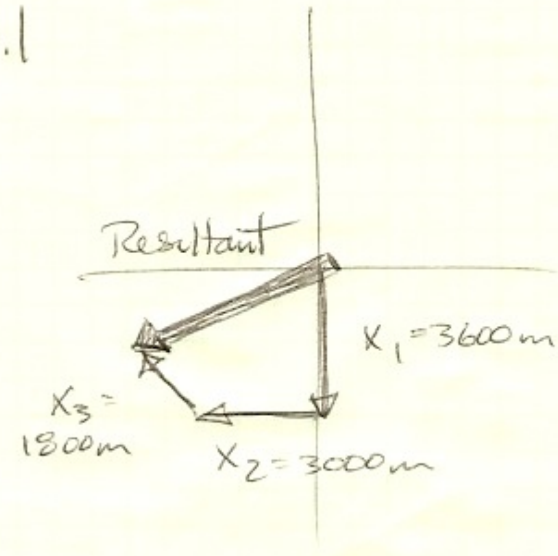
4.1

$x_1 = (20\frac{m}{s})(180s) = 3600m$

$x_2 = (25\frac{m}{s})(120s) = 3000m$

$x_3 = (30\frac{m}{s})(60s) = 1800m$



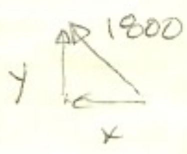
a) Total vector displacement?

$\Delta x = x_1 + x_2 + x_3$

$\Delta x = (0i - 3600j) + (-3000i + 0j) + (-1270i + 1270j)$

$= \boxed{(-4270i - 2330j)m}$

Convert to polar.



$2x^2 = 1800^2$

$x = 1270m$

$c^2 = a^2 + b^2$

$c = \sqrt{4270^2 + 2330^2}$

$c = \boxed{4860\,m}$

@ $\theta = \tan^{-1}\frac{-2330}{-4270}$

$= \boxed{-28.6^\circ} + 180$

$\boxed{= 209^\circ}$

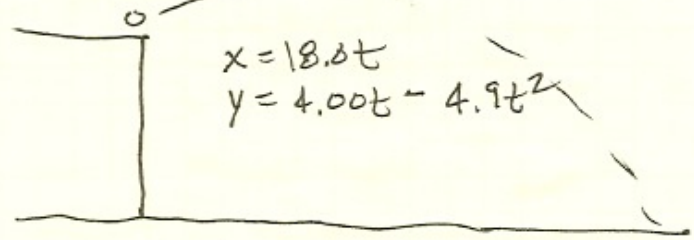
b) Average speed $= \frac{\text{total distance}}{\text{total time}}$

$= \frac{3600 + 3000 + 1800}{180 + 120 + 60} = \boxed{23.3\,m/s}$

c) Average velocity $= \frac{\text{total displacement}}{\text{total time}}$

$= \frac{4860m\ @\ -28.6^\circ}{360\,s} = \boxed{13.5\,m/s\ @\ \cancel{28.6^\circ}\ 209^\circ}$

4.3



a) Balls' position as a function of time, in i, j notation:

$$\vec{r} = \boxed{((18.0t)i + (4.00t - 4.9t^2)j)\ m}$$

b) Velocity as a function of time:

$$v = \frac{\partial r}{\partial t} = \frac{\partial}{\partial t}(18.0t)i + \frac{\partial}{\partial t}(4t - 4.9t^2)j$$
$$= \boxed{(18.0i + (4 - 9.8t)j)\ m/s}$$

c) Acceleration as a function of time:

$$a = \frac{\partial v}{\partial t} = \frac{\partial}{\partial t}(18)i + \frac{\partial}{\partial t}(4 - 9.8t)j$$
$$= \boxed{(0i + -9.8j)\ m/s^2}$$

d) Solving for r, v, & a at t = 3.0s:

$$\vec{r} = ((18)(3))i + (4(3) - 4.9(3^2))j$$
$$= \boxed{(54.0i \mp 32.1j)\ m}$$
$$v = 18i + (4 - (9.8)(3))j$$
$$= \boxed{(18.0i + -25.4j)\ m/s}$$
$$a = \boxed{(0i - 9.8j)\ m/s^2}$$

4.5 $\vec{r} = 3.00i + -6.00t^2 j$

a) Velocity as a function of time:

$$v = \frac{dr}{dt} = \frac{d}{dt}(3i - 6t^2 j)$$

$$= \boxed{(0i - 12t j)\ m/s}$$

b) acceleration as a function of time:

$$a = \frac{dv}{dt} = \frac{d}{dt}(0i - 12t j)$$

$$= \boxed{(0i - 12j)\ m/s^2}$$

c) position & velocity at $t = 1s$.

