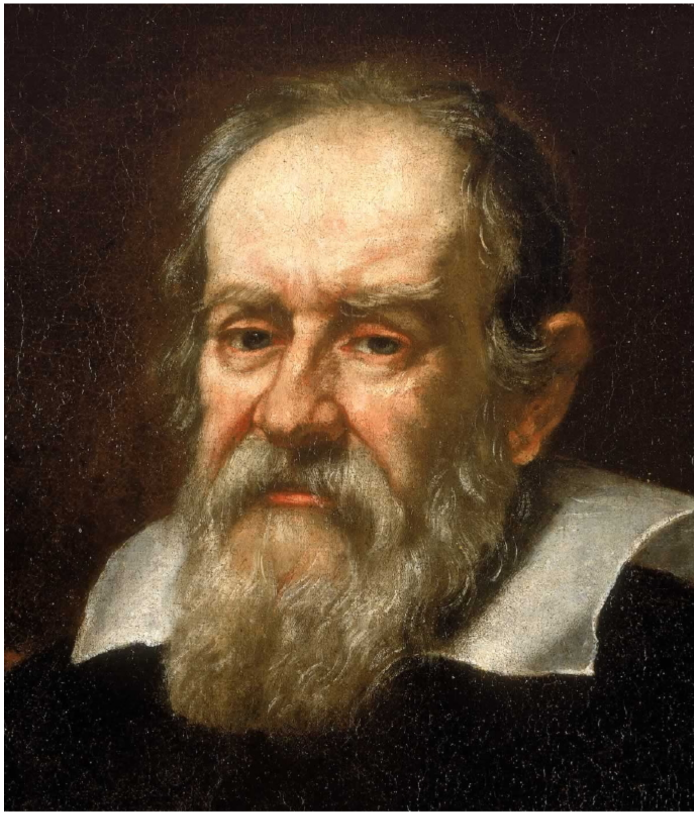
1.) Man was the center of God's creation;
2.) Only through the church could one get to God;
3.) The church was infallible.

In addition, the church endorsed the "findings" of Aristotle, especially liking the notion that the Sun and, essentially, all else, rotated around the earth, this rather naturally fitting into the idea that man was the center of God's creation. What's more, individuals who did not agree with the church were deemed heretics and, if they did not recant their view, were often burned at the stake (this action being necessary to "save their souls").

Enter Galileo Galilei (1564-1642): Galileo started out by using the recently invented telescope to impress the Doge of Venice, thereby receiving a stipend for life to "go out and be scientific." He then used the device to view the sun (a thoroughly bad idea—he was blind by the time he died), seeing sun spots moving on its surface, and he viewed the planet Jupiter along with the four moons (now called *the Galilean moons*) that clearly orbited, not the earth, but Jupiter itself (an observation Aristotle wouldn't have liked).

Galileo also mess with Aristotle directly. He concocted a series of demonstrations, shown below in a rather nice graphic created by the good Mr. White)



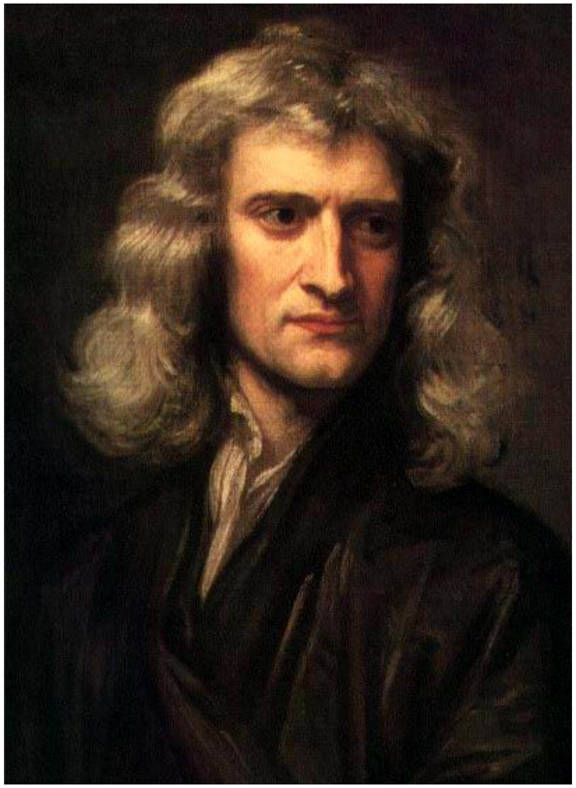


in which he showed that objects tend to roll almost up to their starting level when released on a ramp, the consequence being that if they were never given the opportunity to get back up to that original level (assuming whatever retarding force that kept them from getting *all the way back up* was eliminated), the objects would *never* stop, thereby contradicting Aristotle's claim that objects "come to rest because it is their natural state to do so."

The Vatican, with Aristotle being their man and ever mindful of their doctrine of infallibility (you can't very well tell people what to do under the mantle of infallibility if you are shown to be fallible), was not amused when Galileo published his findings, not in Latin (which was something very few could read) but in Italian. The consequence was two summonses during his lifetime. During the first, he took his telescope to show the moons of Jupiter, thinking that when they saw that not *everything* rotated around the earth, they'd be surprised by intellectually fascinated. They weren't. They apparently told him he might be right, but talking about it would just confuse the common folks. It was best that he not continue publishing.

When he didn't comply, a second summons was sent out, this one with a heresy charge attached (in fact, it wasn't until the 1950's that the Vatican formally acknowledged that it had been wrong about him). He was given the choice of recanting or being burned at the stake as a heretic. He recanted (who wouldn't?).

Put under house arrest, he died nine years later.



The year after Galileo died, Isaac Newton was born (1642-1726/7). In 1665, Cambridge University closed due to the "Great Plague," Newton had his supposed *apple on the head* experience and the idea of gravity emerged.

What was, maybe, more significantly in a cosmic sense was by that time, the Vatican had realized it was not going to be able to stifle scientific exploration, and that Aristotle wasn't the end-all, be-all. Their solution? They made an agreement with the scientific community that said, *As long as you, the scientific community, don't delve into "metaphysical topics" (i.e., topics "beyond the physical"—talking about God or what happens after death), you can do anything you wanted in the way of scientific research.* In other words, you could do research without fear of being burned for it.

This change allowed Newton to be Newton. He was brilliant (not very pleasant to be around, but brilliant). When he didn't have the math required to justify an assumption he wanted to make about how the bits and pieces of stuff making up the whole of the earth gravitationally affect a single body on the earth's surface, he *made up* the math out of whole cloth. Today, we call it *Calculus*. (Leibnitz independently paralleled him—it's Leibnitz's notation we actually use today, but that hardly diminishes the feat.) In 1687, he wrote the definitive work, his Principia Mathematica, on classical mechanics, which included his *analysis of motion*. *That* is what you will spend the first semester studying.

Newton's Three Laws

NEWTON'S FIRST LAW: *In an inertial frame of reference,* objects in motion stay in motion in a straight line, and object at rest stay at rest, unless impinged upon by a net force. This is sometimes called the Law of Inertia.

NEWTON'S SECOND LAW: *The net force* acting on an object is *proportional to* the acceleration of the object, with the proportionality constant being the object's mass. In short, $\vec{F}_{net} = m\vec{a}$.

NEWTON'S THIRD LAW: *For every action* there is an *equal and opposite action* somewhere in the universe.

Newton's First Law

Wait a minute: *Called "the law of inertia," Newton's First Law* starts out with:

In an *inertial frame of reference,*

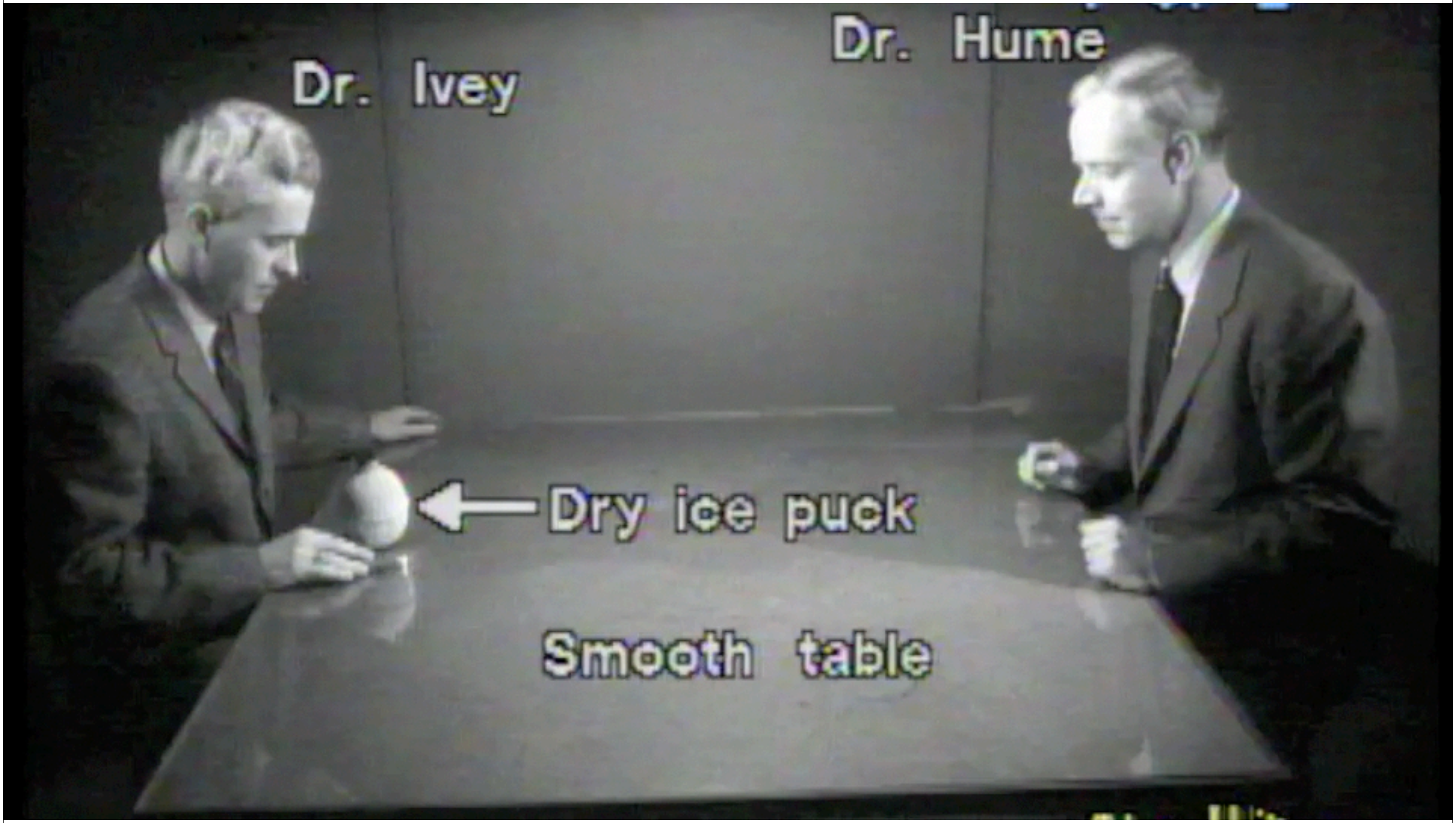
What *is* that?

An *inertial frame of reference* is an UNACCELERATED frame of reference. That is, a *non-inertial frame of reference* is a frame that IS accelerated.

Newton's Laws and his laws of motion are predicated on having a frame of reference that is *NOT ACCELERATED*.

What happens when you view motion from an accelerated frame?

Non-inertial frame: https://youtu.be/zqYHaulHzLI start at 16:45, go to 18:40

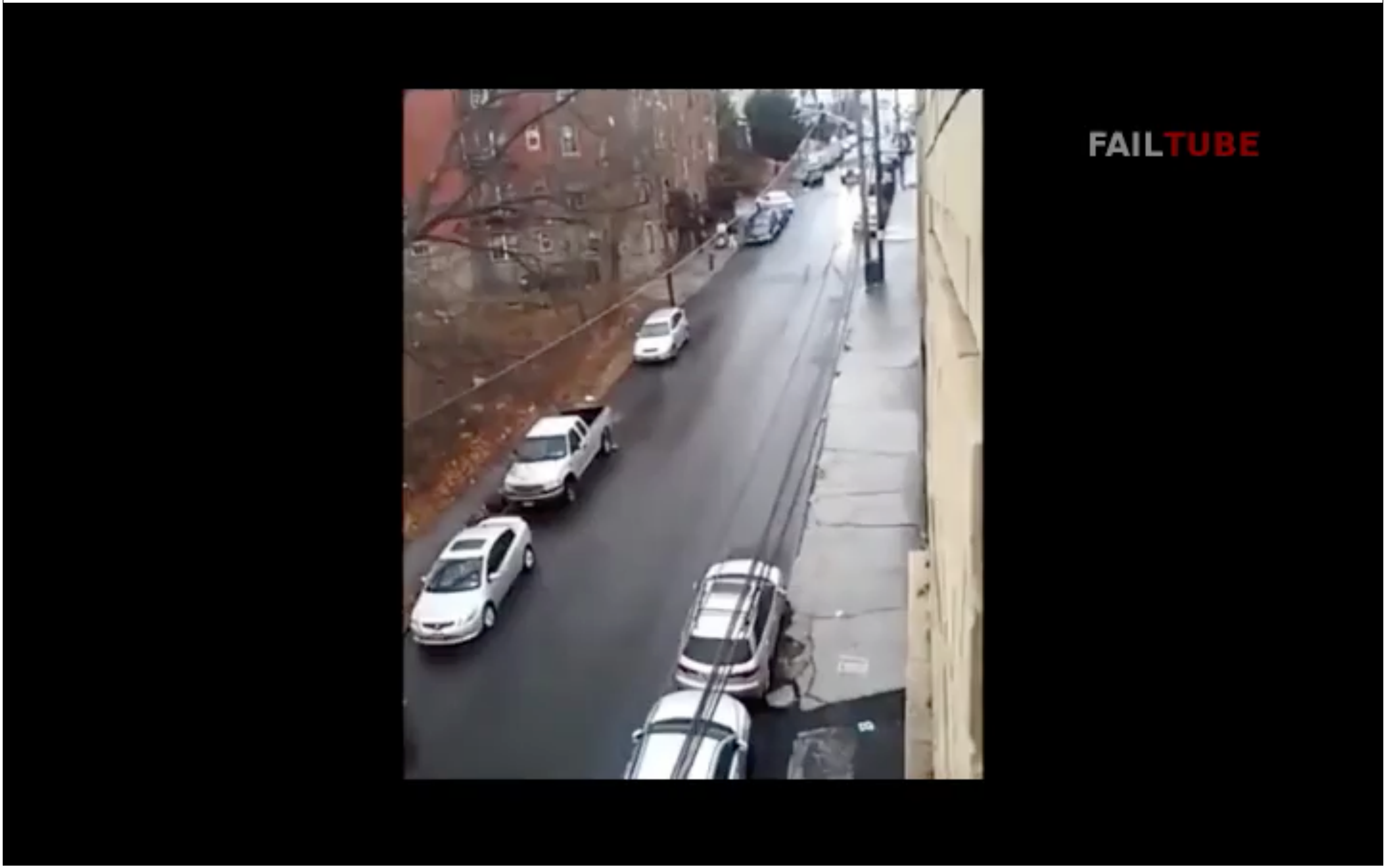


So back at the ranch:

In an *inertial frame of reference,*

Objects in motion stay in motion unless impinged upon by a force,

https://youtu.be/DrOO_HcQngg start 0:48 to 1:03



and objects at rest stay at rest unless impinged upon by a force.

https://youtu.be/mfEMW0sXwiw https://youtu.be/1hDv5pib89s

(more) *and objects at rest* stay at rest unless impinged upon by a force.
https://media.giphy.com/media/loHlOOGrVZEEhojAI/giphy.gif

So what's wrong with this picture?

https://youtu.be/b6DNlshrrVY (this YouTube version cuts out the crucial 3 second part at the end . . .)

