Ch 4 – Motion in 2-D

Projectile Motion

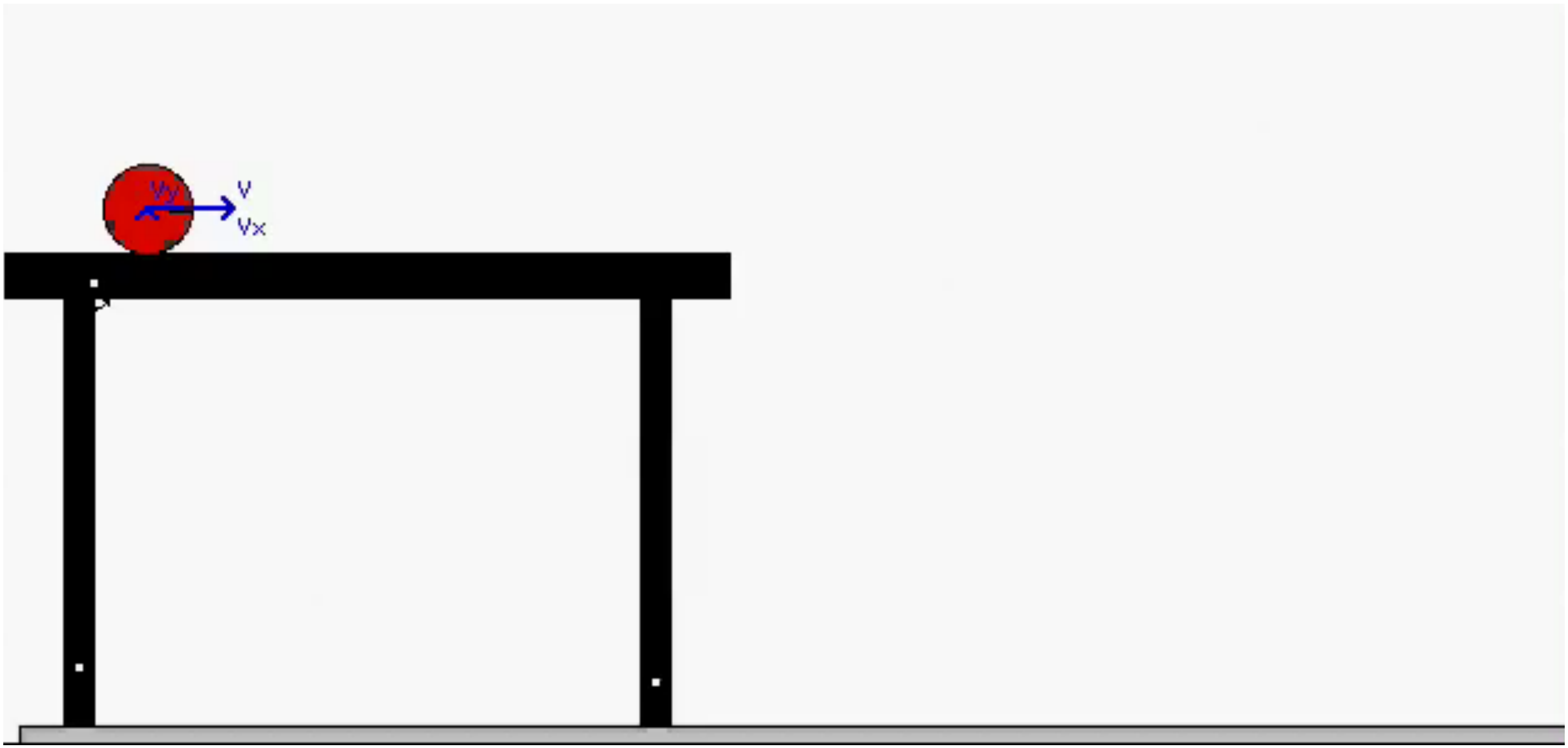
We've already discussed projectile motion briefly. The general idea is this:

Objects thrown in the air are considered *projectiles*; objects thrown at some angle to the vertical follow a curved parabolic path called a *trajectory*.

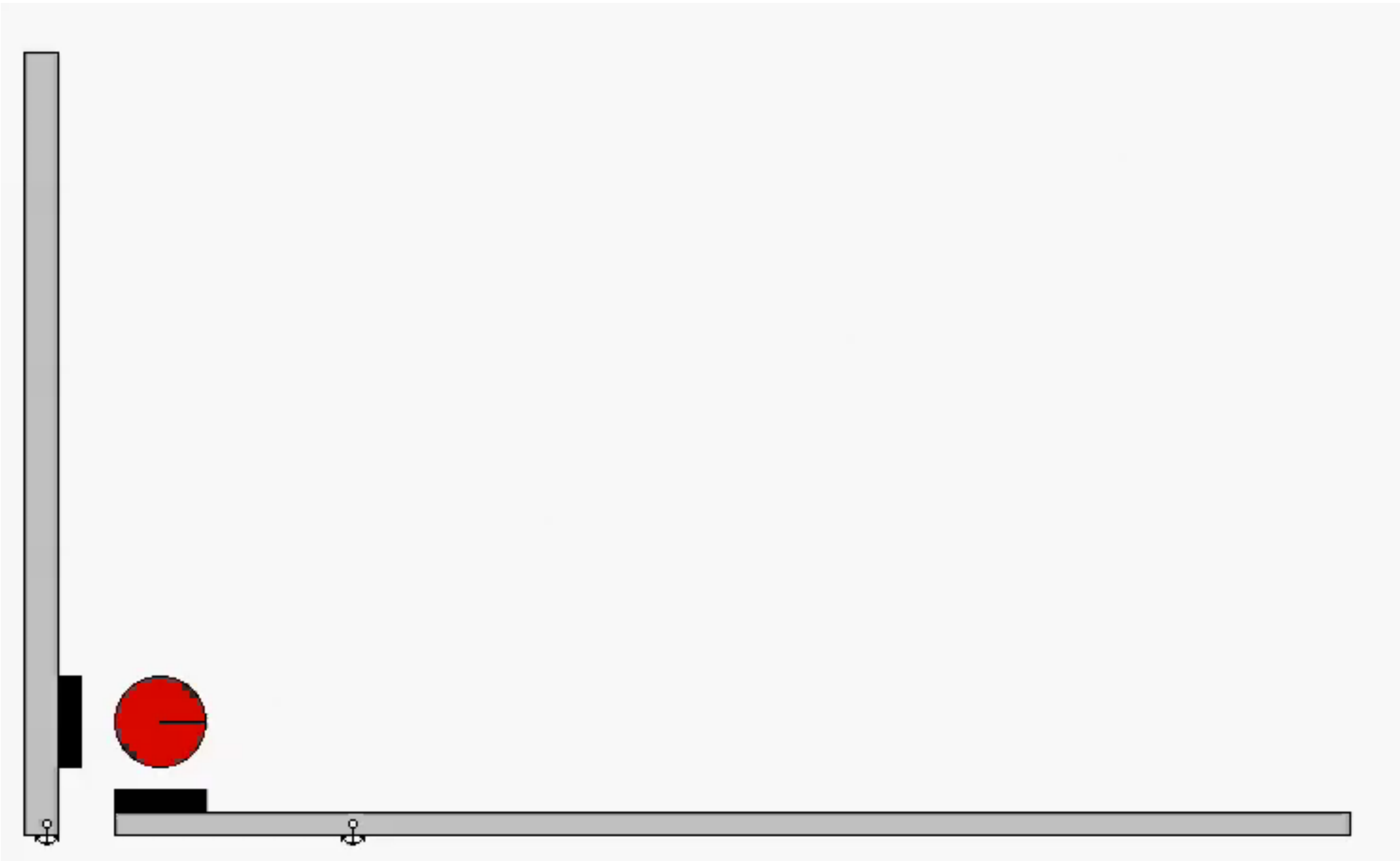
We can easily solve all sorts of projectile problems if we make two assumptions: free-fall acceleration is constant throughout the trajectory, and air resistance is negligible.