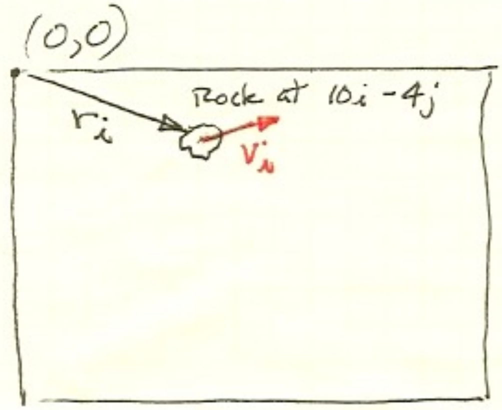
$$\vec{v} = 0i - 12(1) j$$

$$= \boxed{(0i - 12j)\ m/s}$$

$$\vec{r} = 3i - 6(1)^2 j$$

$$= \boxed{(3i - 6j)\ m}$$

4.7



$r_i = (10i - 4j)\,m$

$v_i = (4i + 1j)\,m/s$

After $\Delta t = 20s$,

$v_f = (20i - 5j)\,m/s$

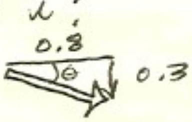
a) What is acceleration of fish?

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t} = \frac{(20i - 5j) - (4i + 1j)}{20}$$

$$= \frac{16i - 6j}{20}\,m/s^2 = \boxed{0.8i - 0.3j\ m/s^2}$$

b) Direction of acceleration w/ respect to $\hat{i}$?

$\hat{i}$ is at $0°$ ($\longrightarrow$).



Direction of a, $\theta = \tan^{-1}\left(\frac{-0.3}{0.8}\right)$

$= \boxed{-20.6°}$

c) If fish continues with this acceleration, what are position & direction of motion (ie. direction of velocity!) after 25.0s?

$$r(25s) = r_i + v_i t + \tfrac{1}{2}at^2$$

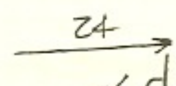
$$= (10i - 4j) + (4i + 1j)\,25 + \tfrac{1}{2}(0.8i - 0.3j)(25)^2$$

$$= \boxed{(360i - 71.8j)\,m}$$

$$v(25s) = v_i + at$$

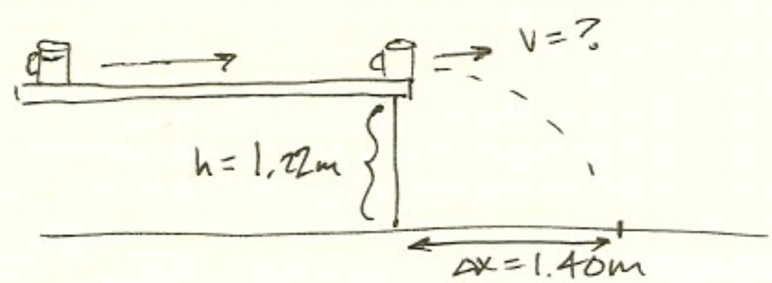
$$= (4i + 1j) + (0.8i - 0.3j)(25)$$

$$= (24i - 6.5j)\,m/s$$



$$\theta = \tan^{-1}\left(\frac{-6.5}{24}\right) = \boxed{-15.1°}$$

4.9



a. With what velocity does mug leave the counter?

Solve for horizontal & vertical analyses separately.