Minor Point--MASS

So what is mass?

A relative measure (relative to what? . . . to a standard found at the Bureau of Weights and Measures in Paris, France) of a body's *resistance to changing its motion* (i.e., its inertia) is called the body's inertial mass.

Example: A Volkswagen sitting out in space is going to be inherently more difficult to move than will be a box of Kleenex. Why? Because the Volkswagen has more inertia, hence more mass.

A relative measure of a body's *willingness to be attracted to other bodies* is called the body's gravitational mass.

Example: Place an object on a scale and the body's attraction to the earth is what provides the force that depresses the scale effecting the measurement. This is a measure of the body's attraction to the earth. The scale is measuring *gravitational mass*.

The standard, located in Paris, is defined as having *1 kilogram of inertial mass*. If we define the SAME STANDARD to having *1 kilogram of gravitational mass*, a very odd and seemingly unexplainable thing is observed. A mass that is determined to be twice as inert as the standard (hence, a 2 kg inertial mass) will also be found to have twice the willingness to be attracted to other objects (hence, a 2 kg gravitational mass). The two mass values will be the same. THERE IS NO GOOD REASON (at least within the Newtonian model) for this to be the case . . . but it is.

That means, if you want the *inertial mass* of an object, which is the kind of mass you need in $\vec{F}_{net} = m\vec{a}$ (think about it—a body's inertia determines how much acceleration it will feel due to a net force, and inertia is what is going to dampen the acceleration), you don't have to use an *inertial balance* to measure it (that is the lab device designed to do that). All you have to do is put the body on a scale and measure it's gravitation mass. Knowing that will give you its inertial mass because they will be numerically the same.

Consequence: Nobody delineates between the two masses, they just refer to a body's *mass*.

Newton's Second Law

A net force applied to a body will accelerate the body in proportion to the force. The proportionality constant between the two will be the body's inertial mass.

More succinctly, and in a considerably more useful form:

$$\vec{F}_{net} = m\vec{a}$$

Note that there are really *three equations* here.

So according to Newton's Second, apply a net force, get an acceleration!

https://youtu.be/_AW0qGGcfbI

Newton's Third Law

For every action there is an equal and opposite action somewhere in the universe. These action/action pairs ALWAYS EXIST.

In other words, *there will always be a force of response when a force of action occurs.*

Example: You push on the wall; the wall pushes with equal and opposite force back on you.

Example: The earth exerts a gravitational force on the moon; the moon exerts a gravitational force back on the earth.

Example: The earth exerts a gravitational force on *you*; *you* exert an EQUAL AND OPPOSITE gravitational force back on the earth.

Example: While standing on a table, the table exerts an upward "normal" force on you; you exert an equal, downward force on the table.

The trick: The *same* language used in identifying the *action* should be used to identify the *response*, but with the objects reversed. *You* apply a force to *it*; *it* applies a force to *you*, etc.

The hand feels a force; the face feels a force. *SAME FORCE MAGNITUDE!*

https://youtu.be/YZSRszXGo80

This is important: (you should be able to pick the right one)

A bug runs into the windshield of a car that is traveling at 60 mph.

a.) The bug experiences a greater force and acceleration than the car.

b.) The bug experiences a greater force but lesser acceleration than the car.

c.) The bug experiences a greater force but same acceleration as the car.

d.) The bug experiences a lesser force and lesser acceleration than the car.

e.) The bug experiences a lesser force but same acceleration as the car.

f.) The bug experiences a lesser force but greater acceleration than the car.

g.) The bug experiences the same force and same acceleration as the car.

h.) The bug experiences the same force but greater acceleration as the car.

h.) The bug experiences the same force but lesser acceleration than the car.

Gravity in general

Newton realized that the attraction between *any* two object was proportional to the masses involved and inversely proportional to the distance between the objects. With G as proportionality constant:

$$\vec{F}_{grav} = G\frac{m_1 m_2}{r^2}(-\hat{r})$$

(Notice that this is written in radial unit vector notation.)

If one of the masses is the earth with the second is an object on or near the earth's surface, like *you*, the "r" term becomes the *radius of the earth* and one of the masses becomes the earth's mass, and we can re-write the force magnitude as:

$$\left|\vec{F}_{grav}\right| = m_{you}\left(G\frac{m_e}{{R_e}^2}\right)$$

But we know "G," and we know both the *radius* and *mass* of the earth, so putting those numbers in and we get:

$$\left|\vec{F}_{grav}\right| = m_{you}\left(9.8\ m/s^2\right)$$

Called a your "weight," and defining $g = 9.8\ m/s^2$, (which is POSITIVE), we can define the magnitude of the *force of gravity on an object near the earth* to be

$$\left|\vec{F}_{grav}\right| = m_1 g$$

Force Types

There are five types of forces you will need to deal with when negotiating Newton’s Second Law (N.S.L.). They are:

Gravitational force:

--Usually *downward*;

--Near the earth’s surface, the magnitude denoted as:

F_g or mg

where “g” is always positive and, near the surface of the earth, is numerically equal to $9.8\ m/s^2$

https://youtu.be/Gq_bjaI0NTo

Wiley coyote--gravity being inappropriate

Normal force:

--Force of support;
--can be provided by floors, walls, other objects;
--always directed *perpendicularly away from* object that provides it;

--Magnitude denoted as: F_N or N

when normal forces go bad . . .

Normal force:

--Force of support;

--can be provided by floors, walls, other objects;

--always directed *perpendicularly away from* object that provides it;

--Magnitude denoted as: F_N or N

when normal forces go bad . . . https://youtu.be/-1X1o4tQqCw

Tension force:

--Provided by rope or cable;
-- Always directed *away from* the object that provides it;

--Magnitude denoted as:

F_T or T

(the guy was OK)

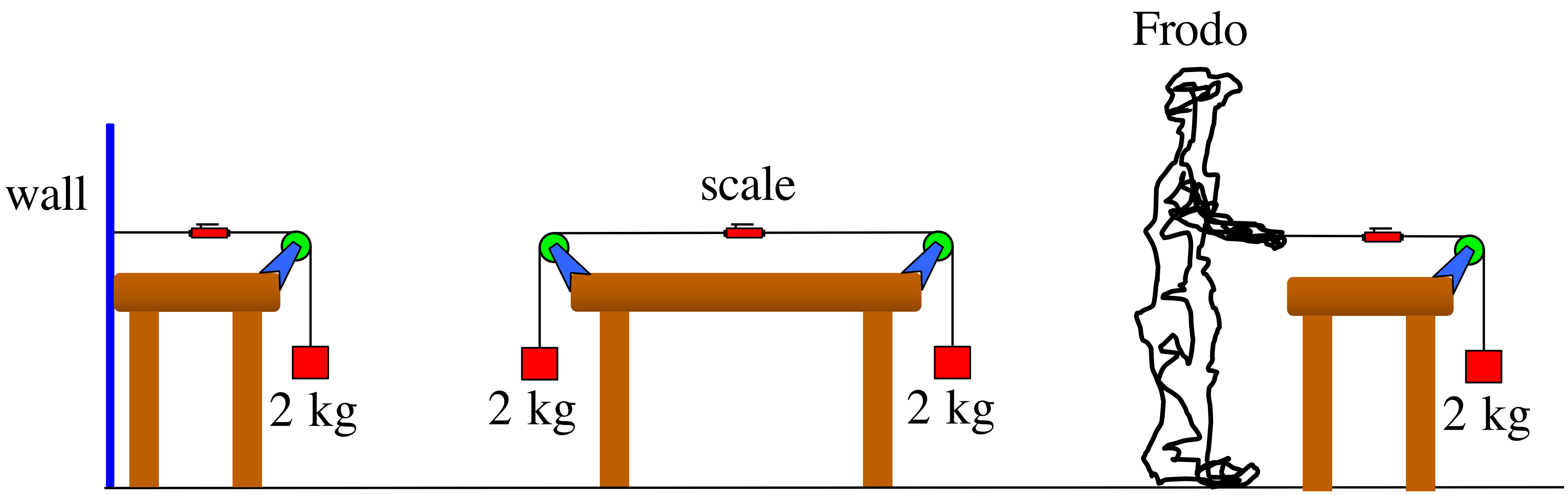
when tension goes bad . . .

https://youtu.be/xxrM5tv_RNI (start at 1:35)

--Magnitude denoted as:

Tension force (con't):

What is the difference in the tensions in the three situations? That is, how will the scale values differ?



Kinetic and Static frictional force:

There are a number of types of frictional force. Friction always involves two bodies in contact with one another. Although we will deal with *rolling friction* later, this chapter will only discuss and use what is called *kinetic friction*, sometimes referred to as *sliding friction* (think *pushing a crate across a floor*), and *static friction*, which occurs when two bodies are in contact but are not slipping, relative to one another (think *holding traction* as you drive through a curve on a freeway).

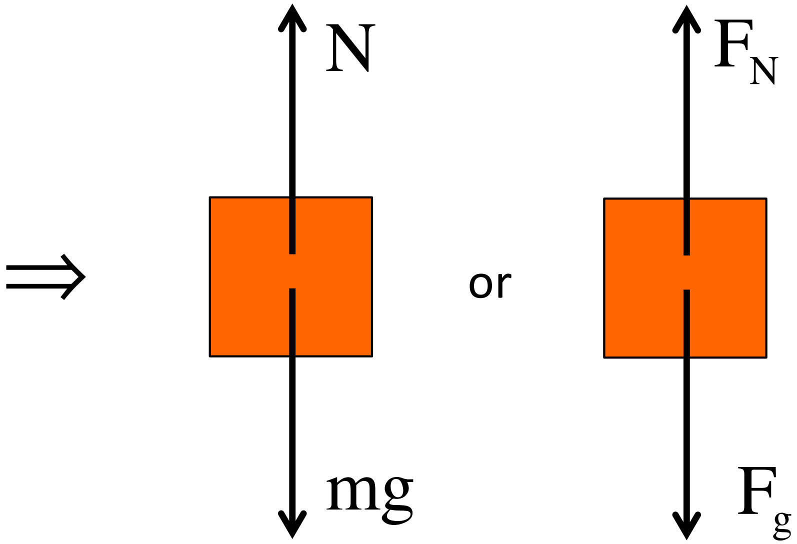
The first set of example problems do not require the use of friction, so we will be putting off our discussion of frictional forces until later in the chapter.

Newton's Laws and Problem Solving

Free body diagrams:

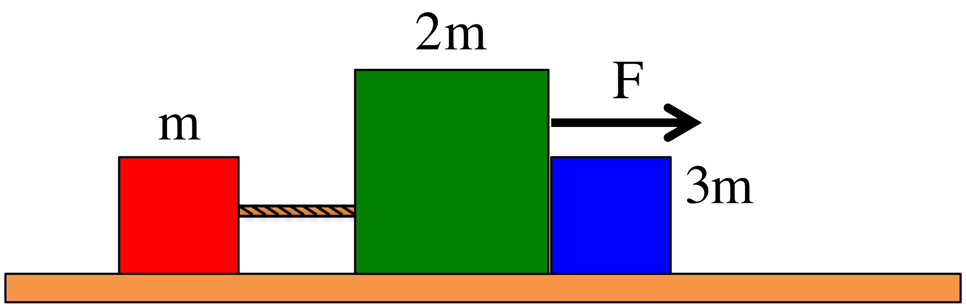
A free body diagram is a sketch upon which all of the forces acting on a body are identified. Each force DIRECTION is shown as an arrow and each force MAGNITUDE is presented algebraically. F.b.d.s are always oriented as they naturally exist in the problem (i.e., if the body is on a slant, the f.b.d. will be on a slant).

Example: draw the f.b.d. for the forces acting on the bowling ball (appreciation to Mr. White for the ball).



Note that the forces in non-rotational situations don't have to be drawn where they ACT, but do have to be oriented in the correct direction. More precise representations will be required when we begin to work with rotational motion

A Simple Newton's Second Law Problem



Consider: Three blocks of known mass, all in terms of "m," are arranged as shown. A *F =60 nt* force is applied.

What is the acceleration of the system?

The solution to this problem may or may not be obvious. There is a formal approach, though, that will always take you to a conclusion. Called *the formal approach*, it follows:

The Formal Approach,

According to Fletch

Step 0: Begin by thinking to yourself, “I couldn’t possibly do this problem.” Having acknowledged that, begin the process:

Step 1: Pick one object in the system and draw a free-body diagram (FBD) depicting all the forces acting on the body (options: gravity, tension friction, normal, push-me pull-you);

Step 2 and 3: Identify the line of the acceleration. Once identified, place a coordinate axis along that line and a coordinate axis perpendicular to that line;

Step 4: If there are any off-axis forces acting on the body, break them into components along the defined axes;

Step 5: Sum the forces, keeping track of signs, along one axis, and put that sum equal to “ma,” where “a” is the acceleration along that axis;

Step 6: If you have enough to solve, do so. If not, either sum the forces along the other direction or repeat the process on another body in the system.

Notes on the formal approach:

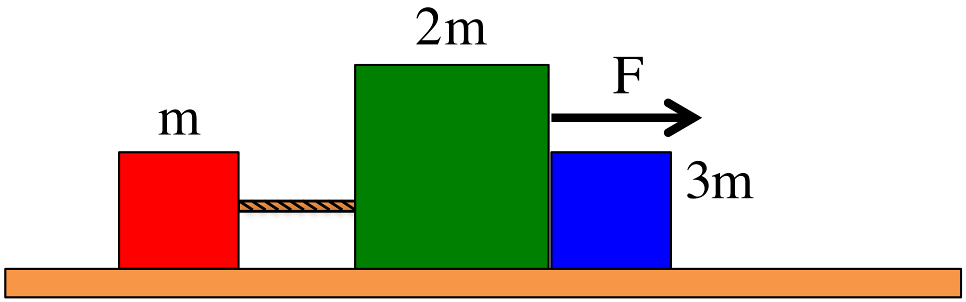
--*Assume nothing* that *can* be derived using f.b.d.s and N.S.L.