The reason these problems are easy to solve is because ***we can consider the horizontal and vertical motions independently!!!*** In fact, we're *forced* to this by the nature of the problem: horizontally, there is no force that causes the velocity of the object to change, while vertically, the force of gravity causes the object to accelerate toward the earth.

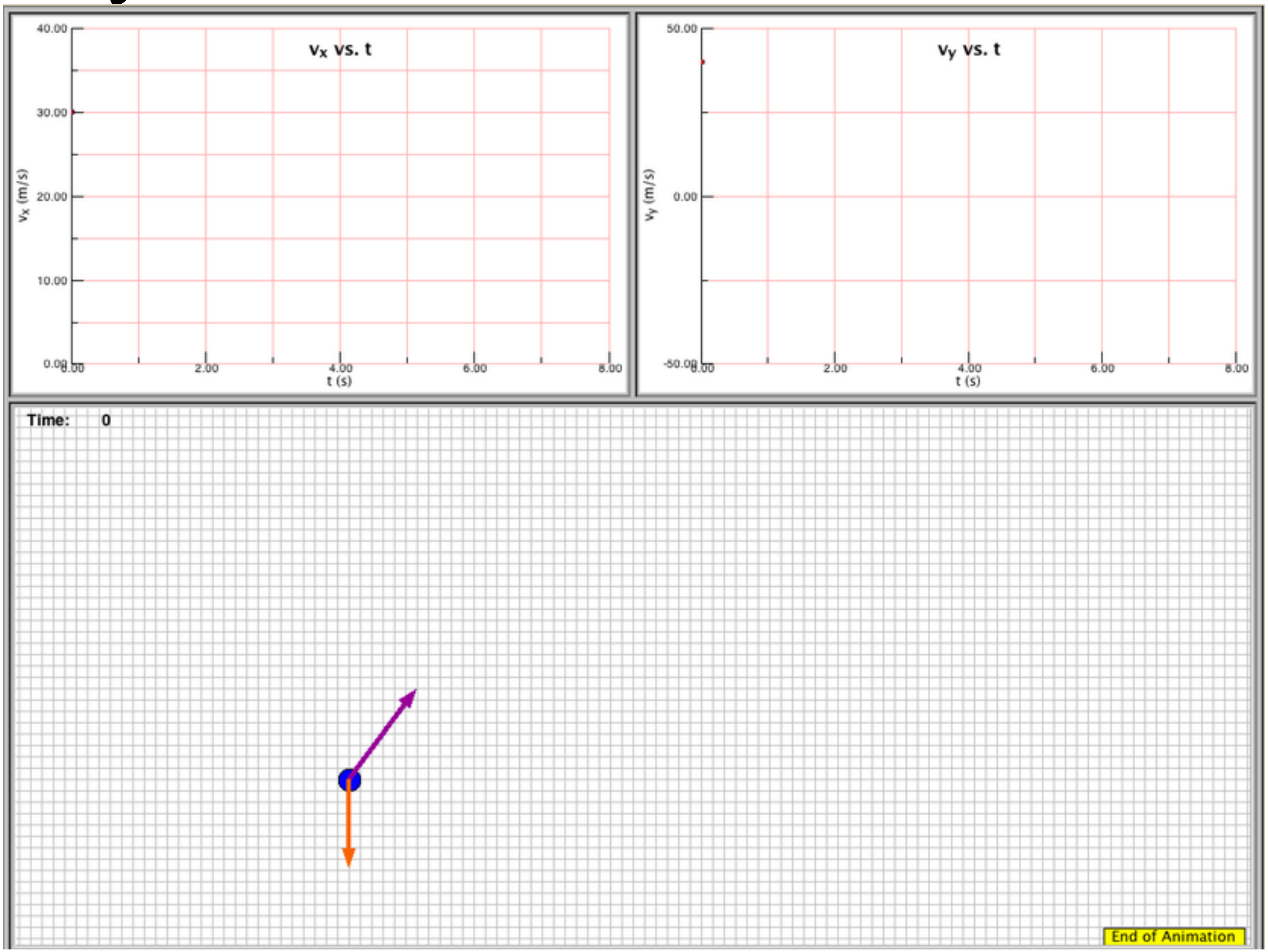
Projectile Animation 1



Projectile Animation 2



Projectile Animation 3



Projectile Fail

To solve 2-d problems:

- ***Strategy 1***
  Break given vectors into **x-** and **y-**components; then apply kinematics in **x** and **y** directions separately. Finally, resolve components as needed for final solution (in polar notation?)

- ***Strategy 2***
  Using vector-based kinematic equations (like $\Delta\mathbf{r}=\mathbf{v_i}t+(1/2)\mathbf{a}t^2$) with vectors written in unit-vector (**i, j**) notation.

Projectile Fail

Examples of projectile motion, our favorite 2-D motion, are everywhere.

Example 1 – Projectile Motion

A long jumper leaves the ground with a velocity of 11.0m/s at 20.0° above the horizontal.

a. What is the maximum height he reaches?

b. How far horizontally does he jump? Solve using components.

c. Assuming you've determined time, solve again using **i, j** notation.

Example 1 – Solutions

a. What is the maximum height she reaches?

Vertically:

$$v_i = 11\sin 20 = 3.76m/s$$

$$v_f = 0$$

$$a = -9.80m/s^2$$

$$v_f = v_i + at \rightarrow t = \frac{v_f - v_i}{a} = 0.384s$$

$$\Delta y = ?$$

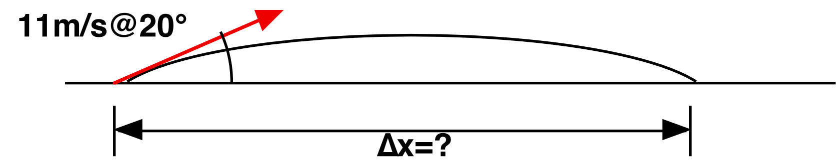
$$\Delta y = v_i t + \frac{1}{2}at^2$$

$$\Delta y = (3.76m/s)(0.384s) + \frac{1}{2}(-9.80m/s^2)(0.384s)^2$$

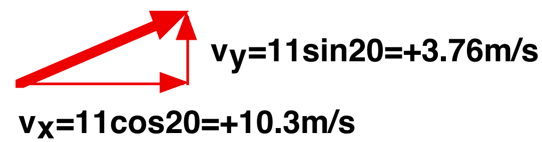
$$\Delta y = 0.721m$$

Example 1 – Solutions

b. How far does she jump?