Vertically: $\left.\begin{array}{l} a_y = -9.8\,m/s^2 \\ \Delta y = -1.22\,m \\ v_i = 0 \\ t = ? \end{array}\right\}$ $\quad \Delta y = v_i t + \frac{1}{2}at^2$

$-1.22 = 0t + \frac{1}{2}(-9.8)t^2$

Solve to get

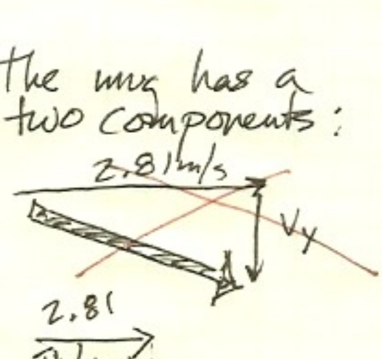
$\underline{t = 0.499s}$

Horizontally: $V_x = \frac{\Delta x}{\Delta t}$

$V_x = \frac{1.40m}{0.499s} = \boxed{2.81\,m/s}$

b. Just before it hits the floor the mug has a net velocity composed of its two components:

We need to find v_y just before it hits.



$V_f = V_i + at$ (vertically)

$V_f = 0 + (-9.8)(0.499s)$

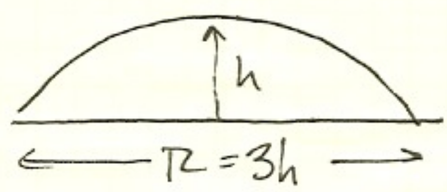
$V_f = -4.89\,m/s$

$V = \sqrt{2.81^2 + 4.89^2} = \boxed{5.64\,m/s}$

$\theta = \tan^{-1}\left(\frac{-4.89}{2.81}\right) = \boxed{-60.1°}$

4.11 A projectile is fired so that range $R = 3\,h_{max}$. What is θ?

From Section 4.3, there are two equations that describe the height & range of a projectile as a function of v & θ.



$$h = \frac{v_i^2 \sin^2\theta_i}{2g} \quad \& \quad R = \frac{v_i^2 \sin 2\theta_i}{g}$$

Using $R = 3h \ldots$

$$\frac{\cancel{v_i^2} \sin 2\theta_i}{\cancel{g}} = \frac{3\cancel{v_i^2} \sin^2\theta_i}{2\cancel{g}}$$

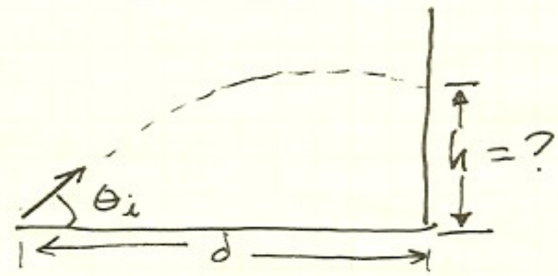
Trig identity: $\sin 2\theta = 2 \sin\theta \cos\theta$, so

$$2 \cancel{\sin\theta} \cos\theta = \frac{3}{2} \sin\theta \cancel{\sin\theta}$$

$$\frac{4}{3} = \frac{\sin\theta}{\cos\theta}$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = \boxed{53.1^\circ}$$

4.15 Find h as a function of other variables.



The time that it takes the water to travel horizontally is the same as the time it takes to travel vertically.

Horizontally: $d = \underbrace{(v\cos\theta)}_{v_x\text{, x-component of } v} t$

Vertically: $h = v_i t + \frac{1}{2}at^2$

$$h = (v\sin\theta)t + \frac{1}{2}(g)t^2$$

Solving first eqn for t: $t = \frac{d}{v\cos\theta}$

Sub in to second eqn:

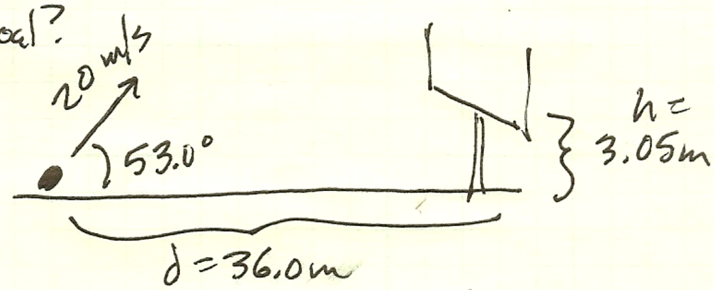
$$h = (\cancel{v}\sin\theta)\left(\frac{d}{\cancel{v}\cos\theta}\right) + \frac{-g}{2}\left(\frac{d}{v\cos\theta}\right)^2$$

$$h = \boxed{\frac{d\sin\theta}{\cos\theta} + \frac{-d^2 g}{2v^2\cos^2\theta}}$$ or

$$= \boxed{\frac{d}{\cos\theta}\left(\sin\theta + \frac{-dg}{2v^2\cos\theta}\right)}$$

4.17 a. Does ball clear goal?