--*When in doubt,* once you have acknowledged the claim made in Step 0, *FOLLOW THE BLINKING PROCEDURE* and it will take care of you!!!!!!!!

--*It is important* that you consciously identify the *line of the acceleration* in each problem as there will come a time when dealing with bodies traveling around curved paths when that line will NOT be along the line of motion, and misidentifying the *line of acceleration* in those instances will be disastrous (more about that later).

--*You need* to understand how to use *the formal approach*, but there is another way, a shortcut *seat of the pant* approach, you can use to check yourself. It will be presented to you shortly.

So back at the ranch: Three blocks of known mass, all in terms of "m," are arranged as shown. A *F =60 nt* force is applied.

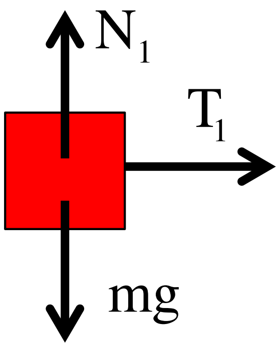


By the numbers:

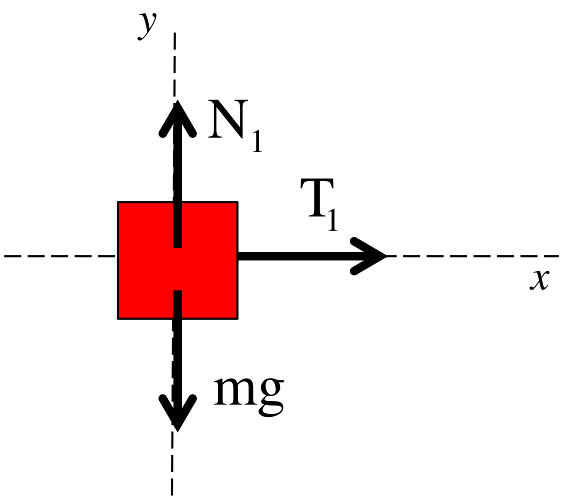
Step 0: Lordy, lordy, "I couldn't possibly figure this out."

Step 1: Pick one body in the system and draw a f.b.d. for it (blurbing well!).

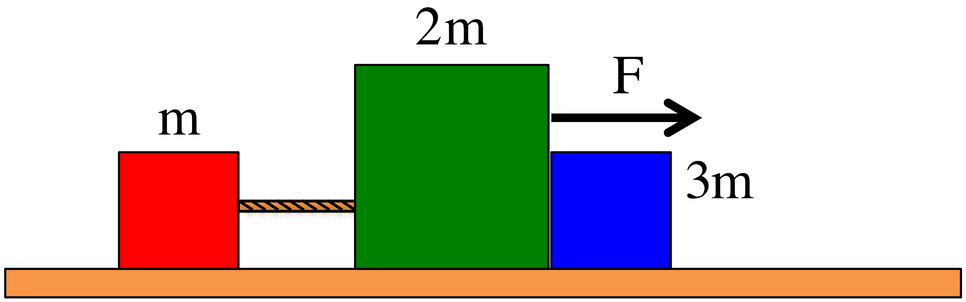
f.b.d. on "m"



Step 2: Identify the body's line of acceleration and put a coordinate axis along that line. Put a second axis perpendicular to the first.



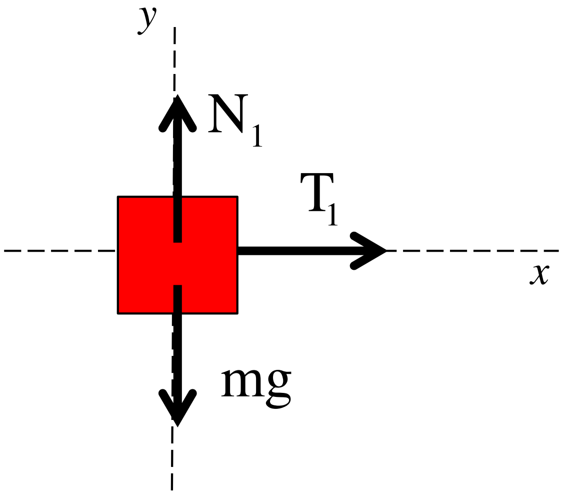
Step 3: If there are any off-axis forces, break them into components along your two coordinate axes. (There aren't any.)



Step 4: Sum the forces running along one of the axes and put that sum equal to the body's mass "m" times its acceleration along that line (include blurbs).

$$\underline{\sum F_x :}$$

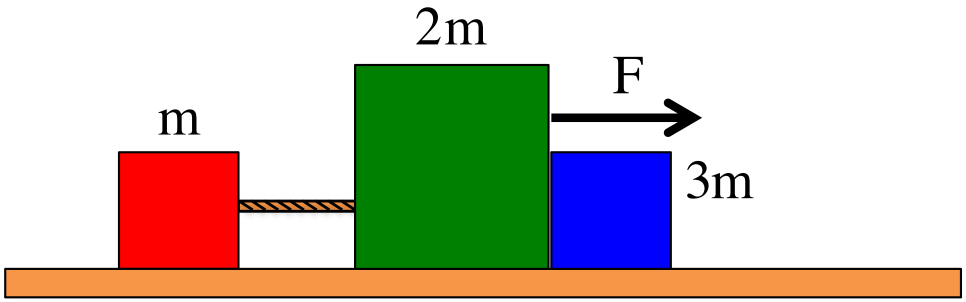
$$T_1 = ma$$



Note that with no friction in the system, summing along the y-direction is not needed.

Step 5: Repeat *the process* if you don't have enough information to solve the problem.

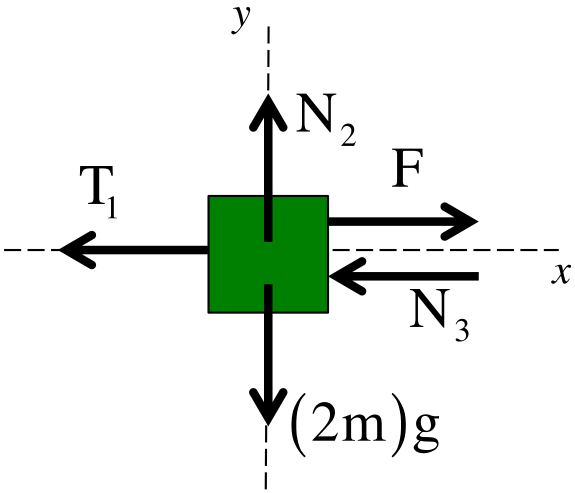
Steps 1, 2, 3 and 4: Pick another body and draw a f.b.d. and axes for it (blurbing well!). With no off-axis forces, sum the forces in the x-direction.



--Note that there is a *normal in the horizontal* due to *3m* being jammed up against *2m*, the force F acting *only* on the *2m* and a tension force on 2m.

--Note also that because T_1 was the *magnitude* of the tension force on *m*, I need to call the magnitude of that same tension acting on *2m* the *same variable*

f.b.d. on the 2m body

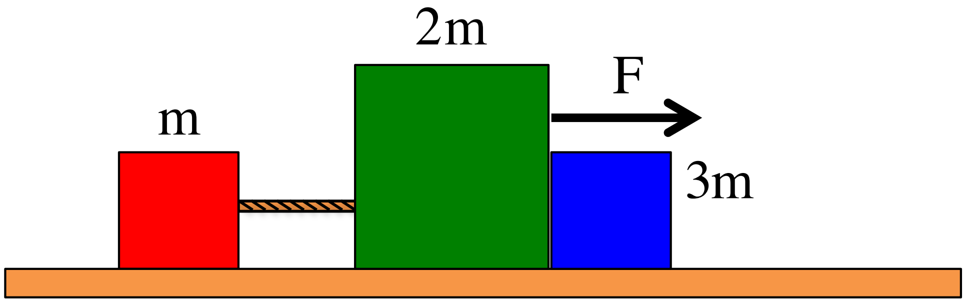


$$\underline{\sum F_x :}$$

$$-N_3 - T_1 + F = (2m)a$$

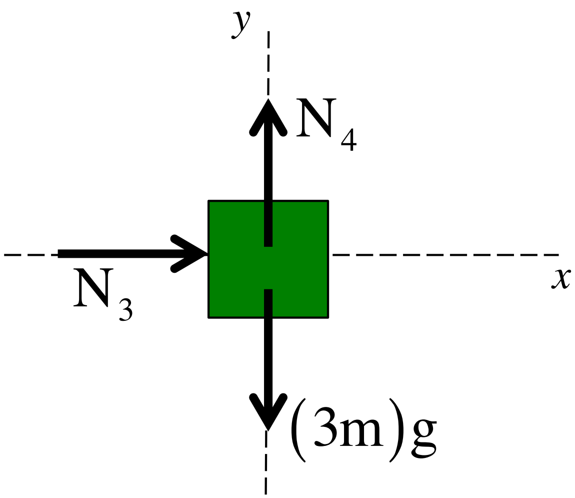
Still not enough . . .

Steps 1, 2, 3 and 4 again:



--Note that there is a *normal in the horizontal* due to *2m* being jammed up against *3m*. I've already identified the magnitude of that force as N_3 on the previous f.b.d., so I have to use that same designation on this f.b.d.!

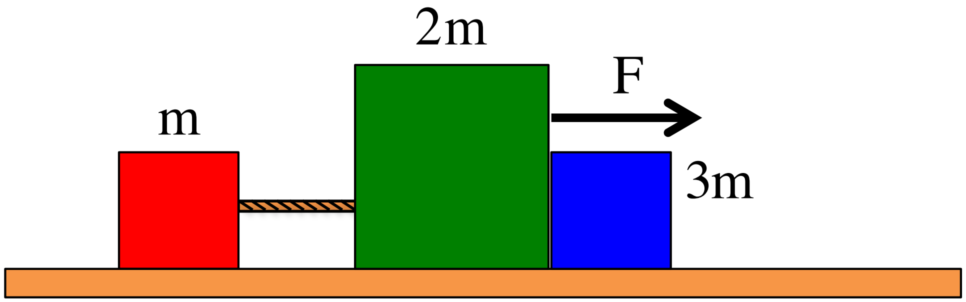
f.b.d. on the 3m body



$$\underline{\sum F_x :}$$

$$-N_3 = (3m)a$$

Now we need to solve equations simultaneously:



Our equations are:

$$T_1 = ma \quad \text{Equ. A}$$

$$-N_3 - T_1 + F = (2m)a \quad \text{Equ. B}$$

$$-N_3 = (3m)a \quad \text{Equ. C}$$

Substituting Equ's A and C into B yields:

$$-N_3 \;-\; T_1 \;+F = (2m)a$$
$$-(3ma)-(ma)+F=(2ma)$$
$$\Rightarrow F = 6ma$$
$$\Rightarrow a = \frac{F}{6m}$$

The "quick and dirty" approach

If the magnitude of the acceleration of all of the bodies in a system is the same, we can examine the system from a holistic perspective and the total, net force acting on the system in the direction of acceleration will equal the *total mass* in the system times the system's acceleration.

If you know how to use this approach, it can take a classic, five minutes, by the book Newton's Second Law problem and turn it into a 30 second romp.

The technique follows:

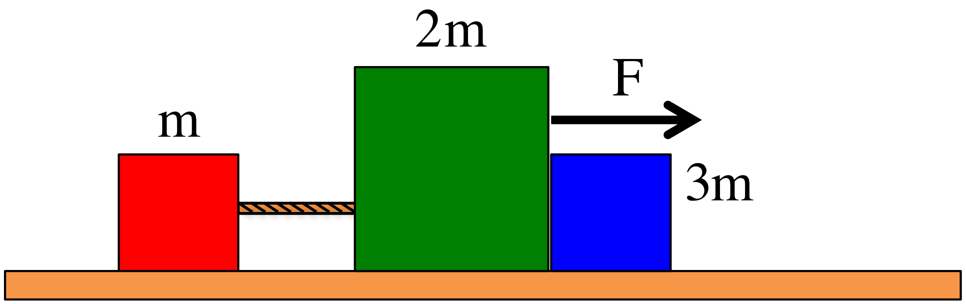
Step 1: Identify all the forces that help the system accelerate (an FBD may help).

Step 2: Define the forces that make the system accelerate in one direction as POSITIVE and the forces that make the system accelerate in the opposite direction as NEGATIVE.

Step 3: Sum the forces, signs included, and put them equal to the TOTAL MASS of the system times the acceleration of the system.

Step 4: Solve for the system's acceleration.

For Our Problem—Quick and Dirty



Step 1: The only force motivating the system to acceleration is F. All other forces acting are "internal" to the system.

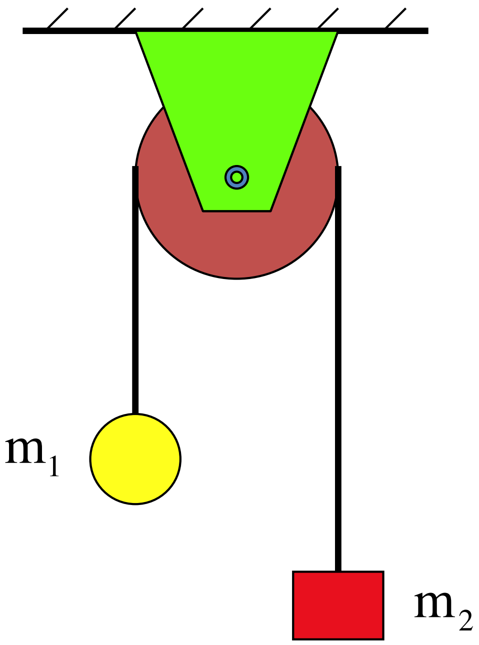
Step 2: As there is only one motivating force, we don't have to worry about assigning positiveness and negativeness to forces.

Step 3: Summing the motivating forces (signs included) and putting them equal to the TOTAL MASS OF THE SYSTEM times the *acceleration of the system* yields:

$$F = (m + 2m + 3m)a$$
$$\Rightarrow a = \frac{F}{6m}$$

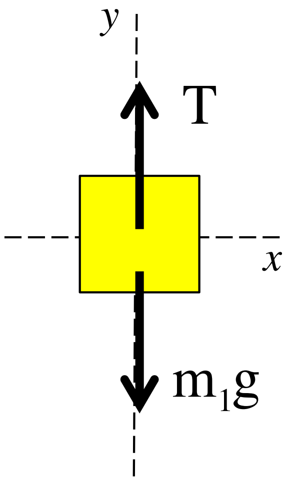
Like I said, quick and dirty!