Get components of initial velocity:



Consider vertical and horizontal situations independently! Let's start with vertical situation: how long will she be in the air?

Vertically:

$$v_i = 3.76 m/s$$

$$\Delta x = 0$$

$$a = -g = -9.80 m/s^2$$

$$\Delta x = v_i t + \frac{1}{2} a t^2$$

$$0 = 3.76t + \frac{1}{2}(-9.80)t^2$$

Quadratic eqn: $t = \{0,\ 0.767s\}$

Horizontally:

$$v = 10.3 m/s \ \text{(constant)}$$

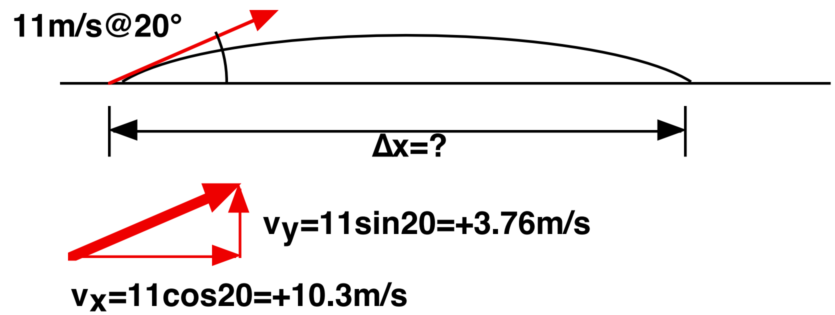
$$t = 0.767s$$

$$\Delta x = vt$$

$$\Delta x = (10.3 m/s)(0.767s) = 7.90 m$$

Example 1 – Solutions

c. If we calculate (using *y* direction) that the time for the leap is t=0.767s, let's solve for horizontal distance.



$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

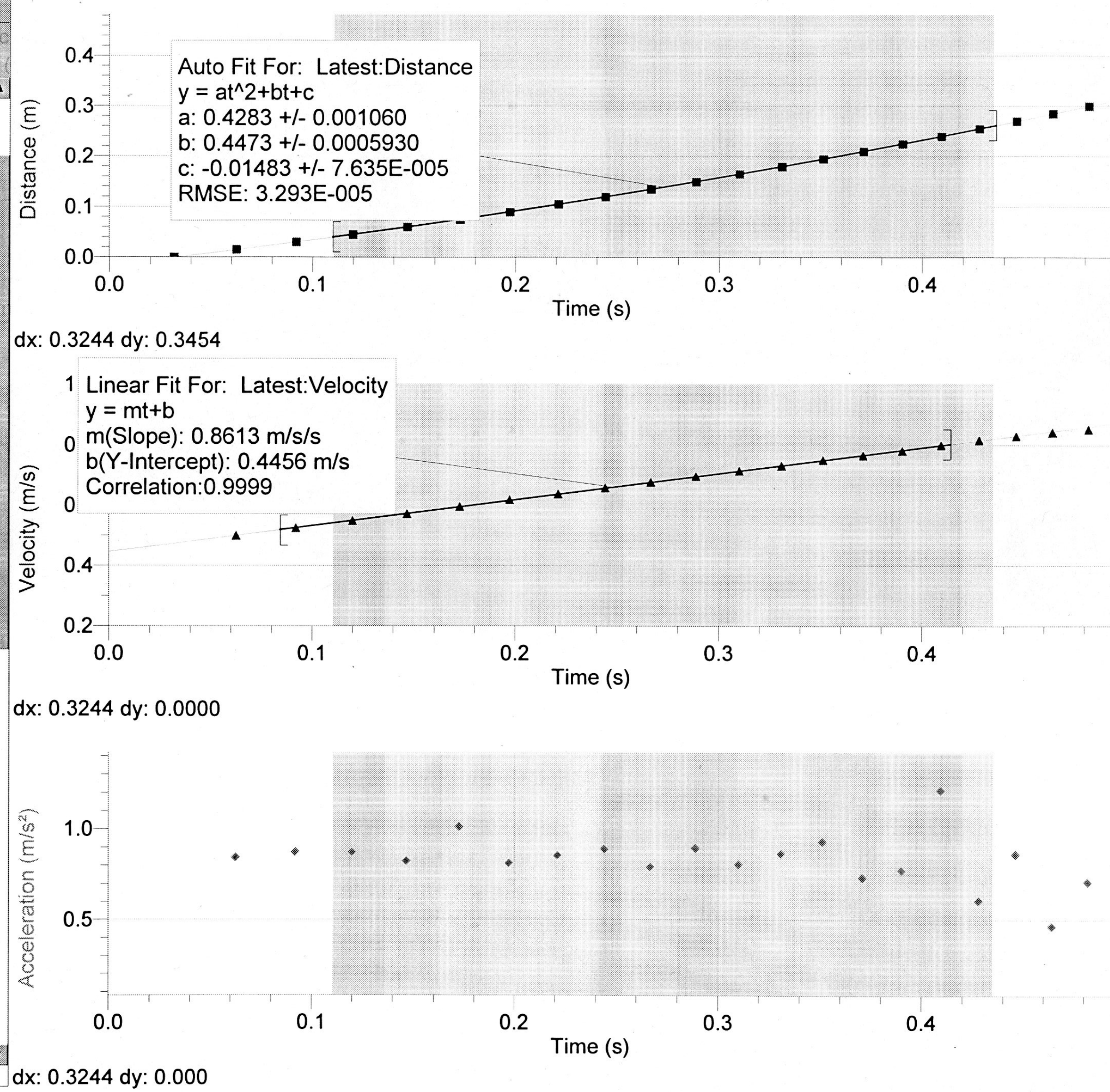
$$(x\mathbf{i} + 0\mathbf{j}) - (0\mathbf{i} + 0\mathbf{j}) = (10.3\mathbf{i} + 3.76\mathbf{j})(0.767) + \frac{1}{2}(0\mathbf{i} - 9.8\mathbf{j})(0.767)^2$$

$$(x\mathbf{i} - 0\mathbf{i}) + (0\mathbf{j} - 0\mathbf{j}) = (7.9\mathbf{i} + 2.88\mathbf{j}) + -2.88\mathbf{j}$$

$$x\mathbf{i} = (7.9\mathbf{i})m$$

Lab-due Tue, beg. of class

	Time (s)	ta	Distance (m)	Velocity (m/s)
5	0.091883	1	0.030	0.523
6	0.105283	0		
7	0.119883	1	0.045	0.548
8	0.132722	0		
9	0.146684	1	0.060	0.571
10	0.159028	0		
11	0.172483	1	0.075	0.595
12	0.184284	0		
13	0.197193	1	0.090	0.617
14	0.208630	0		
15	0.221122	1	0.105	0.637
16	0.232219	0		
17	0.244306	1	0.120	0.657
18	0.255025	0		
19	0.266783	1	0.135	0.676
20	0.277203	0		
21	0.288684	1	0.150	0.695
22	0.298784	0		
23	0.309984	1	0.165	0.713
24	0.319883	0		
25	0.330783	1	0.180	0.730
26	0.340412	0		
27	0.351083	1	0.195	0.748
28	0.360522	0		
29	0.370884	1	0.210	0.765
30	0.380183	0		
31	0.390316	1	0.225	0.779
32	0.399392	0		
33	0.409382	1	0.240	0.798
34	0.418184	0		
35	0.427911	1	0.255	0.815
36	0.436614	0		
37	0.446188	1	0.270	0.829
38	0.454724	0		
39	0.464124	1	0.285	0.840
40	[illegible]			



Mr. White, 2010-09-13 hanging mass = 49.88g, cart = 499.43g (1) 9/13/2010 11:33:39

Lab-due Tue, beg. of class

Not sure of how to do a lab writeup for this course?