Split ball's motion up into horizontal & vertical analyses:



Vertical position is what we're trying to determine, so let's start w/ horizontal analysis.

Horizontally: $d = v_x t \rightarrow t = \frac{d}{v_x} = \frac{d}{v\cos\theta}$

$$t = \frac{36\text{ m}}{20(\cos 53)} = \underline{2.99\text{s}}$$

Vertically, where is ball at 2.99s?

$$\Delta y = v_i t + \tfrac{1}{2}at^2$$

$$\Delta y = (v\sin\theta)t + \tfrac{1}{2}(-9.8)t^2$$

$$= (20\sin 53)(2.99) - 4.9(2.99)^2$$

$$= \boxed{3.95\text{m}}$$ It does clear bar!

b. Is the ball rising or falling? Find direction of velocity at that moment in time.

Vertically: $v_f = v_i + at$

$$= \cancel{20} + (-9.8)(2.99)$$

20 sin 53

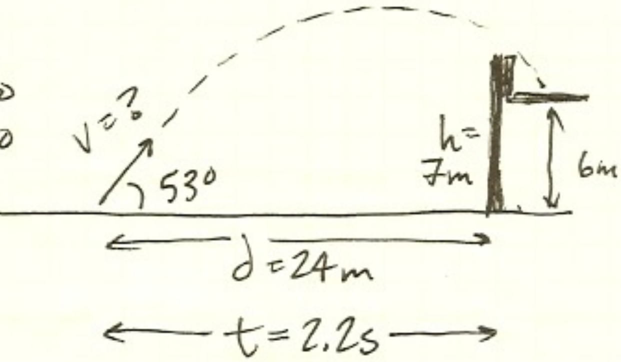
$$= \cancel{-9.30\text{ m/s}}$$ Answer is negative,

-13.3 m/s

so $\boxed{\text{ball is falling.}}$

4.19 a.

It takes 2.20s for ball to travel 24m horizontally so we can easily determine the ball's horizontal component of velocity:



$$V_x = \frac{d}{t}$$

$$V\cos\theta = \frac{d}{t}$$

$$V_{net} = \frac{d}{t\cos\theta} = \frac{24\,m}{(2.2s)(\cos 53)} = \boxed{18.1\,m/s}$$ This is net velocity V.

b. Ball's vertical position at $t = 2.2s$ is:

$$\Delta y = v_i t + \tfrac{1}{2}at^2$$

$$\Delta y = (V\sin\theta)t + \tfrac{1}{2}(g)t^2$$

$$\Delta y = (18.1)(\sin 53)(2.2) + \tfrac{1}{2}(-9.8)(2.2)^2$$

$$\Delta y = 8.13m$$

So ball clears the wall by $8.13 - 7 = \boxed{1.13\,m}$

c. To find where ball lands horizontally, we need to know time of travel, which we can determine by analyzing vertical motion:

$$\Delta y = v_i t + \tfrac{1}{2}at^2$$

$$+6 = (18.1\sin 53)t + (-4.9)t^2$$

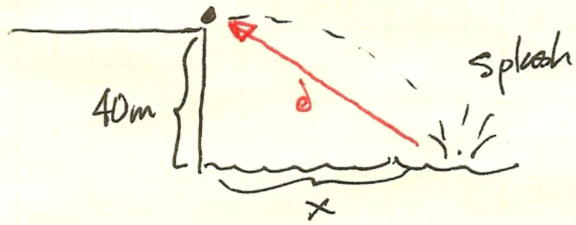
Going up Coming down

Solve quadratic eqn to get $t = 0.500s, 2.45s$

$$\Delta x = vt = (18.1\cos 53)(2.45s) = \boxed{26.7m} - 24 =$$

$\boxed{2.7m \text{ from wall}}$

4.21



Total time between kick & hearing splash is 3.00s.