Another Example: Called an Atwood Machine, a massless, frictionless pulley is suspended from the ceiling with a string threaded through it. Two unequal masses are attached to either end of the string and the system is allowed to free fall.



Without identifying the steps, but executing them:

f.b.d. on m_1



(from the f.b.d.)

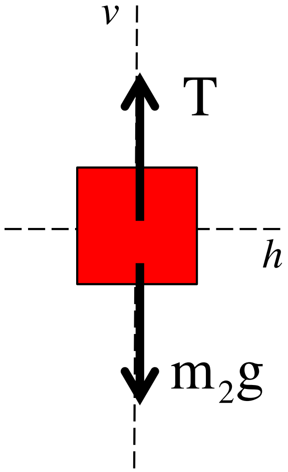
$$\underline{\sum F_y :}$$

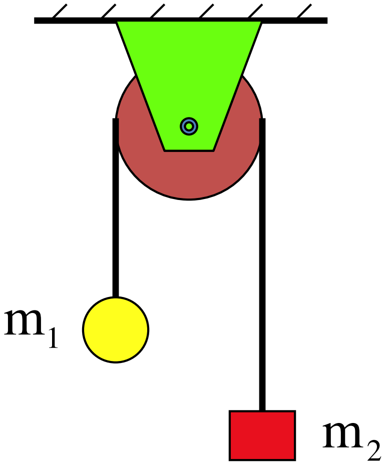
$$T - m_1 g = m_1 a$$

$$\Rightarrow \quad T = m_1 g + m_1 a$$

Big observation: Because +y is UP and "a" has been defined as + in our equation, we are assuming m_1 is accelerating UPWARD. This is important as it tells us m_2's acceleration direction.

f.b.d. on m_2





$$\underline{\sum F_v:}$$

$$T - m_2g = -m_2a$$

Another observation: Because the +v-direction is UP and "a" for m_2 has been deduced to be *down*, we must *unembed the negative sign* in the acceleration term so that "a" can be a magnitude.

Combining the previous equation $T = m_1g + m_1a$ with our new relationship:

$$T \qquad - m_2g = -m_2a$$

$$(m_1g + m_1a) - m_2g = -m_2a$$

$$\Rightarrow \quad m_1g - m_2g = -m_2a - m_1a$$

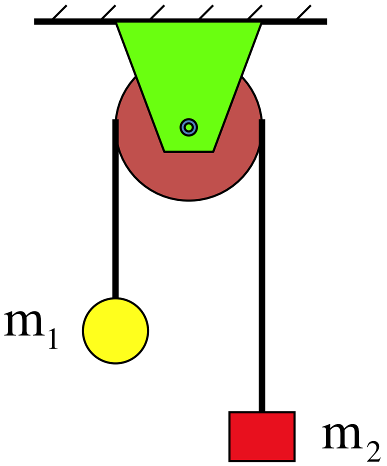
$$\Rightarrow \quad m_1g - m_2g = (-m_2 - m_1)a$$

$$\Rightarrow \quad a = \frac{m_1g - m_2g}{(-m_2 - m_1)}$$

Note 1: If you put in numbers and solve to get a negative "a," it just means you've assumed the wrong direction for "a."

Note 2: If the numerator's variables have different signs, you've messed up one of the "ma" signs when you used N.S.L.

Quick and dirty:

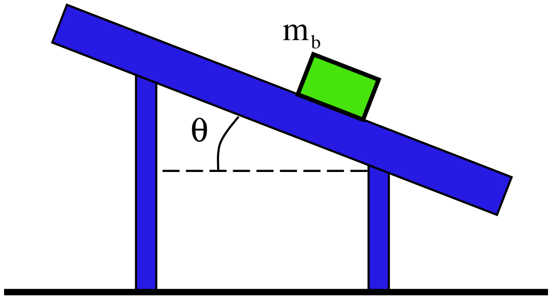


$$m_1g - m_2g = (m_1 + m_2)a$$

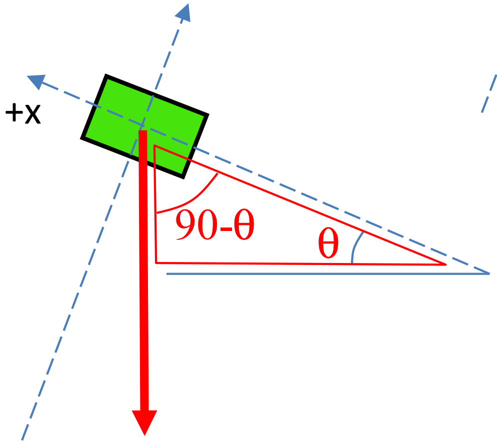
$$\Rightarrow \quad a = \frac{m_1g - m_2g}{(m_1 + m_2)}$$

Note: I’ve picked the mass I think will dominate and made its acceleration positive (even though that acceleration in this case will be downward). If, once numbers have been included, the math turns out to yield a negative acceleration, the negative sign simply means I have assumed the wrong direction for the acceleration and the opposite direction is the correct one. Nothing needs to be done beyond a statement to that effect.

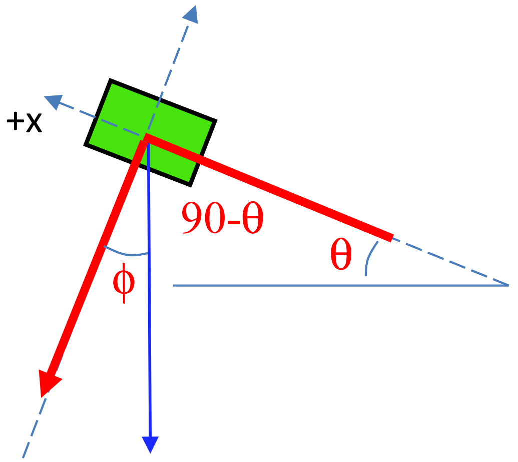
Note about inclines and angles: A block of mass m sits stationary on a slanted table of known angle θ. For whatever reason, you decide you want to know the component of gravity acting on the block *along the line of the table*, and the component of gravity *perpendicular to the table*. How would the incline's angle fit into that calculation?



A right triangle with θ in it and one side in the vertical looks like:



A normal (in red) at right-angles to the table looks like,

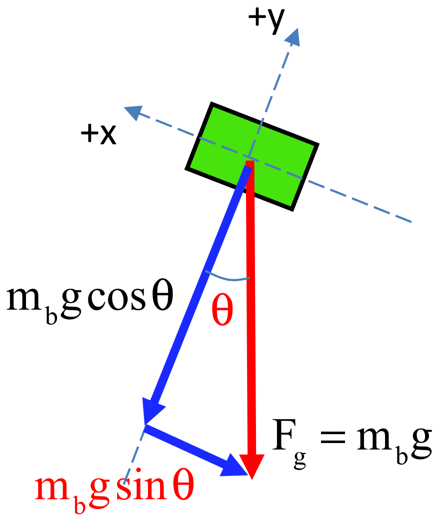


so summing angles (see diagram) yields:

$$90^{\circ} = \phi + \left(90^{\circ} - \theta\right)$$

$$\Rightarrow \quad \phi = \theta$$

and our force diagram looks like:

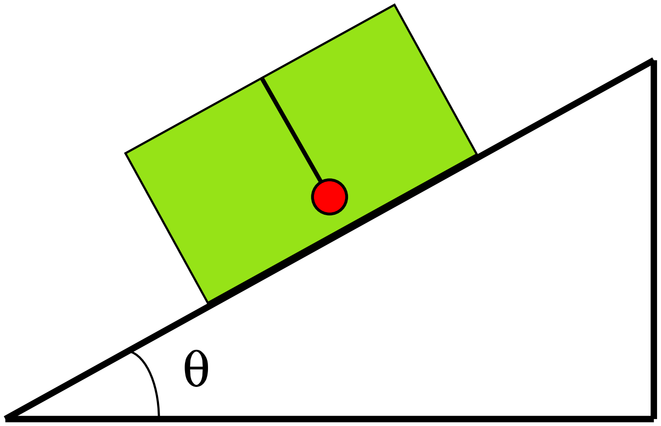


with components (as shown):

Conclusion:

For incline problems, the incline's angle θ will always be found between the vertical and the normal to the incline.

The car on the incline goes from zero to 30 m/s in 6 seconds. The 0.1kg toy hangs at 90° to the incline.



a.) In what direction is the car accelerating?

Think about what the toy would be doing if the car was stationary, then accelerated up the incline . . . (The answer is *down the incline*.)

b.) What is the incline's angle and the tension in the line?

$$\sum F_y :$$

$$T - mg\cos\theta = m\overset{0}{\cancel{a_y}}$$

$$\Rightarrow \quad T = mg\cos\theta$$

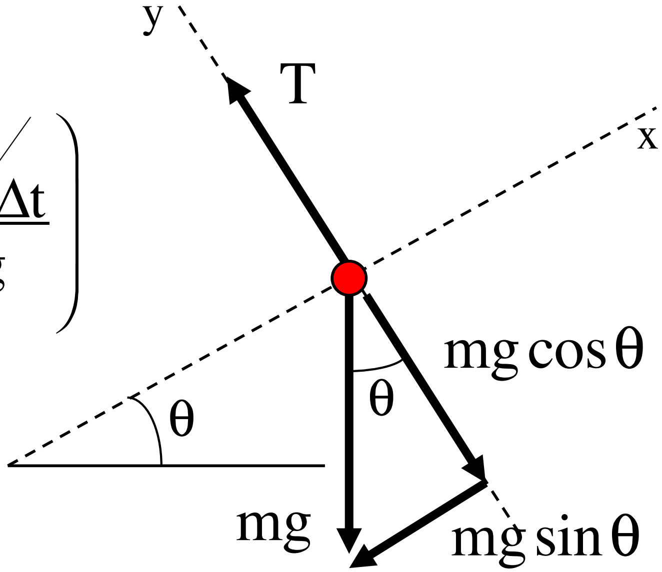
$$\sum F_x :$$

$$-mg\sin\theta = -ma$$

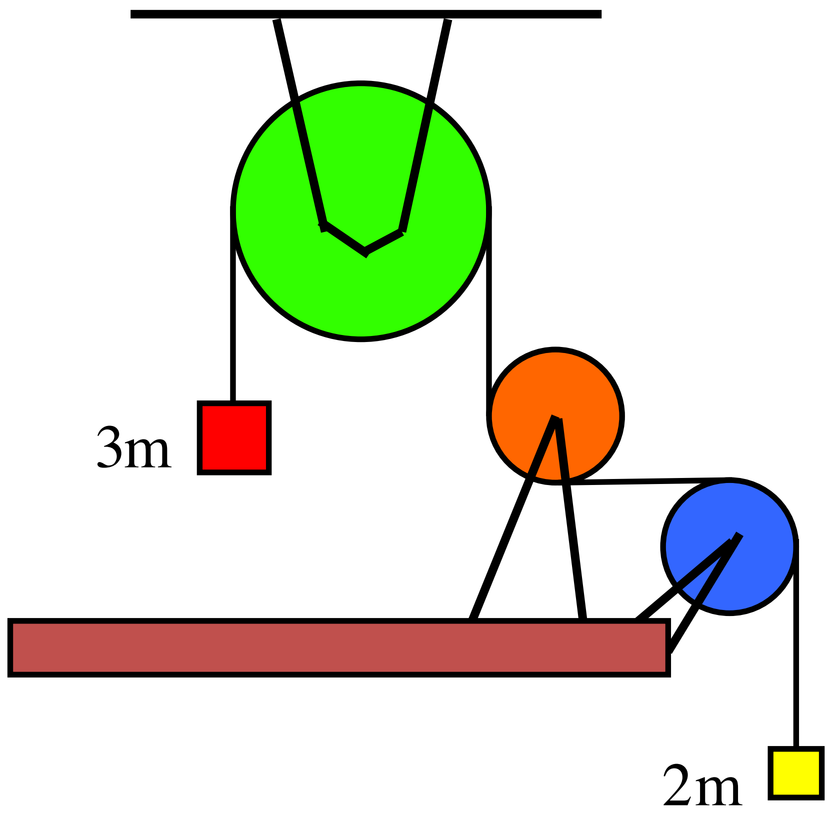
$$\Rightarrow \quad \theta = \sin^{-1}\left(a/g\right) = \sin^{-1}\left(\frac{v_f/\Delta t}{g}\right)$$

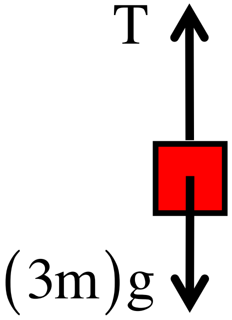
$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{30\ \text{m/s}/6\ \text{s}}{9.8\ \text{m/s}^2}\right)$$

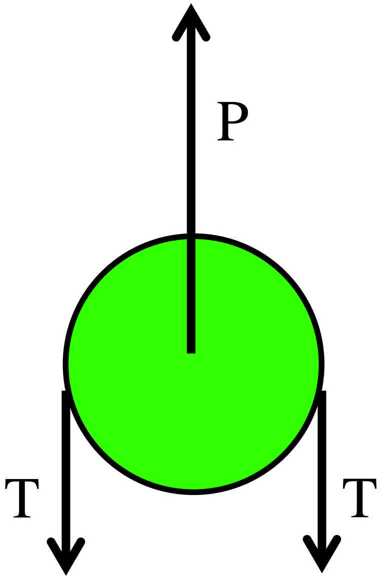
$$= 30.7^\circ$$

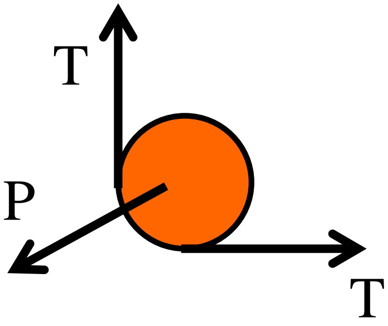


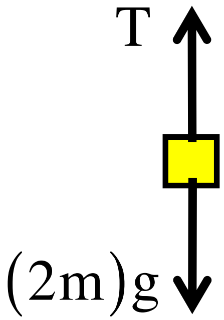
Draw f.b.d.s for each of the *masses* and *massless pulleys* below.

