CHAPTER 3:
VECTORS AND SCALARS

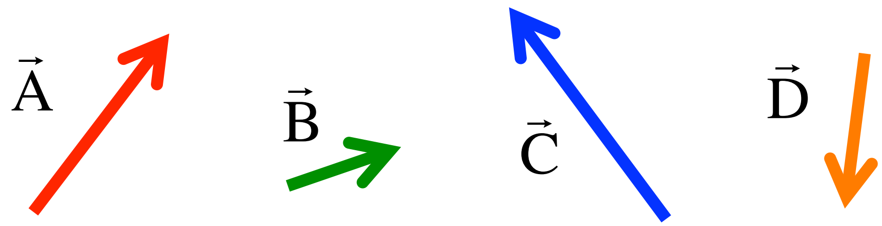
Scalars (magnitude only)

$T = 78^{\circ}F$ (not 78 degrees *down* or *up,* just 78 degrees—just a relative measure of the average amount of kinetic energy per molecule of air)

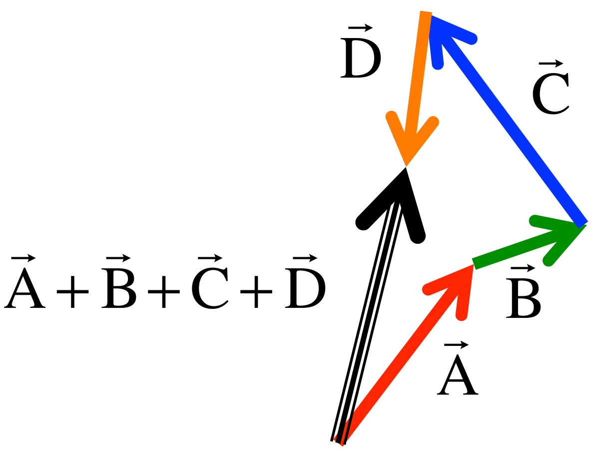
Vectors (magnitude and direction . . . direction matters!)

There are two ways to deal with vectors, graphically and in conjunction with a coordinate axis. We'll start with the graphical approach first.