$$t_{fall} + t_{sound} = 3.00\,s$$

t_{fall}:

$$\Delta y = v_i t + \tfrac{1}{2} a t^2$$

$$-40 = 0t + \tfrac{1}{2}(-9.8)t^2$$

$$t = 2.86\,s$$

t_{sound}:

$$t = \frac{d}{v}$$

$$t = \frac{\sqrt{40^2 + x^2}}{343}$$

$$3 - 2.86 = \frac{\sqrt{40^2 + x^2}}{343}$$

$$(0.14)^2 = \frac{40^2 + x^2}{343^2}$$

$$x = \cancel{28.3\,m}\ \ 26.6\,m$$

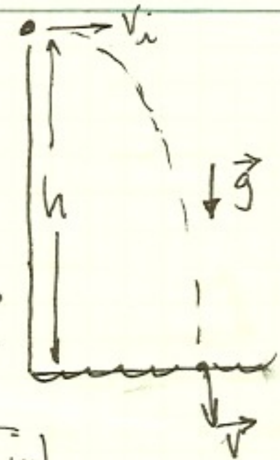
$$\text{So } v_{initial} = \frac{x}{t} = \frac{\cancel{28.3\,m}\ 26.6\,m}{2.86\,s} = \boxed{\cancel{9.63}\ m/s}$$

$$\cancel{9.90}$$

$$\boxed{9.29\,m/s}$$

Answer will vary depending on how many sig figs you carry through in calculations!

4.23 $v_i = 18.0\,\text{m/s}$, $h = 50.0\,\text{m}$



a) Initial coordinates of stone, relative to water at cliff (0,0) are $\boxed{(0, 50)\text{ m.}}$

b) v_{initial} components are $\boxed{(18.0i + 0j)\text{ m/s}}$

c) To analyze vertical motion, assume $\boxed{\text{constant acceleration down} = -g.}$

d) For horizontal motion, $\boxed{\text{constant velocity in x direction.}}$

e) $v_{x_f} = \boxed{v_{x_i}} = 18.0\,\text{m/s}$ ← symbolic.

$v_{y_f} = v_{y_i} + at = \boxed{0 + (-g)t}$

f) Position equations.

$x_f = x_i + v_i t$

$x_f = \boxed{0 + v_{xi} t}$

$y_f = y_i + v_i t + \frac{1}{2} a t^2$

$\underbrace{y_f = \cancel{50}^{\,y_i} + 0t + \frac{1}{2}(-g)t^2}_{\text{Symbolic}}$

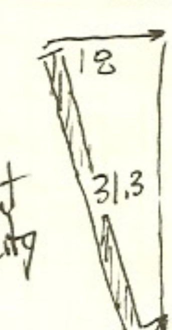
g) $\Delta y = v_i t + \frac{1}{2} a t^2$

$-50 = 0t + \frac{1}{2}(-9.8)t^2$

$t = \boxed{3.19\,\text{s}}$ to fall

h) Horizontally: $v_{xf} = v_{xi} = 18\,\text{m/s}$

Vertically $v_{yf} = v_{yi} + at = 0 + (-9.8)(3.19) = 31.3\,\text{m/s}$



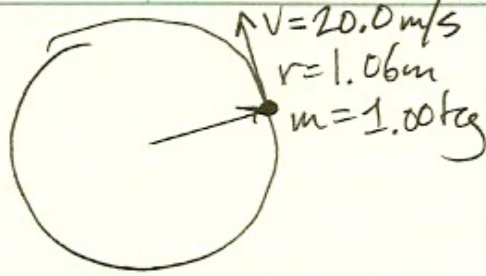
Resultant velocity

$v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{18^2 + 31.3^2} = \boxed{36.1\,\text{m/s}}$