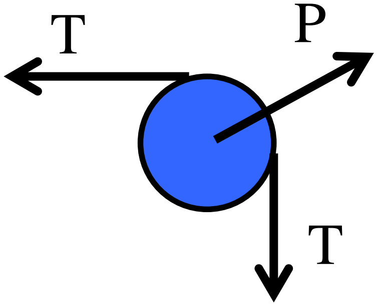






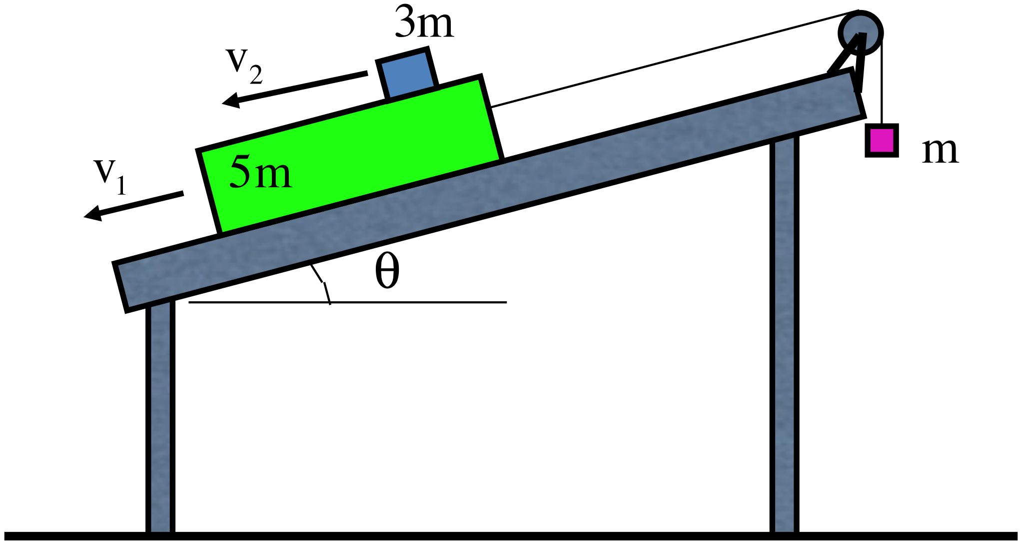


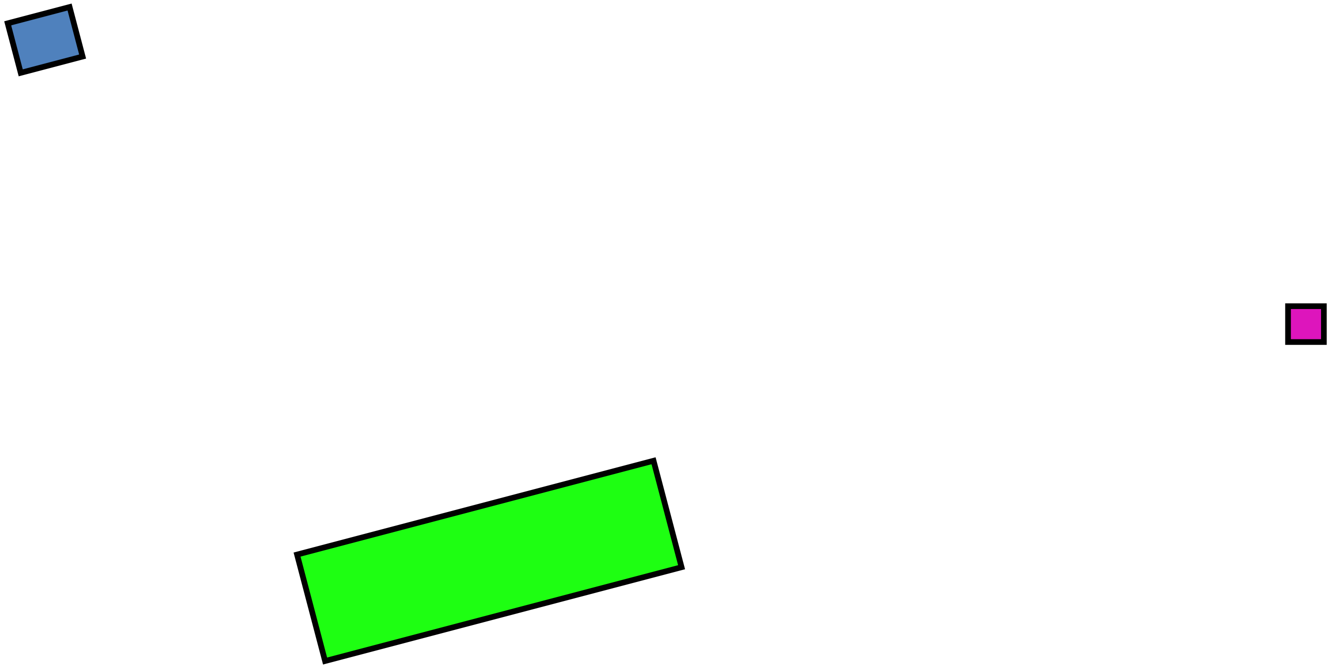




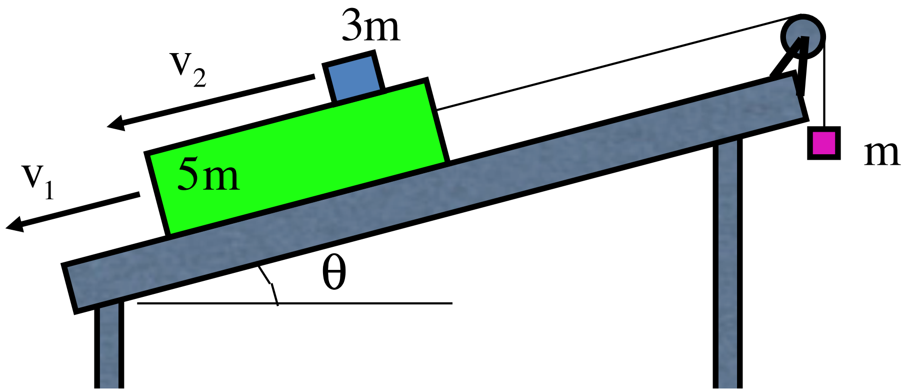
13.)

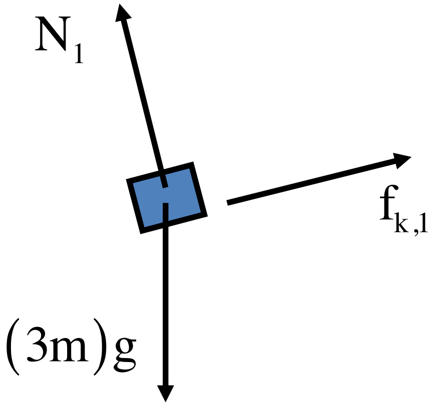
frictional on both surfaces; (velocities of 5m and 3m masses are down incline--acceleration assumed UP the incline with 3m mass *breaking loose*)

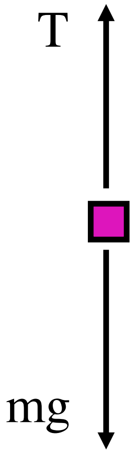




Friction on both surfaces; (velocity of 5m and 3m masses are down incline—acceleration assumed UP incline and 3m mass has broken loose). Do f.b.d.s on all bodies

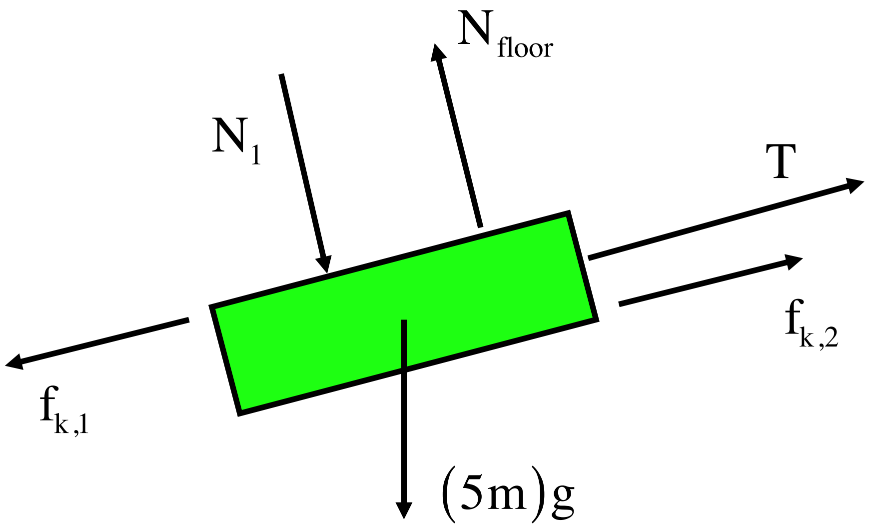






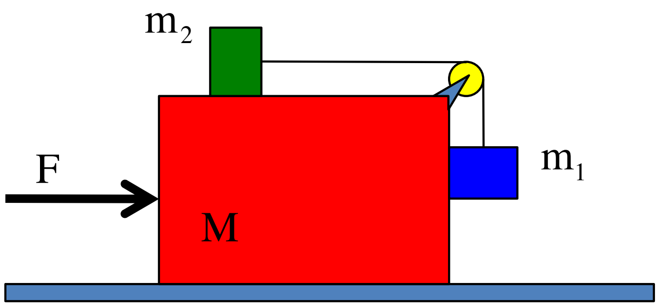
Comment: You are told that the acceleration of the 3m blocks is UP the incline. Gravity will provide a component of force *down* the incline, so where does the force *up* the incline come from?

It *has* to come from friction between the 3m and 5m masses. This observation is important because it tells you the direction of the frictional force on the 3m mass.

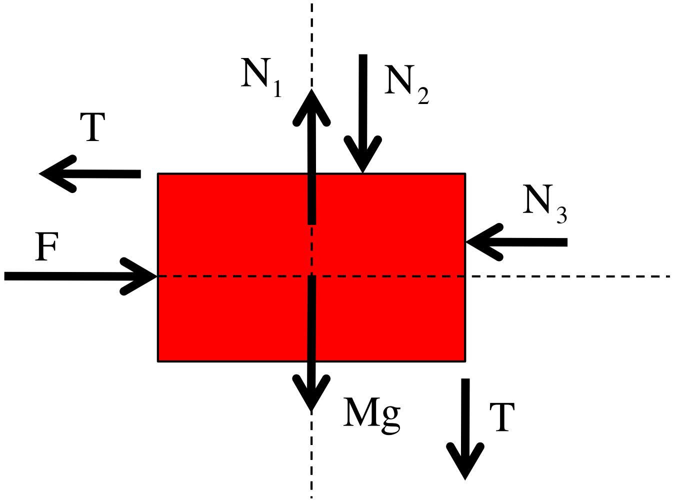


Comment: If the 5m mass produces a frictional force on the 3m mass up the incline, the 3m mass must produce an equal frictional force on the 5m mass *down* the incline. There are a total of 6 forces on this f.b.d.

What force F will keep m_2 stationary with respect to M?



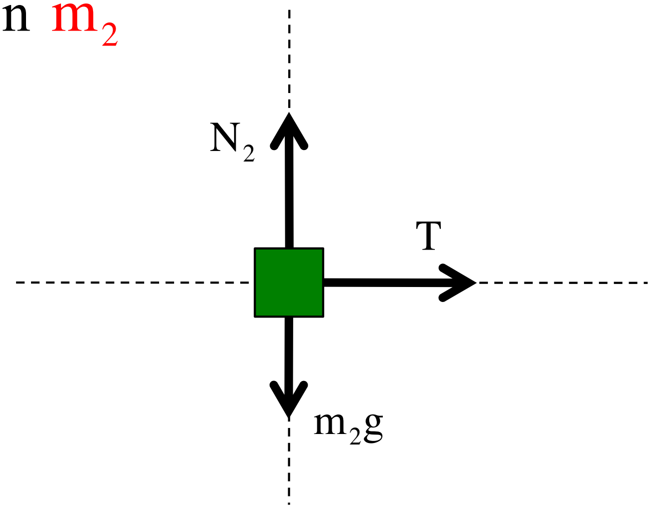
f.b.d. on M

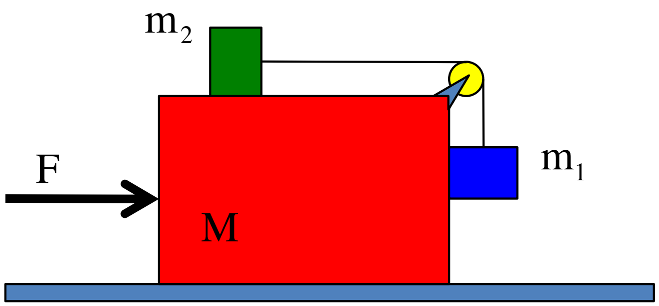


$$\sum F_x :$$

$$F - T - N_3 = Ma$$

f.b.d. on m_2

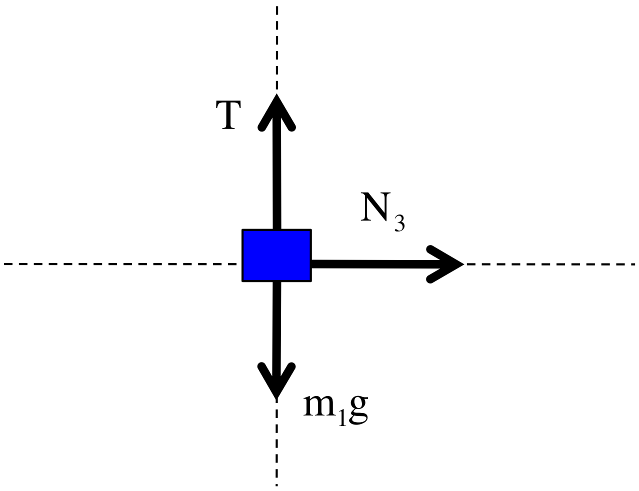




$$\sum F_x :$$

$$T = m_2 a$$

f.b.d. on m_1



$$\sum F_y :$$

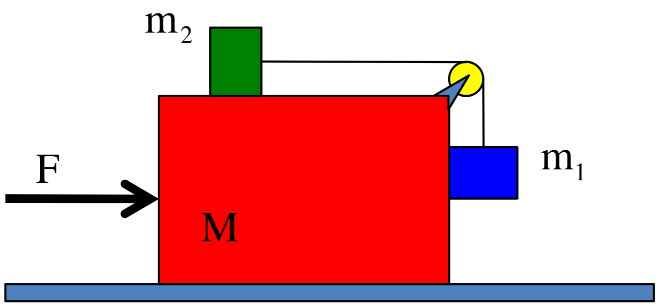
$$T - m_1 g = m_1 \cancelto{0}{a}$$

$$\Rightarrow T = m_1 g$$

$$\sum F_x :$$

$$N_3 = m_1 a$$

Accumulating and combining the equations:



$$N_3 = m_1 a$$

$$T = m_2 a$$

$$T = m_1 g$$

$$\Rightarrow \quad (m_2 a) = m_1 g$$

$$\Rightarrow \quad a = \frac{m_1}{m_2} g$$

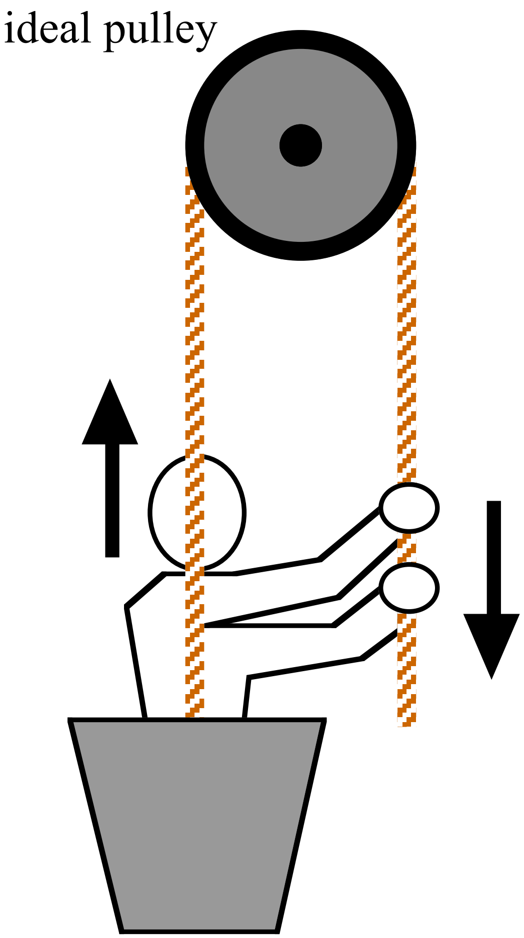
$$F - T - N_3 = Ma$$

$$\Rightarrow \quad F - m_2 a - m_1 a = Ma$$

$$\Rightarrow \quad F = (M + m_2 + m_1) a$$

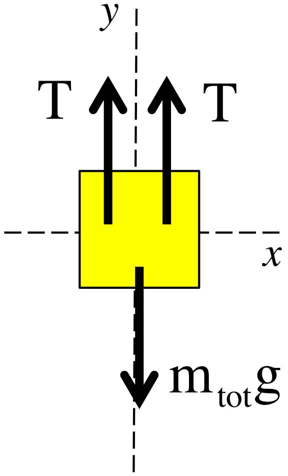
$$\Rightarrow \quad F = (M + m_2 + m_1)\left(\frac{m_1}{m_2} g\right)$$

One last bit of nastiness with a nice picture courtesy of Mr. White. An individual in a bucket pulls herself upward with constant acceleration. How much pulling force must she apply to do this?