See

http://www.crashwhite.com/apphysics/materials/assignments/lab/index.html

for specific information.

Example 2

A stone is thrown from the top of a building , with an initial velocity of 20.0 m/s at 30.0° above the horizontal. If the building is 45.0 m tall...

a. how long is the stone in flight?

b. What is the speed of the stone just before it hits the ground?

c. Where does the stone strike the ground?

a. $\Delta y = v_i t + (1/2)at^2$
   $-45 = 20\sin 30 t + (1/2)(-9.8)t^2$
   $t = 4.22s$

b. Vertical:
   $v_f = v_i + at$
   $v_f = 20\sin 30 + (-9.8)(4.22)$
   $v_f = -31.4m/s$
   Horizontal:
   $v_x = (20\cos 30) = 17.3m/s$
   Speed $= \sqrt{(v_x^2 + v_y^2)} = 35.9m/s$

c. $\Delta x = vt$
   $\Delta x = (20\cos 30)(4.22)$
   $\Delta x = 73.1m$ away (horz)

Nice acting, Keanu

1. There is a gap in the freeway. Convert the gap's width to meters.
2. How fast is the bus traveling when it hits the gap? What is its velocity in m/s?
3. Keanu hopes that there is some "incline" that will assist them. Assume that the opposite side of the gap is 1 meter lower than the takeoff point. Also, the stunt drivers that launch this bus clearly have the assistance of a "takeoff ramp" from which the bus launches at an angle. Assume that the ramp is angled at 3.00° above the horizontal. Prove whether or not the bus will make it to the opposite side. (Consider bus as a particle.)

Example 3 – More theoretical

A theoretical problem: A particle starts from the origin at time t=0, with initial velocity v_x=-10m/s, and v_y=+5m/s. The particle is accelerating at 3 m/s^2 in the x direction.

a. Find v components as a function of time.

b. Find **v** (in **i,j** notation)as a function of time.

c. Find speed at time t=5.00s.

d. Find position coordinates x and y as a function of time.

e. Find **r** relative to origin as a function of time.

f. Find displacement vector(in **i,j** notation) at time t=5.00s.

a. $v_x = -10+3t$; $v_y = 5$

b. $\mathbf{v}=((-10+3t)\mathbf{i} + 5\mathbf{j})$m/s

c. v_{inst}=7.1m/s

d. $x=v_i t+1/2at^2 = (-10)t+1/2(3)t^2$; $y=5t$

e. $r=((-10)\mathbf{i}+5\mathbf{j})t + 1/2(3\mathbf{i})t^2$

f. $\mathbf{r}=-12.5\mathbf{i}+25\mathbf{j}$

Example 4

What is the initial velocity of the ball in this video?

Uniform Circular Motion

We've defined acceleration **a** to be a change in velocity over a period of time:

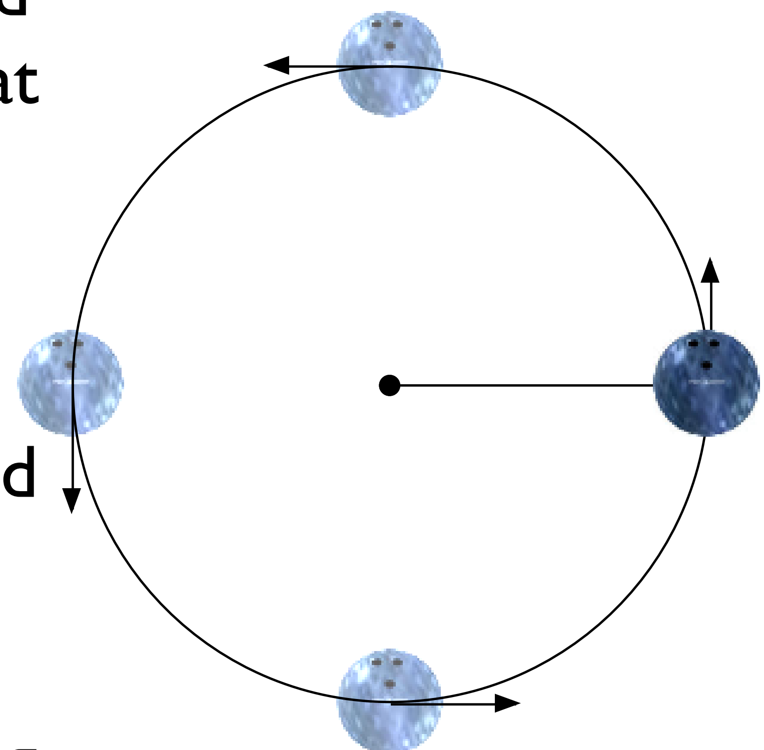
$$\vec{\mathbf{a}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} \qquad \vec{\mathbf{a}}_{inst} = \frac{d\vec{\mathbf{v}}}{dt}$$

Because velocity is a vector, it should be clear that there are two ways that we can change velocity:

a. changing speed (speeding up or slowing down)

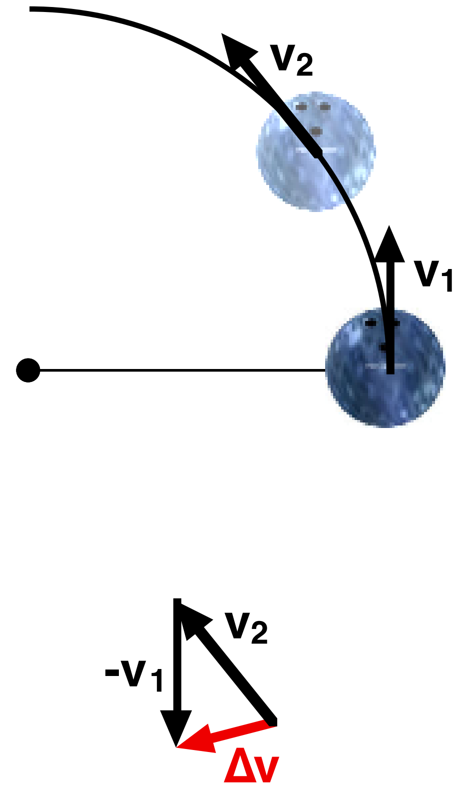
b. changing *direction* (even if the speed remains constant)

Thus, an object traveling in a circle, even at constant speed, is *accelerating*.