$\theta = \tan^{-1}\left(\frac{-31.3}{18}\right) = \boxed{-60.1^\circ}$

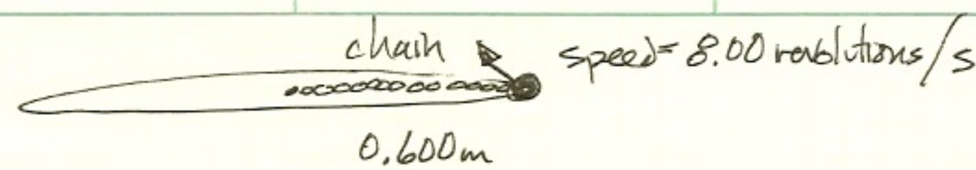
4.27

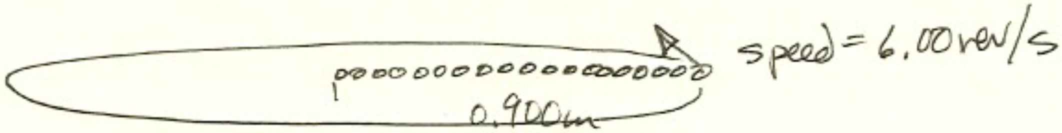


$$a_r = \frac{v^2}{r}$$

$$a_r = \frac{(20.0\,m/s)^2}{1.06\,m} = \boxed{377\,m/s^2}$$

4.30





a. Which speed is greater?

$$\frac{8\text{ rev}}{s} \times \frac{2\pi r}{1\text{ rev}} = \frac{8 \cdot 2 \cdot \pi \cdot (0.6m)}{s} = \boxed{30.2\ m/s}$$

$$\frac{6\text{ rev}}{s} \times 2\pi(0.9m) = \boxed{33.9\ m/s}$$

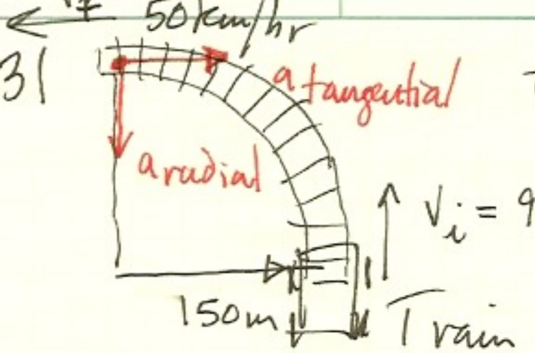
The radius 0.900 m gives the greater velocity.

b. $a_c = \frac{v^2}{r}$

$$a_c = \frac{(30.2)^2}{0.6} = \boxed{1.52e3\ m/s^2}$$

c. $a_c = \frac{(33.9)^2}{0.9} = \boxed{1.28e3\ m/s^2}$

4.31



$t = 15.0\,s$

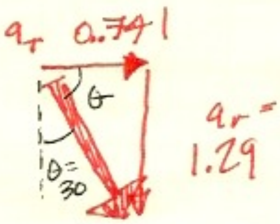
The train is changing both speed (tangential acceleration), & direction (radial acceleration), so we have to account for both of those.

$$a_{tangential} = \frac{V_f - V_i}{t} = \frac{\frac{50e3\,m}{3600\,s} - \frac{90e3\,m}{3600\,s}}{15.0\,s}$$

$$= \boxed{-0.741\,m/s^2}$$

$$a_{radial} = \frac{v^2}{r}$$

At the point indicated $a_r = \dfrac{\left(\frac{50e3\,m}{3600\,s}\right)^2}{150\,m}$



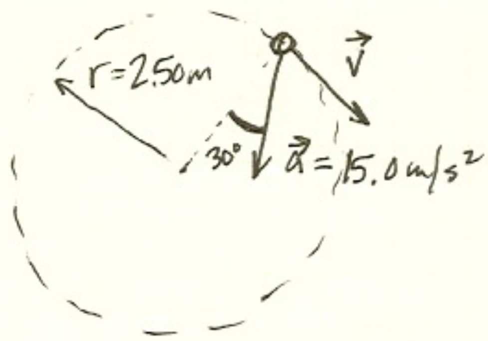
$$= \boxed{1.29\,m/s^2}$$

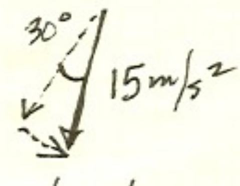
$$a_{net} = \sqrt{0.741^2 + 1.29^2} = \boxed{1.49\,m/s^2} \text{ magnitude}$$

$$\theta = \tan^{-1}\left(\frac{1.29}{0.741}\right) = 60° \rightarrow \boxed{@30° \text{ relative to radius}}$$

4.32

a. Radial acceleration of the particle?