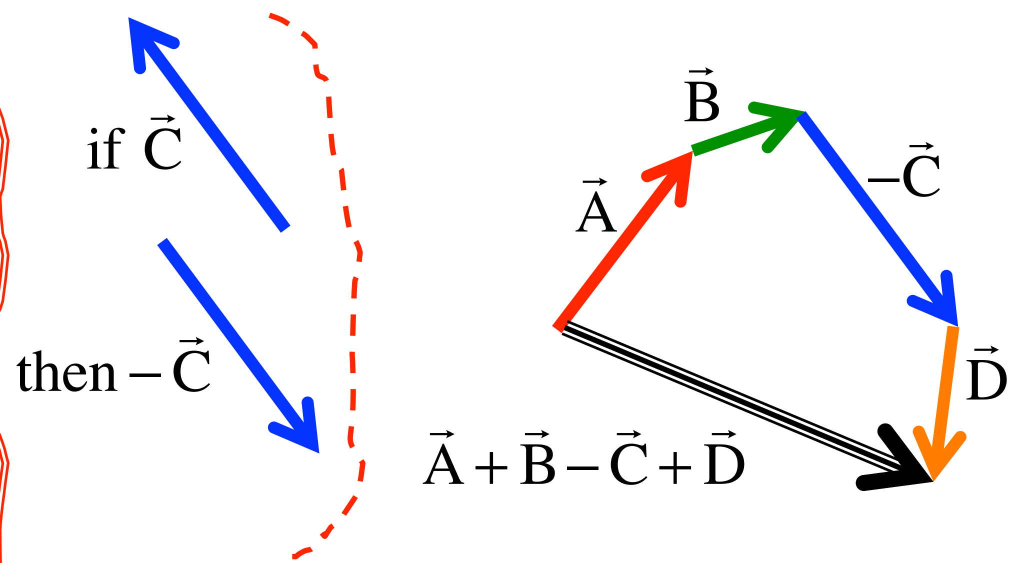
STRAIGHT GRAPHICAL VECTOR ADDITION/ SUBTRACTION



$\vec{A}+\vec{B}+\vec{C}+\vec{D}$



$\vec{A}+\vec{B}-\vec{C}+\vec{D}$



GRAPHICAL MANIPULATION FROM POLAR INFORMATION

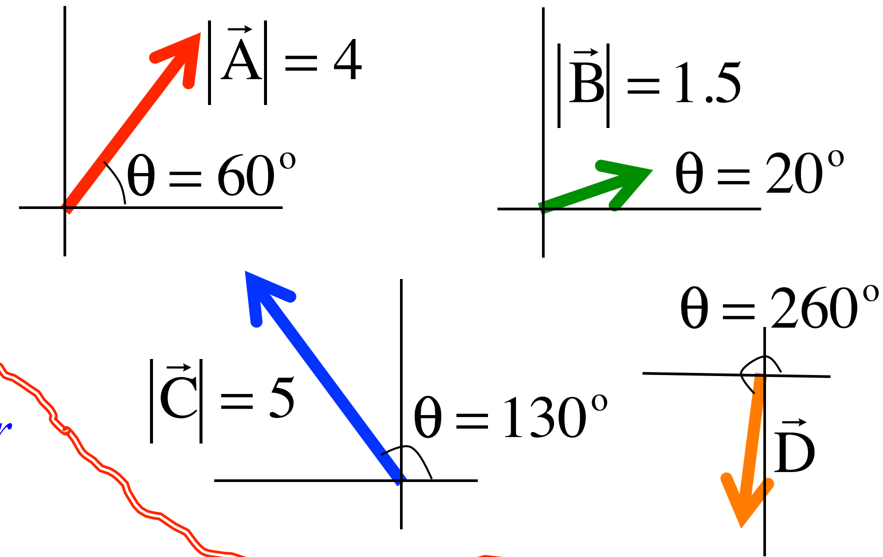
$\vec{A} = 4\measuredangle 60^\circ$

$\vec{B} = 1.5\measuredangle 20^\circ$

$\vec{C} = 5\measuredangle 130^\circ$

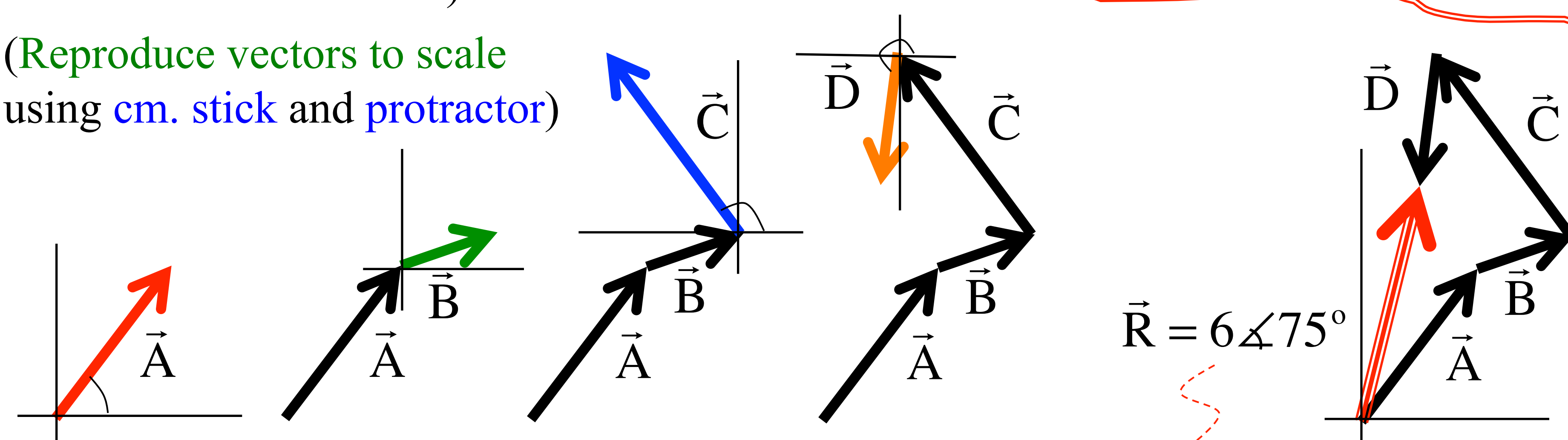
$\vec{D} = 3\measuredangle 260^\circ$

individual vectors:



(*VECTOR ADDITION* using a *centimeter stick* and protractor to measure both the vectors and the resultant)

(Reproduce vectors to scale using cm. stick and protractor)



(using cm. stick and protractor)

VECTORS

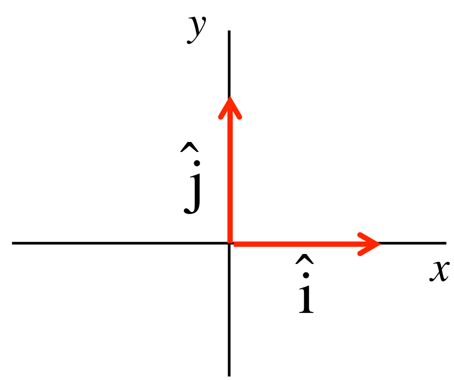
Vectors (defined in *polar notation*)

$$\vec{F} = (8 \text{ nt}) \measuredangle 125^\circ$$

(a force whose magnitude is 8 newtons oriented at an angle of 125 degrees relative to the +x-axis)

Vectors (defined in *unit vector notation*)

A vector with magnitude ONE defined to be in the x-direction is called a UNIT VECTOR in the x-direction. It's symbol is $\hat{i}$ (pronounced "i-hat"). The unit vector in the y-direction is $\hat{j}$.



A vector framed in Cartesian coordinates in *unit vector notation* might look like:

$$\vec{v} = (3 \text{ m/s})\hat{i} + (4 \text{ m/s})(-\hat{j})$$

(this is really the addition of a mini-vector of magnitude 3 m/s in the x-direction and a min-vector of magnitude 4 in the minus y-direction)

CONVERSIONS

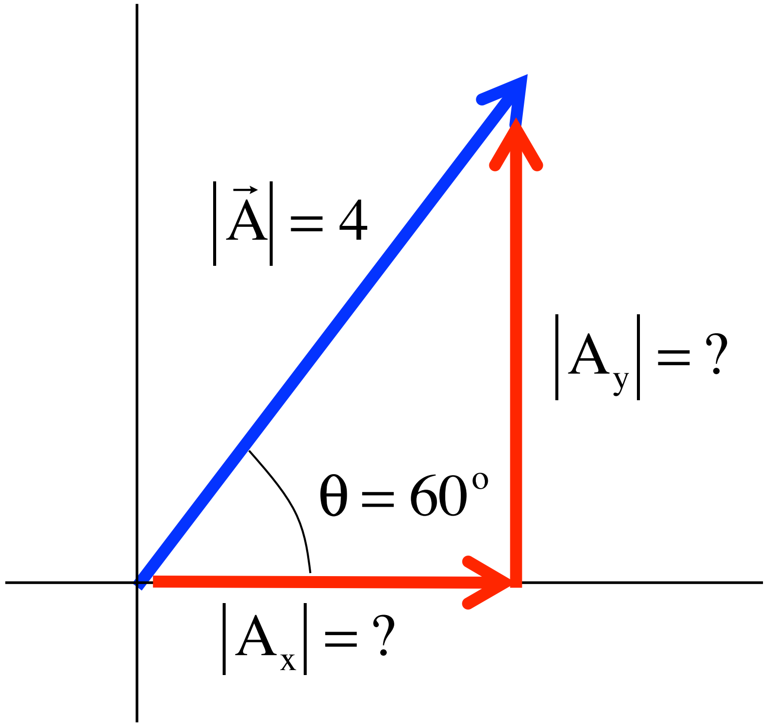
POLAR TO UNIT VECTOR NOTATION

Example 1:

$$\vec{A} = 4\measuredangle 60^\circ \text{ to unit vector}$$

In looking at the sketch, you can use trig to write:

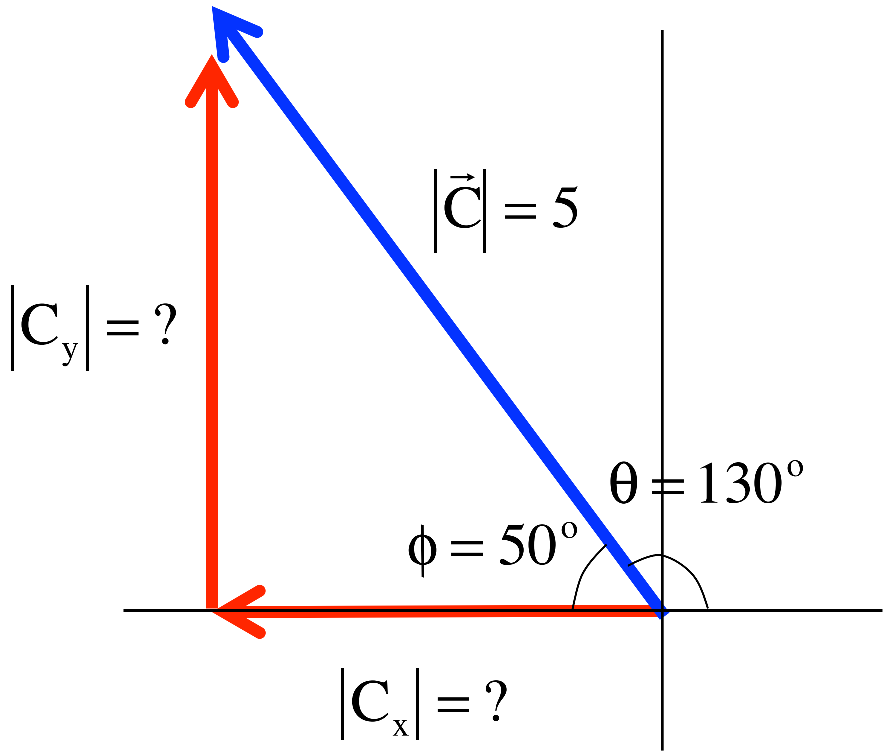
$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$
$$= \left(\left|\vec{A}\right|\cos\theta\right)\hat{i} + \left(\left|\vec{A}\right|\sin\theta\right)\hat{j}$$
$$= \left(4\cos 60^\circ\right)\hat{i} + \left(4\sin 60^\circ\right)\hat{j}$$
$$= 2\hat{i} + 3.46\hat{j}$$



Example 2:

$$\vec{C} = 5\measuredangle 130^{o} \text{ to unit vector}$$

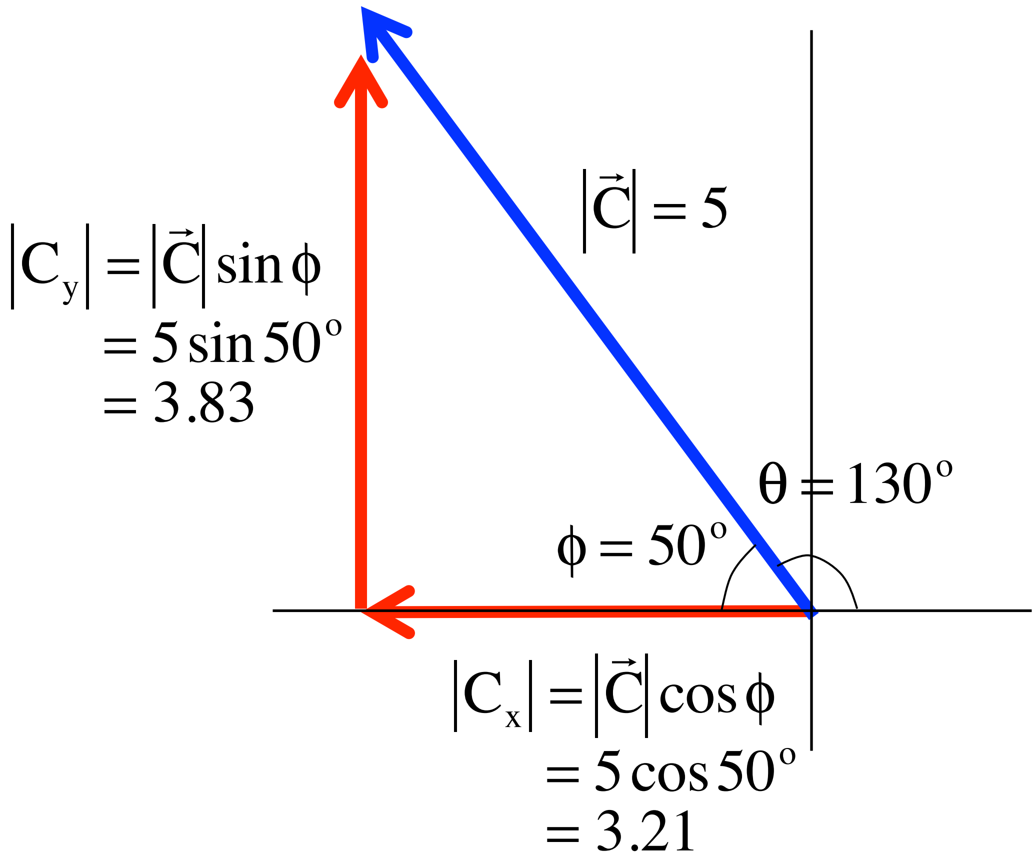
This is a little bit trickier because you are no longer looking at a first-quadrant triangle. There are two ways to do this. The easiest is to create a triangle that IS a right triangle (see sketch), determine it's sides, then add whatever signs and unit vectors are needed to characterize the vector. Remember, what you are doing with unit vector notation is creating mini-vectors, one in the x-direction, one in the y-direction, and adding them.