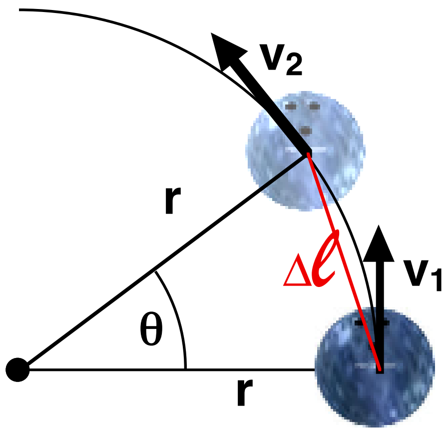
Direction of centripetal *a*

We can determine a formula for calculating centripetal ("center-seeking") acceleration as follows.



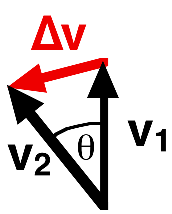
$\Delta v = v_2 + (-v_1)$

Magnitude of centripetal *a*



$$\frac{\Delta v}{v} = \frac{\Delta \ell}{r}$$

As Δt approaches 0, $\Delta \ell$ approaches distance traveled by ball along path of circle during time t.



$$\Delta v = \frac{v\Delta \ell}{r}$$

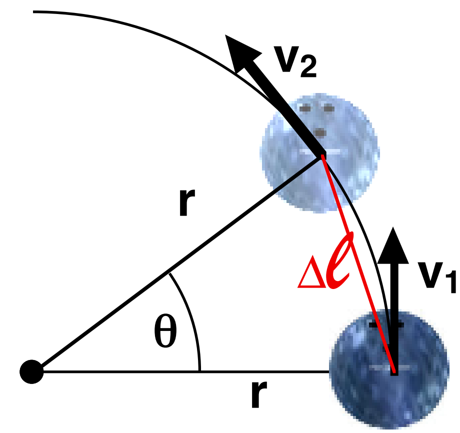
$$\frac{\Delta v}{\Delta t} = \frac{v\Delta \ell}{r\Delta t}$$

$$a_c = \frac{v}{r} \bullet \frac{\Delta \ell}{\Delta t} = \frac{v}{r} \bullet v = \frac{v^2}{r}$$

Centripetal acceleration

An object moving in a circle of radius *r* with constant speed *v* has an acceleration directed toward the middle of the circle, with a magnitude

$$a_c = \frac{v^2}{r}$$



Example 4 – Hammer throw

At the beginning of this hammer throw, a 5 kg mass is swung in a horizontal circle of 2.0 m radius, at 1.5 revolutions per second. What is the acceleration of the mass (magnitude and direction)?

Example 4 – The Hammer

At the beginning of this hammer throw, a 5 kg mass is swung in a horizontal circle of 2.0 m radius, at 1.5 revolutions per second. What is the acceleration of the mass (magnitude and direction)?

$$v = \frac{1.5rev}{s} \bullet \frac{2\pi r}{1rev} = \frac{3\pi(2.0m)}{s} = 18.8m/s$$

$$a_c = \frac{v^2}{r}$$

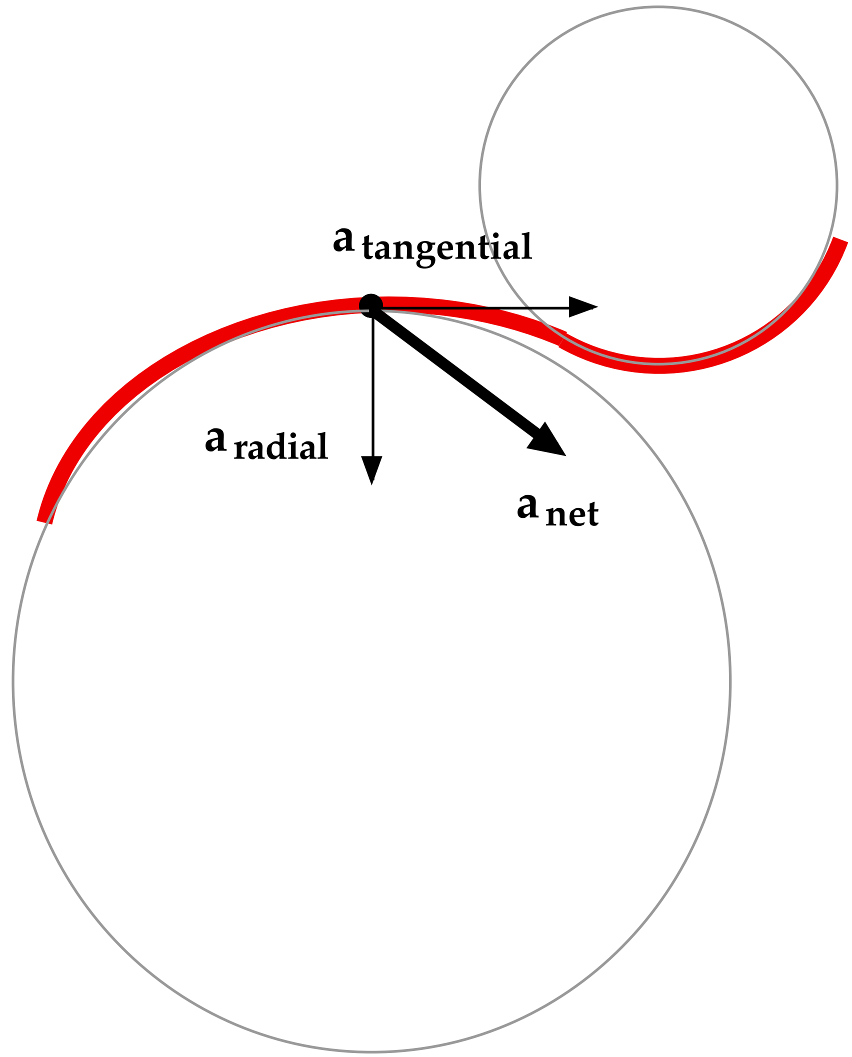
$$a_c = \frac{(18.8m/s)^2}{2.0m} = 177m/s^2, \text{ toward middle of circle}$$

Tangential & radial *a*

Radial acceleration $\mathbf{a}_r$ is due to the change in direction of the velocity vector: $\mathbf{a}_r = v^2/r$

Tangential acceleration $\mathbf{a}_t$ is due to the change in speed of the particle: $\mathbf{a}_t = dv/dt$.

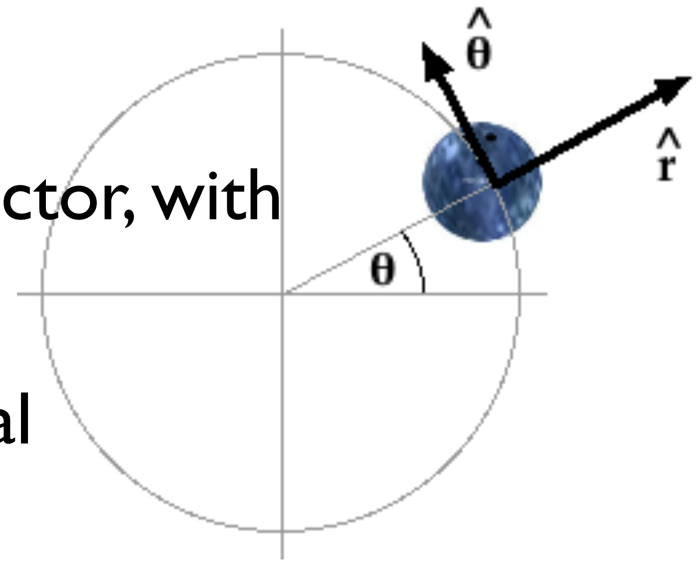
$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_t + \vec{\mathbf{a}}_r$$



More unit vectors?