Take net acceleration & split it up into components:

$a_r = 15 \cos 30 = \boxed{13.0\ m/s^2}$ (a)

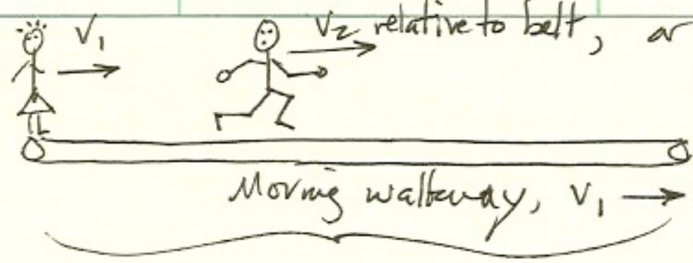
$a_t = 15 \sin 30 = \boxed{7.50\ m/s^2}$ (c)

$a_r = \frac{v^2}{r} \Rightarrow v = \sqrt{a_r \cdot r}$

$= \sqrt{13 \cdot 2.5}$

$v = \boxed{5.70\ m/s}$ (b)

4.36

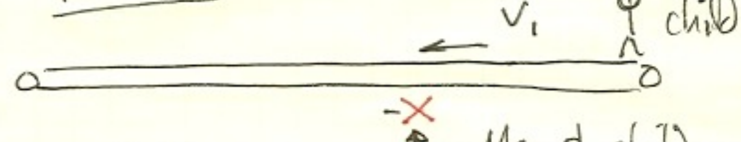


v_2 relative to belt, or $v_1 + v_2$ relative to outside reference frame.

a. $t_♀ = \frac{d}{s} = \boxed{\frac{L}{v_1}}$ L

b. $t_♂ = \boxed{\frac{L}{v_1 + v_2}}$

c.



Man & child meet each other here, perhaps, when they've both traveled the same time t, & they both have the same position x.

Man	Child
$\Delta x_m = vt$	$\Delta x_c = vt$
$= (v_1+v_2)t$	$x_f - x_i = vt$
$x_f - x_i =$	$x_c - L = (-v_1)t$
$x_f - 0 = (v_1+v_2)t$	
$x_m = (v_1+v_2)t$	

starts at end of belt

traveling in opposite direction

$x_m = x_c$

$(v_1 + v_2)t = L - v_1 t$

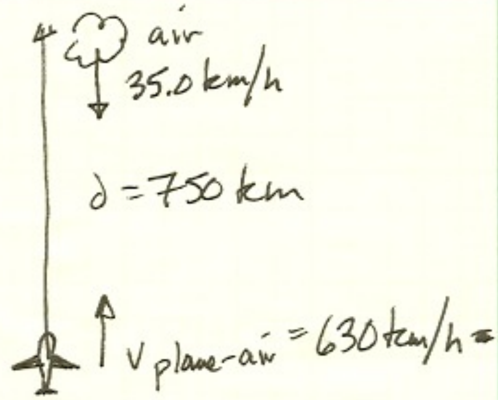
$tv_1 + tv_2 + tv_1 = L$

$t = \boxed{\frac{L}{2v_1 + v_2}}$

4.37

a. $t = \frac{d}{s} = \frac{750\text{ km}}{630 - 35}$

$= \boxed{1.26\text{ hrs}}$

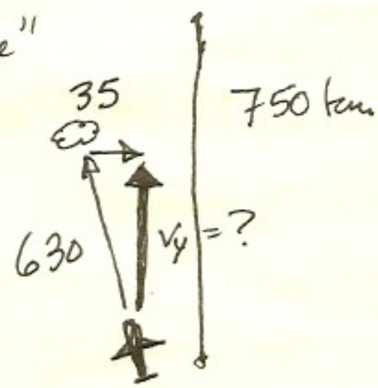


b. $t = \frac{d}{s} = \frac{750\text{ km}}{630 + 35}$

$= \boxed{1.13\text{ hrs}}$

c. Plane has to fly "off course" through air to be able to arrive at destination.