Getting the f.b.d. right, which should view the person and bucket as one, is crucial here. Think about what is happening. Gravity is acting on the two, and there are TWO tensions acting away from contact points (i.e., upward).

f.b.d. on m_{total}



from f.b.d.

$$\underline{\sum F_y :}$$

$$T + T - m_{tot}g = m_{tot}a$$

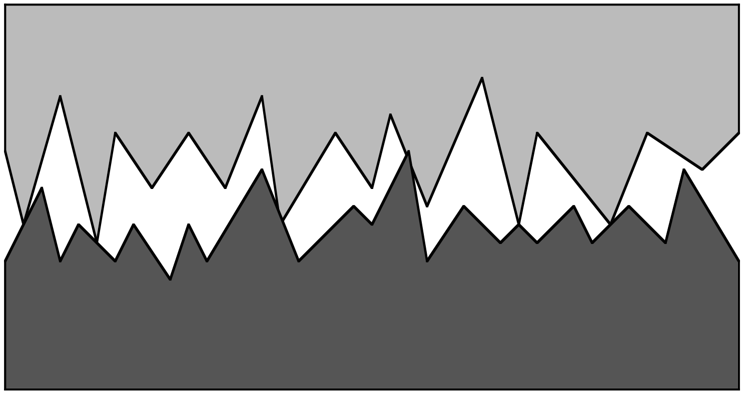
$$\Rightarrow \quad T = \frac{m_{tot}g}{2}$$

Kinetic and Static frictional force:

Time to talk friction. As I said above, there are two types that we are interested in. Kinetic friction occurs when two bodies are in contact and one is moving relative to the other (think *pushing a box across a floor*). Static friction occurs when two bodies are in contact but are not slipping, relative to one another (think *holding traction* as you drive through a curve on a freeway).

There are lots of ways friction can be generated. A dragster, for instance, literally melts its tires, creating a "scotch tape dragged across a surface" effect. That model is NOT going to be ours.

Ours acknowledges the fact that when two objects are in contact with one another, their molecular and atomic structures both jam up against one another and, to some degree, meld into one another. *Shearing that meld* (or attempting to do so) is what causes friction.

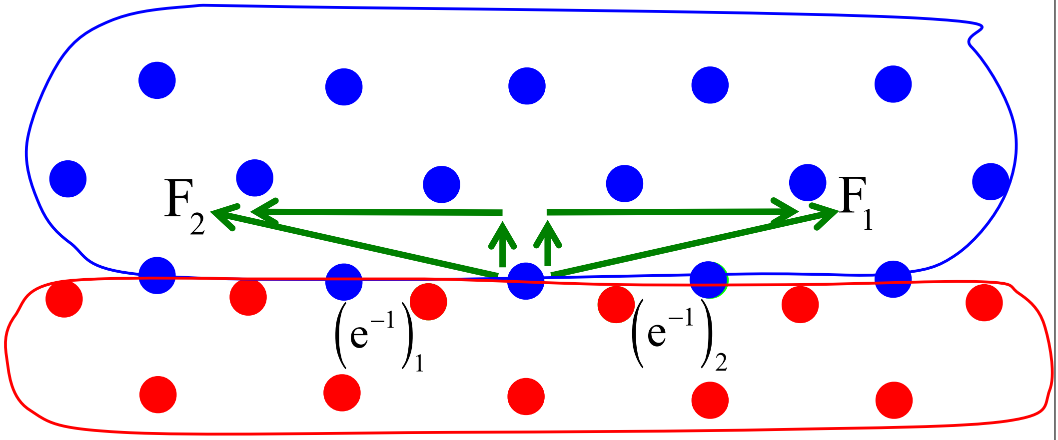


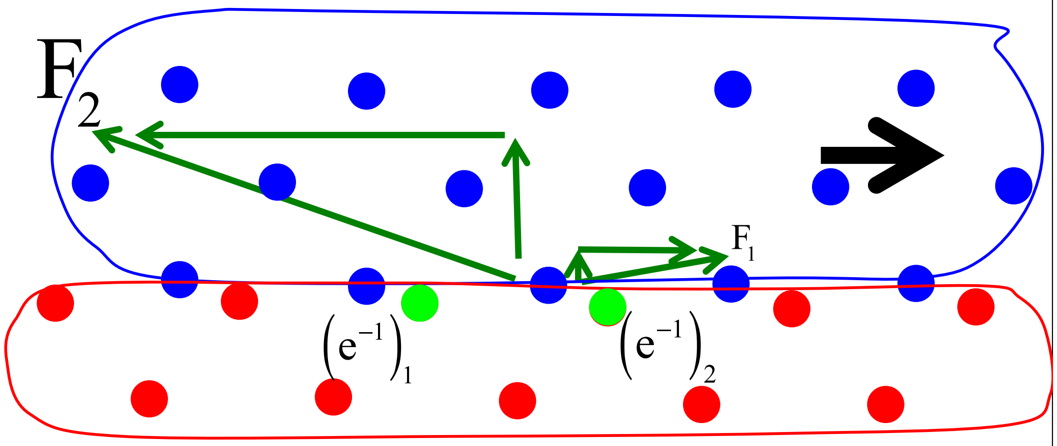
A little closer look is instructive.

As the electrons of the upper object (in blue) nestle into the electron configuration of the lower object (in red), they apply a repulsive force to one another (see sketch).

The horizontal components add to zero. The vertical components produce the normal force that supports the upper object.

But try to move the upper body to the right and the horizontal components will no longer cancel.





This net horizontal force is known as the *static frictional force* between the two bodies. It is the force that has to be overcome before the upper body can actually accelerate to the right. Put a little differently, for the top body to accelerate, an external force to the right that is large enough to effectively shearing the repulsive bonds that exist between electrons has to be applied.

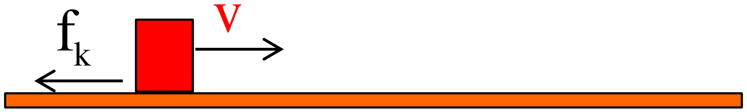
KINETIC FRICTION

--when objects are in contact and moving relative to one another, the shearing of the partial bonding between the two produces a force that is always parallel to the surfaces and directed *opposite the direction of RELATIVE MOTION between the two bodies*

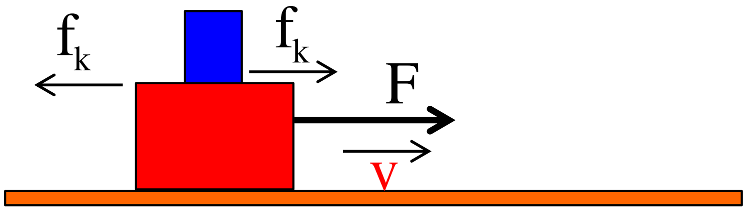
--its magnitude is proportional to the amount of melding, which is measured by the normal force, between the two masses, and is denoted by and is equal to:

$$f_k = \mu_k N$$

where μ_k is a constant called the *coefficient of kinetic friction* and N is the magnitude of the normal force acting between the surfaces.



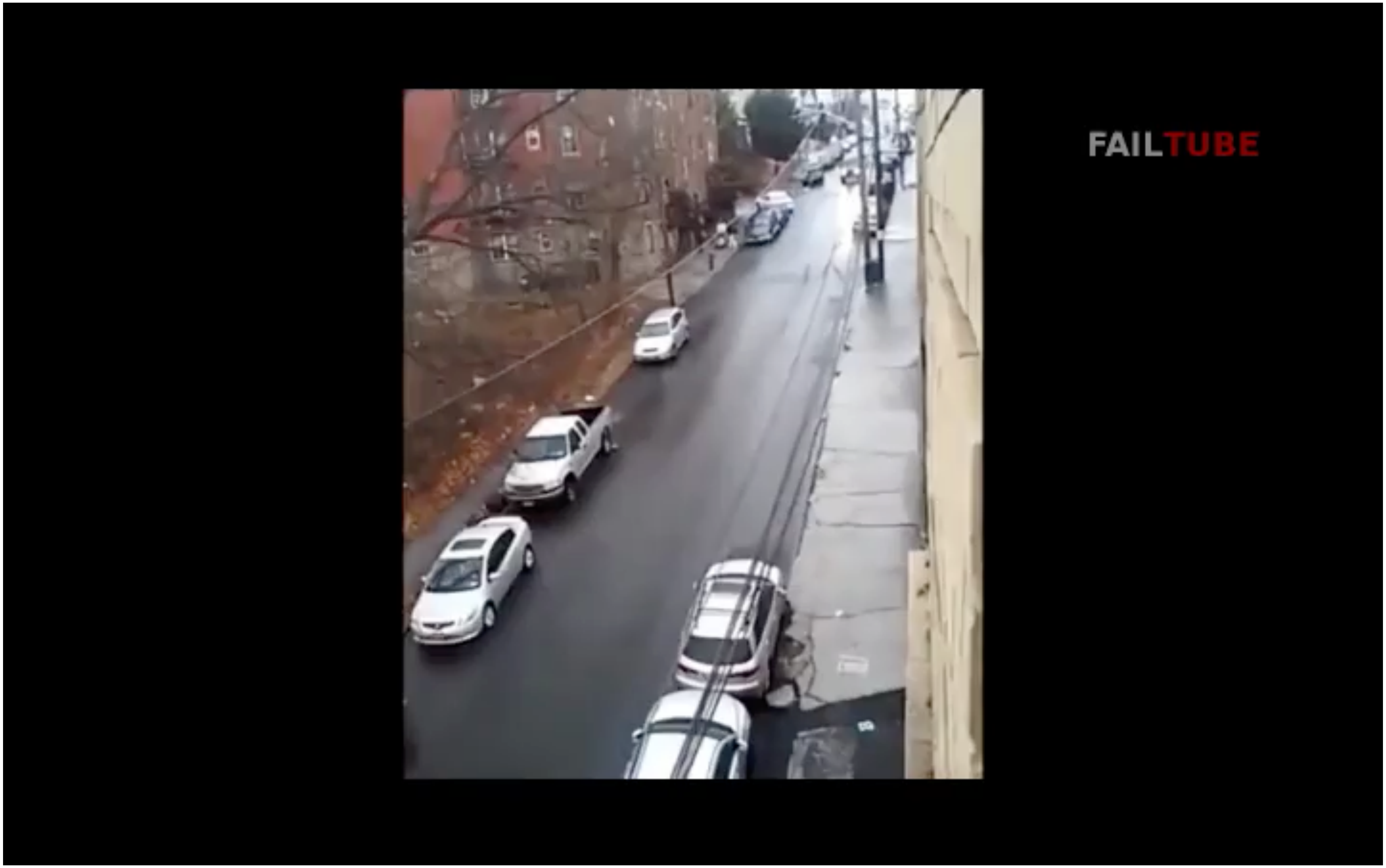
the mass's motion relative to table is to right, so *kinetic frictional force* to left



A force F motivates the red mass to the right. As the red mass tries to slide out from under the blue mass, the blue mass moves to the left RELATIVE TO THE RED MASS. As such, the shearing of the melding between the two masses will produce a *kinetic frictional force* on the blue mass *to the right* . It will ALSO, due to Newton's Third, produce a kinetic frictional force on the red mass *to the left*.

when kinetic friction goes bad . . .

https://youtu.be/DrOO_HcQngg start 0:48 to 1:03



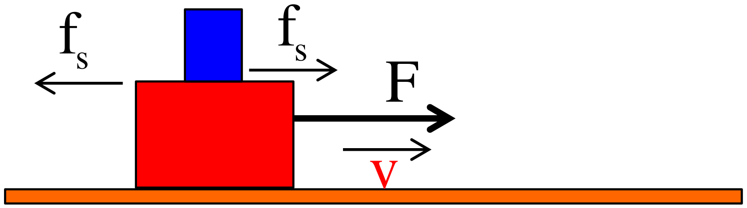
STATIC FRICTION

--if the melding between two objects in contact is great enough, the shearing required for the bodies to break loose won't happen. The stress generated by that opposition produces a force that is parallel to the surfaces and directed *opposite the direction of RELATIVE MOTION the bodies WOULD experience if they broke loose.* There is a continuum of static frictional forces from zero to the break-loose point.

--the magnitude of the MAXIMUM static frictional force is:

$$f_{s,max} = \mu_s N$$

where μ_s is a constant called the *coefficient of kinetic friction* and N is the magnitude of the normal force acting between the surfaces.