It's sometimes convenient to be able to write the acceleration of a particle moving in a circular path in terms of two new unit vectors:

- θ is a unit vector tangent to the circular path, with θ *positive in ccw direction*
- ***r*** is a unit vector along the radius vector, with ***r*** *positive directed radially outward.*



So, if the bowling ball above had a radial acceleration of $4 m/s^2$, and a tangential acceleration of $2 m/s^2$ in the ccw direction what would its net acceleration be at that point?

Example 5 – A little trickier

A bowling ball pendulum is tied to the end of a string 3.00 m in length, and allowed to swing in a vertical arc under the influence of gravity. When the hanging ball makes an angle of 15.0° with vertical, it has a speed of 2.00 m/s.

a. Find the radial acceleration at this instant

b. When the ball is at an angle θ relative to vertical it has tangential acceleration of **g** sin θ. Find the net acceleration of the ball at θ = 15.0°,(using same data as above) and express it in **θ,r** form. Then express that net **a** in polar notation. :(

$-1.33 m/s^2$

$a=(2.54ø-1.33r) m/s^2$

$2.87\ m/s^2$ @167°

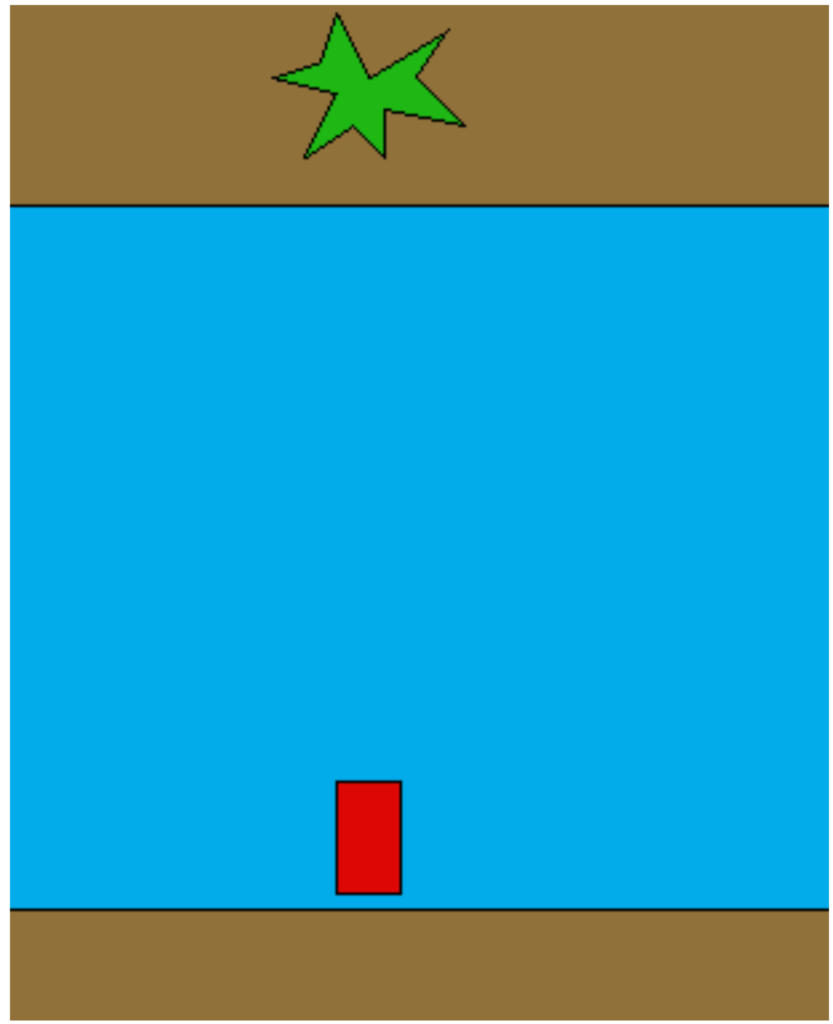
Relative Motion

Observers with different viewpoints ("frames of reference") may measure different displacements, velocities, and accelerations for any given particle, especially if the two observers are moving relative to each other.

Frames of Reference, 1960.

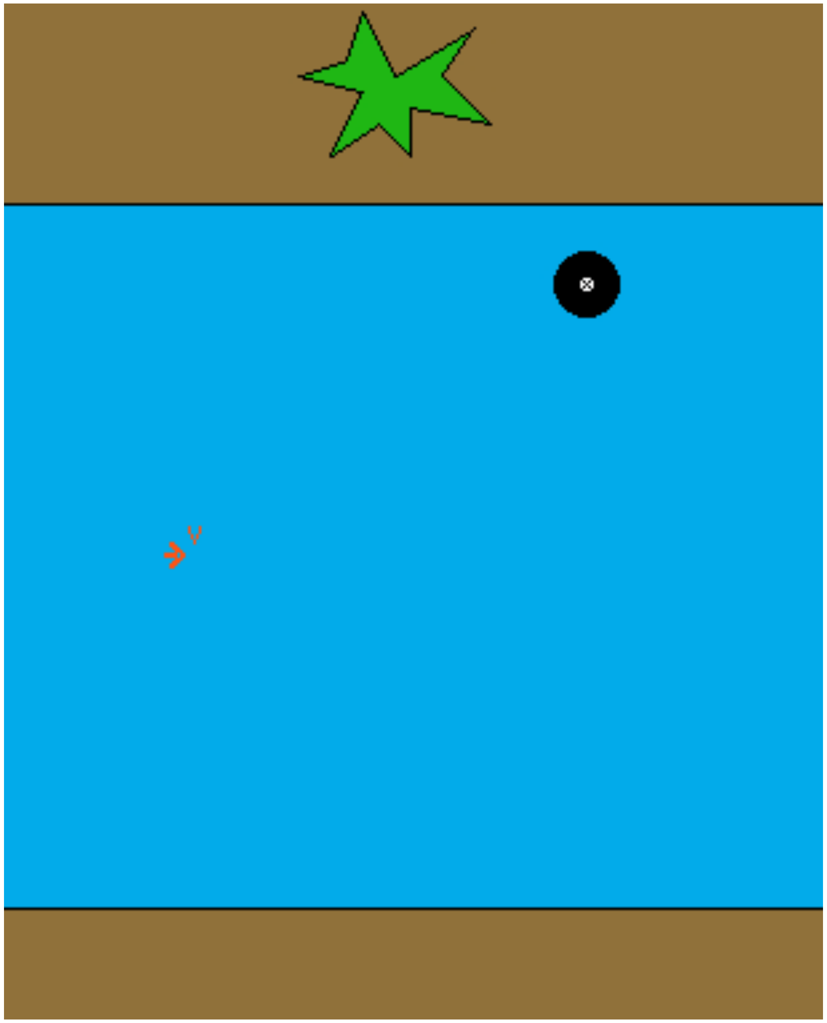
Relative Velocity in 2-D

A boat has the ability to cross a (still water) lake at 3 m/s.