V_{net} in direction of destination:



$a^2 + b^2 = c^2$

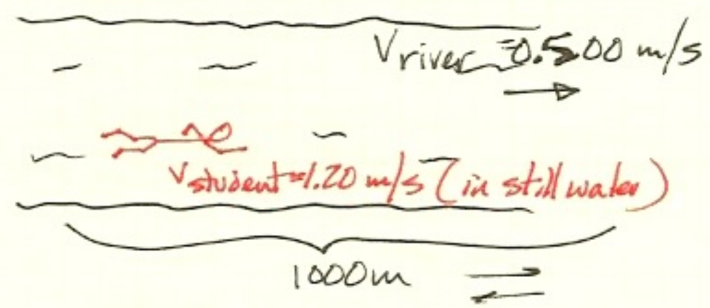
$V_y^2 + 35^2 = 630^2$

$V_y = \sqrt{630^2 - 35^2}$

$= 629\text{ km/hr}$

$t = \frac{d}{s} = \frac{750}{629} = \boxed{1.19\text{ hrs}}$

4.40



a. Time for 1 km roundtrip?

$$t_{there} = \frac{d}{s} = \frac{1000 \text{ m}}{(1.2+0.5)\text{ m/s}} = 588\text{ s}$$

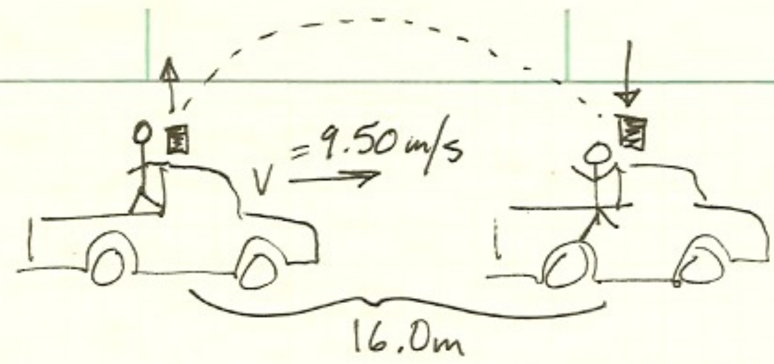
$$t_{back} = \frac{d}{s} = \frac{1000}{1.2-0.5} = 1429\text{ s}$$

$$t_{total} = 2.02e3\text{ s} \approx 34 \text{ minutes}$$

b In still water $t = \frac{d}{v} = \frac{2000\text{ m}}{1.2\text{ m/s}} = \boxed{1.67e3\text{ s}}$

c Intuitively, the swimmer has a higher average speed on the way there, & a lower average speed on the way back, so you might think the times would cancel out. But the swimmer spends more time in the water at the lower speed, so we expect that return leg to have a greater influence on the time.

4.42



a) Relative to the truck, the boy throws the can straight up, & it falls straight back down to him.

b) Initial speed of can up is...?
Let's see how much time it's in the ~~air~~ air.

Horizontally: $t = \frac{d}{s} = \frac{16\,m}{9.50\,m/s} = \underline{1.68s}$

Now, vertically: $v_f = v_i + at$

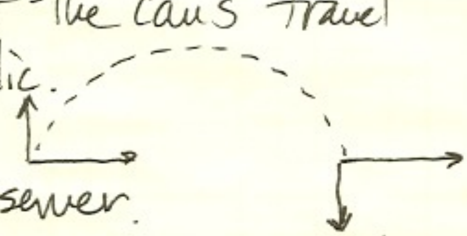
$$0 = v_i + (-9.8)\left(\frac{1.68s}{2}\right)$$

$$v_i = \boxed{8.23\,m/s}$$

At top of path

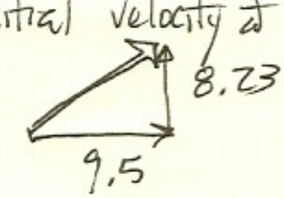
Only half the time to go up, the other half to fall back down.

d) The shape of the can's travel is parabolic to an outside observer.



c) The shape of the can's trajectory is straight up & down, relative to the boy in the truck.

e) Initial velocity of can is combination of v_x & v_y:



$$v = \sqrt{8.23^2 + 9.50^2} = \boxed{12.6\,m/s}$$

$$\theta = \tan^{-1}\left(\frac{8.23}{9.50}\right) = \boxed{@40.9°}$$