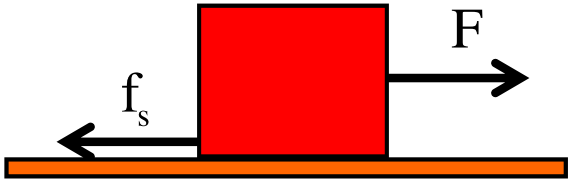


A force F motivates the *red mass* to the right. As the red mass tries to slide out from under the *blue mass*, the blue mass holds on, so to speak, being motivated to the right by the *static frictional force* between the two bodies. How so? If the masses broke loose as the red mass moved right, the blue mass would move left, relative to the red mass (the red mass would slide out underneath the blue mass). What keeps the blue mass from moving leftward, relative to the red guy, is the static frictional force between them *to the RIGHT*.

Additionally, due to Newton's Third, an equal and opposite static frictional force will be applied to the red mass to the left, retarding its motion.

when static friction goes bad . . .

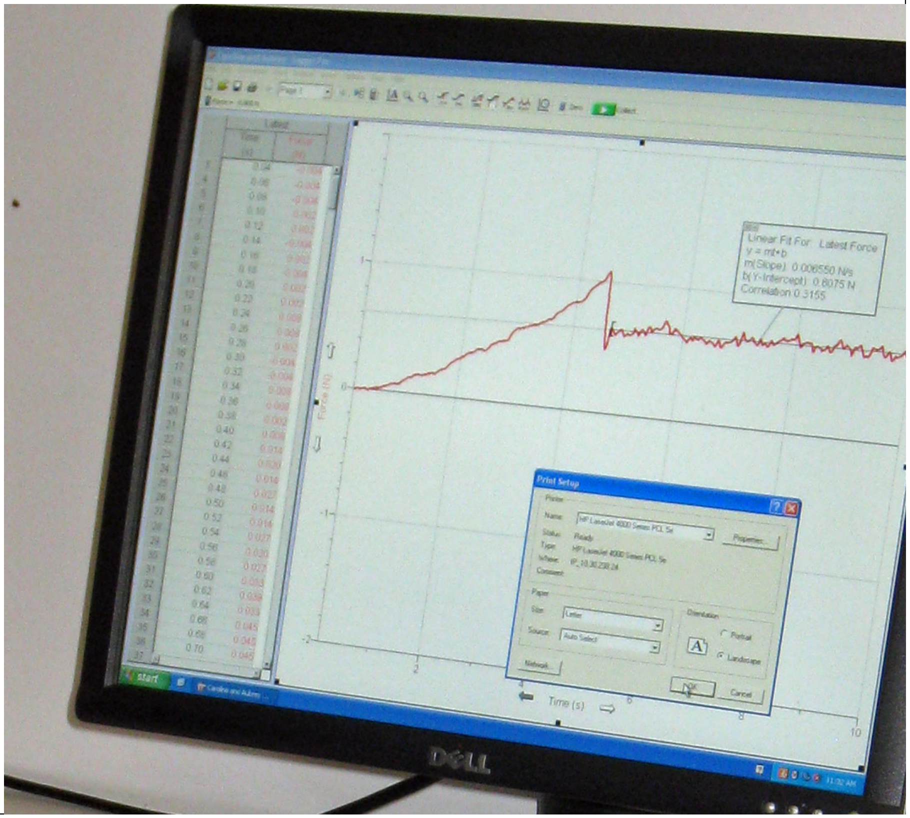
General observation about *kinetic* and *static friction*: Very gently apply a force F to a box sitting stationary on a surface. If the box does not accelerate, it means the static friction is holding it in place. In other words, the force you applied is *equal and opposite* the static frictional force generated between the surfaces.



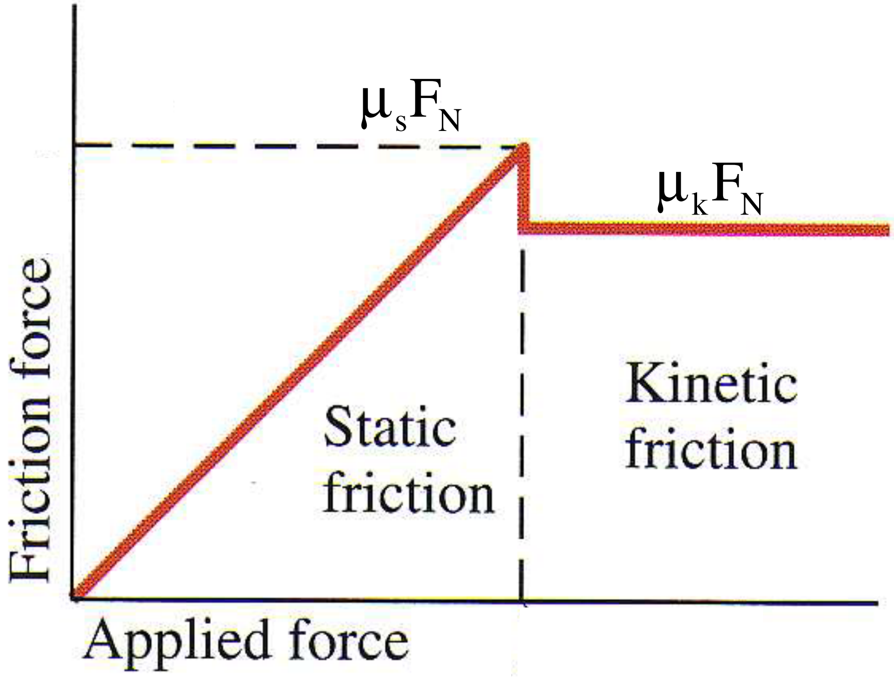
$$f_s - F = m\overset{0}{\cancel{a}}$$

$$\Rightarrow \quad f_s = F$$

If you begin to increase F, at some point the static frictional force will cease to hold, you will have reached the *maximum* static frictional force $f_{s,max} = \mu_s N$ and the box will break loose and begin to slide. At that point, *kinetic friction* will take over and the frictional force on the body will be a constant $f_k = \mu_k N$ (unless you change the characteristics of the surfaces by heating or melting or whatever, which we will assume you won't do).



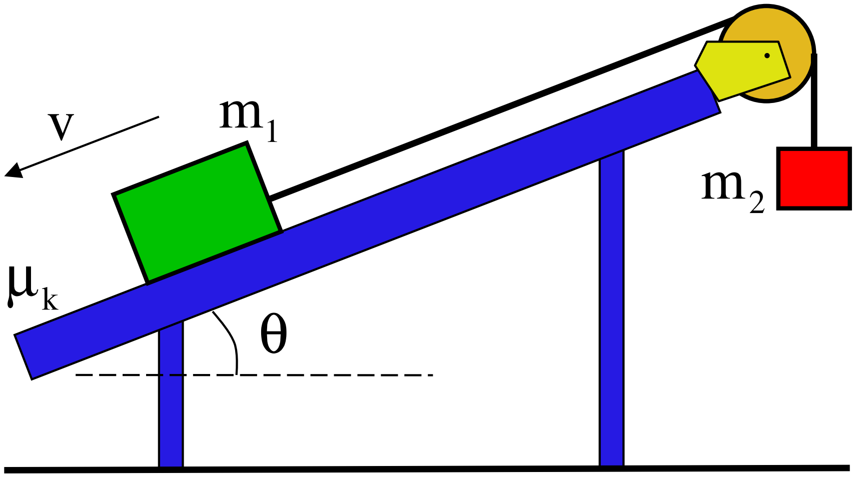
The point is that the two frictional forces are not the same critter. One, for kinetic friction, is constant and always acts while there is sliding between the two surfaces (which makes the denotation for static friction, f_s, unfortunate, as people take the "s" subscript to refer to *sliding* instead of *static*). There are an infinite number of *static* frictional forces with only the MAXIMUM being of much interest, as that is the one that is equal to $\mu_s N$. In any case, the two are summarized graphically (courtesy of Mr. White), below



And as a side point, pretty clearly $\mu_s > \mu_k$ is always true for a given surface.

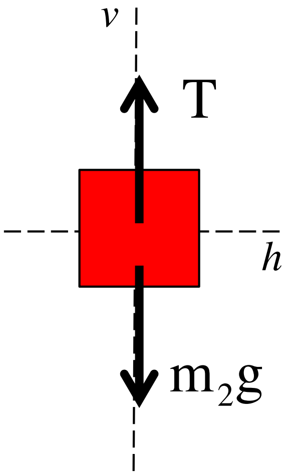
An Example With Friction:

A block on a *frictional incline* of known angle and coefficient of friction is attached to a string that is threaded over a massless, frictionless pulley. The string is attached at its other end to a second mass (see sketch). What is the acceleration of the system?



Without listing the steps, but following them:

f.b.d. on m_2



$$\underline{\sum F_y :}$$

$$T - m_2g = m_2a$$

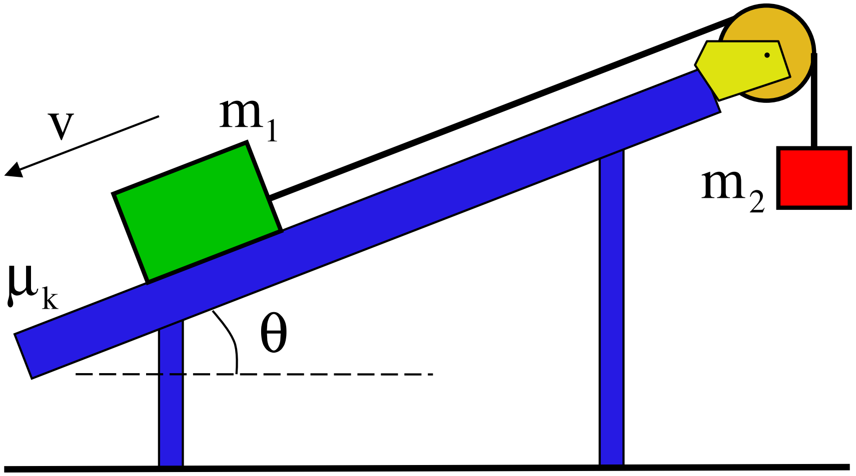
$$\Rightarrow \quad T = m_2g + m_2a$$

Observation: I'm denoting the *vertical axis* with a "*v*" and the horizontal axis with a "*h*."

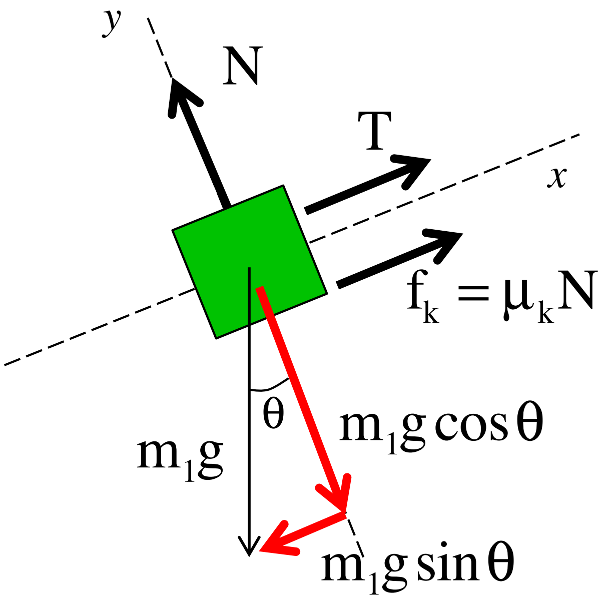
Observation: Because the pulley is massless, the tension T is the same on either side. That is, all the pulley does is redirect the *line of force* due to tension.

Big observation: Because +v is UP and "a" has been defined as + in our equation, we are assuming m_2 is accelerating UPWARD. This means m_1 must be accelerating *down* the incline.

Because m_1 is on a slant, it's f.b.d. must be on a slant. Also, remember that "T" is the same on both sides of the pulley.



f.b.d. on m_1



(from looking at the f.b.d.)

$$\underline{\sum F_y :}$$

$$N - m_1 g \cos\theta = m_1 \cancelto{0}{a_y}$$

$$\Rightarrow \quad N = m_1 g \cos\theta$$

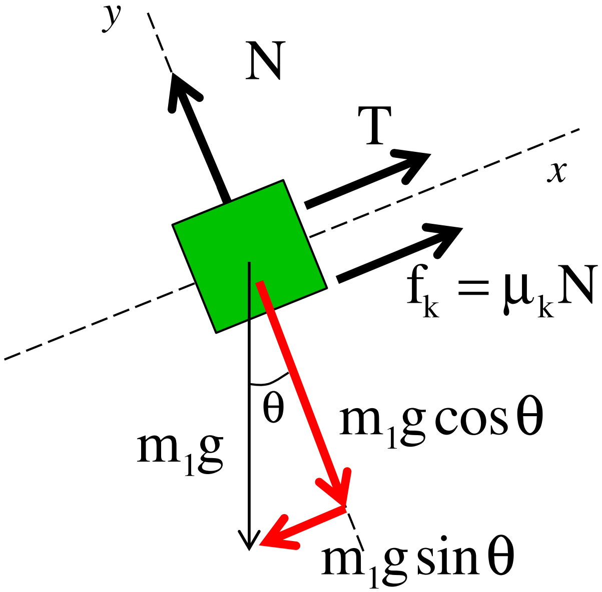
Observation: Notice the f.b.d. is tilted in a manner similar to the block on the incline. This is standard procedure.

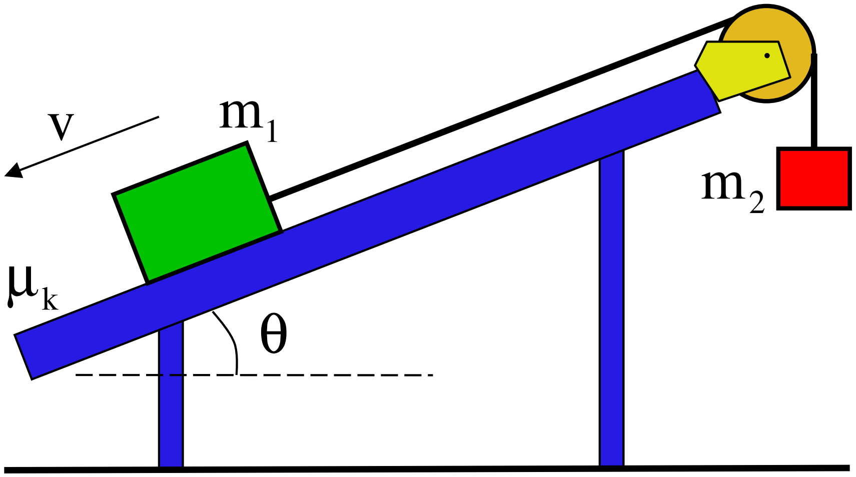
Observation: Notice how the angle in the "mg" force triangle is related to the incline's angle.

Observation: Notice I'm assuming m_1's acceleration is down the incline because I assumed m_2's acceleration was UP, NOT because the m_1 is moving down the incline.