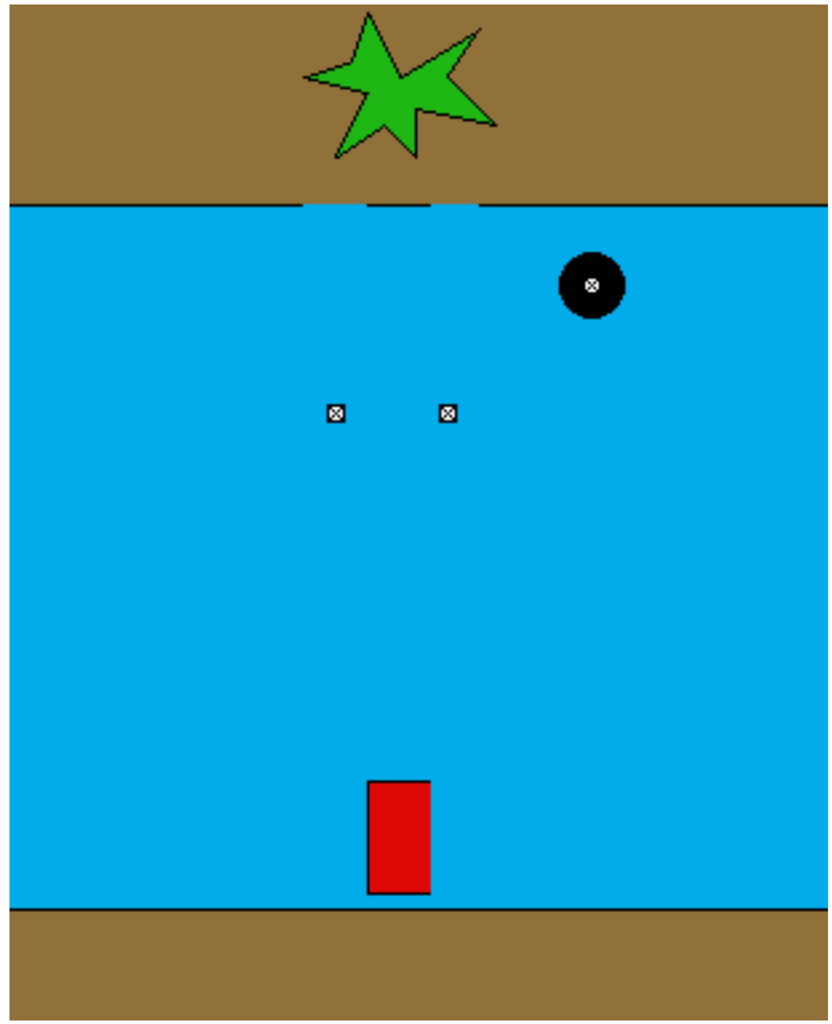
Relative Velocity in 2-D

A river has a velocity of 4 m/s (downstream).



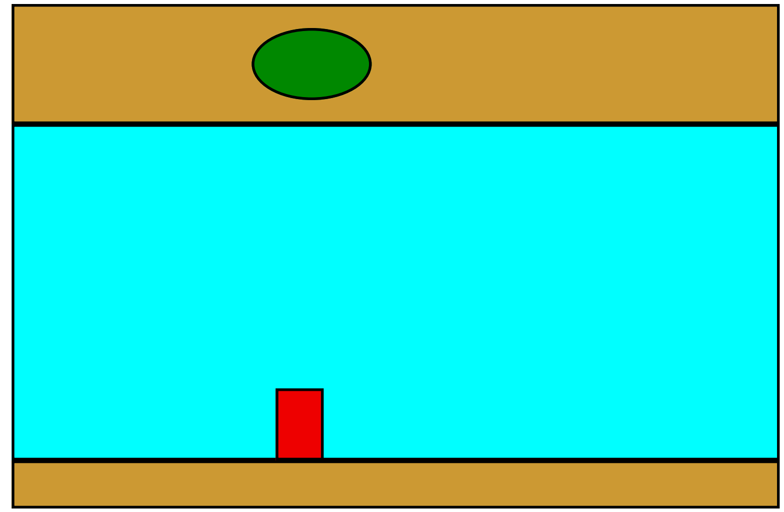
Relative Velocity in 2-D

What happens when we try to aim the boat directly across the river?



Relative Motion Analysis

For relative motion problems, it's useful to label the vectors very clearly, in such a way that we can describe an object's velocity *relative to a certain reference frame*.

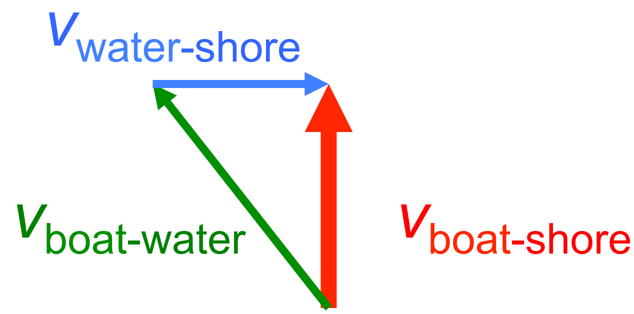


$$\vec{\mathbf{v}}_{br} + \vec{\mathbf{v}}_{rs} = \vec{\mathbf{v}}_{bs}$$

If the boat's velocity (relative to the water) is 3m/s @ 90°, and the river's velocity is 4m/s @ 0° (relative to the shore), what is the boat's velocity relative to the shore?

Example 7

A boat can travel with a velocity of 20.0 km/hr in water.



a. If the boat needs to travel directly North across a river with a current flowing at 12.0 km/hr to the east, at what upstream angle should the boat head?

$$\theta = \sin^{-1}\left(\frac{12}{20}\right) = 37°, \textit{ relative to North}$$

b. If the river is 6 km across, how long will it take the boat to get there?

$$t = \frac{d}{s} = \frac{y}{y_y} = \frac{6km}{20\cos 37} = 0.38hr,$$

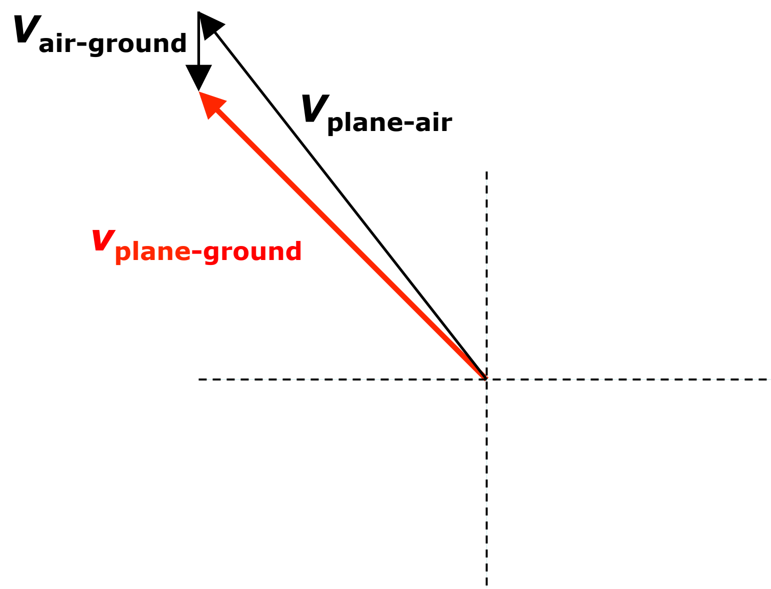
or

$$t = \frac{d}{s} = \frac{diag-distance}{diag-speed}$$

$$t = \frac{6km/\cos 37}{20} = 0.38hr$$

Example 8

A plane *wants* to fly at 300 km/h, 135°, but a 50 km/h wind blows from the north. What velocity should the plane fly at (magnitude & direction) to achieve its desired velocity?



$$v_{plane-ground} = v_{plane-air} + v_{air-ground}$$

$$x: 300\cos 135 = v_{plane-air,x} + 0$$

$$v_{plane-air,x} = -212 km/h$$

$$y: 300\sin 135 = v_{plane-air,y} + -50 km/h$$

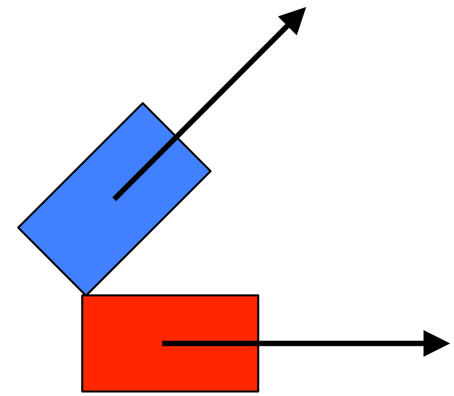
$$v_{plane-air,y} = 212 km/h + 50 km/h = 262 km/h$$

$$v = \sqrt{212^2 + 262^2} = 337 km/h$$

$$\theta = \tan^{-1}\left(\frac{267}{212}\right) = 52°; 180 - 51 = 129°$$

Example 9

The speed of a each constant velocity car is 0.30 m/s. What is the *velocity* of the red car (moving along the x-axis relative to the blue car (@45°)?



(Estimate, then calculate.)

Red car is moving +x and -y relative to blue car's motion.

$$v_{Red\text{-}Blue} = v_{Red\text{-}Ground} - v_{Blue-Ground}$$

$$v_{Red\text{-}Blue} = (0.3\mathbf{i} + 0\,\mathbf{j}) - (0.21\mathbf{i} - 0.21\mathbf{j})$$

$$v_{Red\text{-}Blue} = (0.09\mathbf{i} - 0.21\mathbf{j})m/s$$

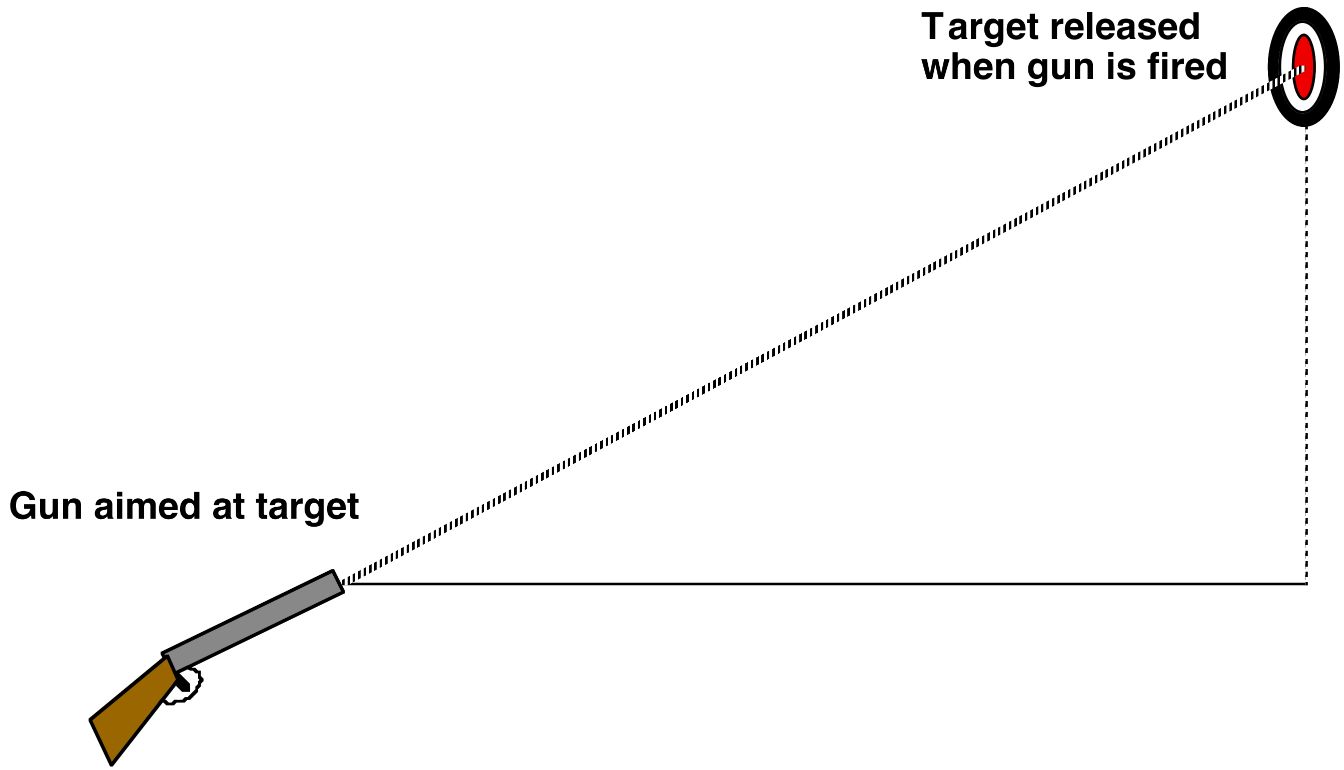
$$v_{Red\text{-}Blue} = 0.23m/s@-67^\circ$$

Feedback on First Lab

1. Name goes at upper right corner of every sheet
2. Blurbs (brief written descriptions) should accompany *all* calculations, especially where source of values used may not be obvious.
3. When using graphs for calculations, identify *on the graph* which data points you're using for your calculations.
4. Follow the outline given in the lab writeup description (online)
5. Space out your writing as much as possible, to leave room for corrections & blurbs (you) & comments (me)
6. In summary, state error percentages as evidence that the lab was (or was not) successful.

Demo-Monkey & Hunter

Will bullet hit: a) above target; b) on target, or c) below target?



Demo-Monkey & Hunter

Solving for the target's position as a function of time, and the bullet's position as a function of time, will allow you to determine the *y*-coordinates of each object when their *x*-coordinates are equal.