Observation: Notice I needed to know the direction of m_1's VELOCITY (not its acceleration) to determine the direction of the *kinetic frictional force* on it, which will be OPPOSITE the (relative) motion.

f.b.d. on m_1





From looking at the f.b.d., and noting that the acceleration of m_1 is in the *negative direction*, as defined by our coordinate axis, we can write (with substitutions):

$$\underline{\sum F_x}:$$

$$\mu_k \quad N \quad + \quad T \quad - m_1 g \sin\theta = -m_1 a$$

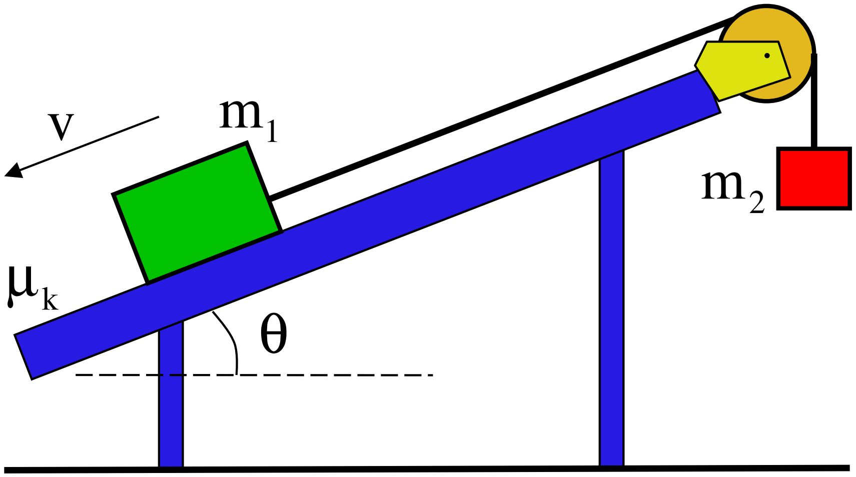
$$\mu_k \left(m_1 g \cos\theta\right) + \left(m_2 g + m_2 a\right) - m_1 g \sin\theta = -m_1 a$$

$$\Rightarrow \quad \mu_k m_1 g \cos\theta + m_2 g - m_1 g \sin\theta = -m_2 a - m_1 a$$

$$\Rightarrow \quad a = \frac{\mu_k m_1 g \cos\theta + m_2 g - m_1 g \sin\theta}{-m_2 - m_1}$$

Quick and Dirty!

Showing more steps than are needed (but doing so to be complete), and remembering we are adding up all the forces that actively motivate the system to accelerate in one direction and subtracting those that actively motivate the system to accelerate in the other (then putting that equal to the total mass times acceleration), we get:



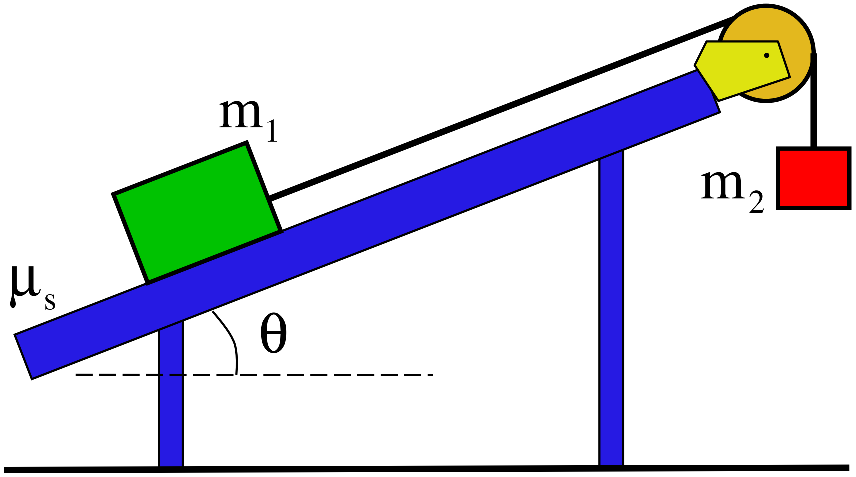
$$m_2 g + \quad f_k \quad - m_1 g \sin\theta = (m_1 + m_2)a$$

$$m_2 g + u_k \quad N \quad - m_1 g \sin\theta = (m_1 + m_2)a$$

$$m_2 g + u_k (m_1 g \cos\theta) - m_1 g \sin\theta = (m_1 + m_2)a$$

$$\Rightarrow \quad a = \frac{m_2 g + u_k (m_1 g \cos\theta) - m_1 g \sin\theta}{(m_1 + m_2)}$$

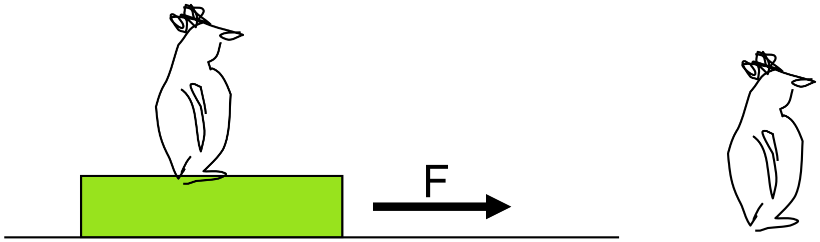
As a slight twist: Same problem, but now you are told that the static frictional force between m_1 and the incline is just large enough to keep m_1 from breaking loose. What can you deduce about the system and what additional information would you need (or would you have to assume) before you could derive an expression for that coefficient of static friction? Is this a big deal?



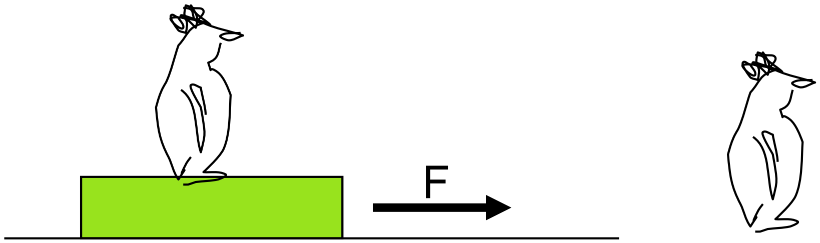
To determine the direction of the static frictional force on the f.b.d., you need to know the direction the body would move if it DID break loose. If m_2 was tiny and the incline's angle large, m_1 would be tugging to accelerate DOWN the incline and the static frictional force would fight that, being oriented UP the incline. If m_2 was large the angle not too big, m_1 would be tugged UP the incline and the static frictional force would be DOWN the incline. Knowing which way the body will be tugged defines the direction of the static frictional force (i.e., opposite the tug). Also, with $a = 0$, the tension T is just the weight of m_2.

Does it matter? You bet. If you do the f.b.d. both ways and use N.S.L. with the wrong direction for f_s, you get different numerically values for μ_s. This makes sense as in the one case, friction will be working with gravity, and in the other, not.

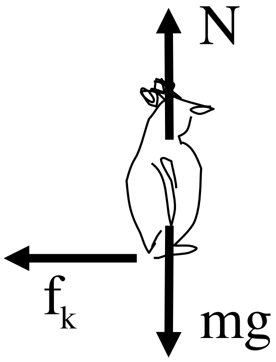
A penguin (which took me a seriously long time to draw) sits on a slightly frictional block. When a large force F is applied to the block, the penguin breaks traction and slides. Draw a f.b.d. for the forces acting on the penguin.



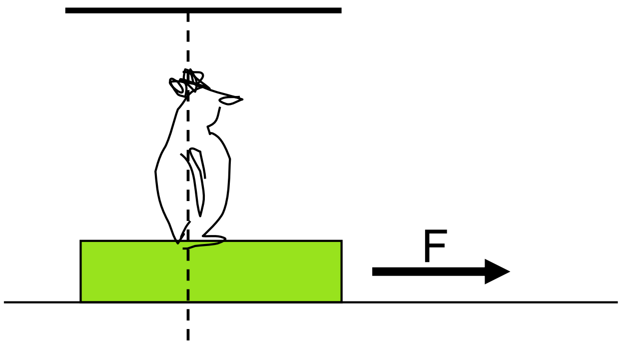
A penguin (which took me a seriously long time to draw) sits on a slightly frictional block. When a large force F is applied to the block, the penguin breaks traction and slides. Draw a f.b.d. for the forces acting on the penguin.

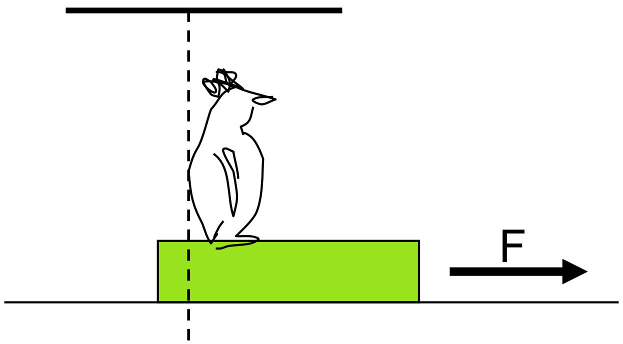


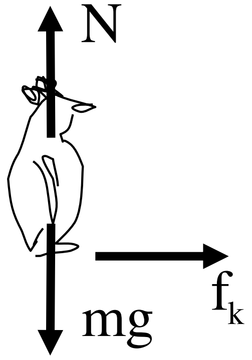
Some will notice that the block moves right, so they will automatically put the frictional force to the left (this will be reinforced by the fact that the penguin will sooner or later slide off the back side of the block). Problem is, relative to the ground—an inertial frame of reference—the penguin is being dragged (accelerated) to the right (see sketches below), which means you need a force to the right. The only force available to do that is friction. Gotta be careful!



wrong

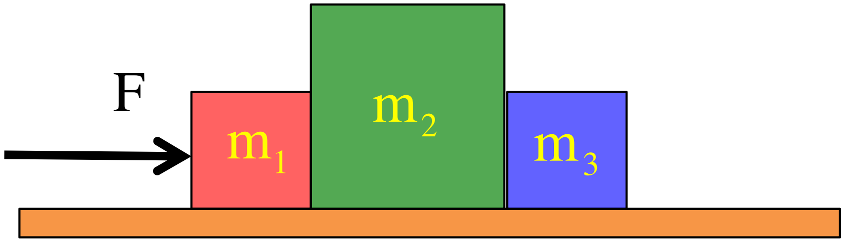






Seat of the pants problem:

Quick! Three blocks on a frictionless surface with force *F* applied as shown.



a.) What's the acceleration of the system (quick like a bunny!).

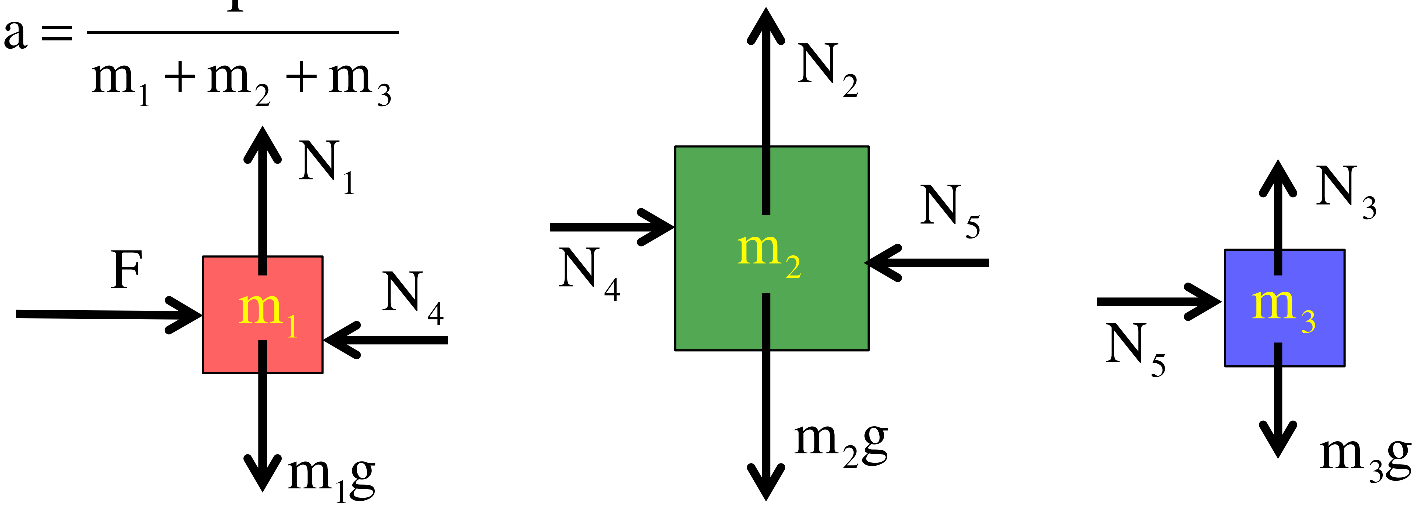
b.) Draw a f.b.d. on each block (quick!).

c.) Determine the net force on each block (quick!).

d.) Determine the contact force on the last block.

a.) $$a = \frac{F}{m_1 + m_2 + m_3}$$

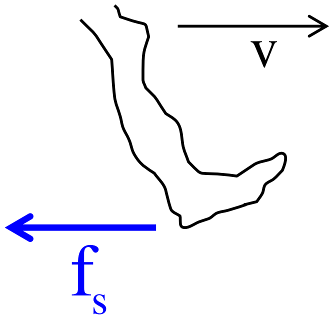
b.)



c.) $F_{net,m_1} = m_1 a$ $F_{net,m_2} = m_2 a$ $F_{net,m_3} = m_3 a$

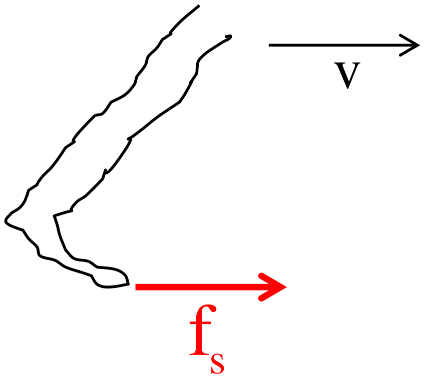
d.) $N_5 = m_3 a$

Walking, Running and Friction: What kind of friction is involved with walking or running?



With walking, when the heal of your foot first hits the ground, *STATIC friction* aims backwards (you aren't *sliding*) keeping your foot from slipping forward.

Once your body has wheeled over the foot and the foot is pushing off the ground, *STATIC friction* aims FORWARD. Why? Think about the direction dirt would fly if you were pushing off hard and the ground was soft and gave way—the ground would fly back due to your push. N.T.L. maintains that the ground must apply an equal and opposite push-back on you *forward* . . .



In the case of running (with the body always forward of the foot), it's all pushing forward! In other words, the direction of the frictional force, being static, is *in the direction of apparent motion*! (Remember, with both static and kinetic friction, it is what a body is doing RELATIVE TO THE BODY IT IS IN CONTACT WITH that determines force direction.)

Finale

https://youtu.be/cWOv7NyOnhY