Example 2:

$$\vec{C} = 5\measuredangle 130^{o} \text{ to unit vector}$$

In looking at the sketch, you can either use the magnitudes, which are always positive, and manually put in the signs for the unit vectors, yielding:



$$\begin{aligned}\vec{C} &= |C_x|\left(\pm\hat{i}\right) + |C_y|\left(\pm\hat{j}\right)\\ &= 3.21\left(-\hat{i}\right) + 3.83\hat{j}\\ &= 3.21\left(-\hat{i}\right) + 3.83\hat{j}\end{aligned}$$

OR write it in terms of components, which carry along the signs with them, yielding:

$$\begin{aligned}\vec{C} &= C_x\hat{i} + C_y\hat{j}\\ &= (-3.21)\hat{i} + 3.83\hat{j}\end{aligned}$$

You get the same result either way.

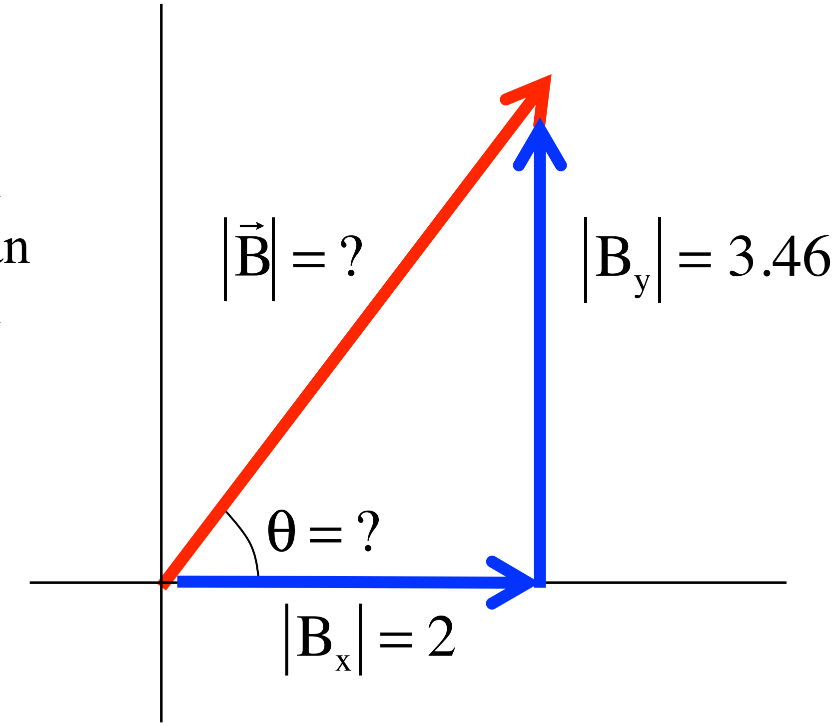
Unit Vector to Polar Notation

Example 1:

$$\vec{A} = 2\hat{i} + 3.46\hat{j} \quad \text{to polar}$$

In looking at the sketch and noting that you can get the magnitude using the Pythagorean relationship and the angle using the tangent function, we can write:

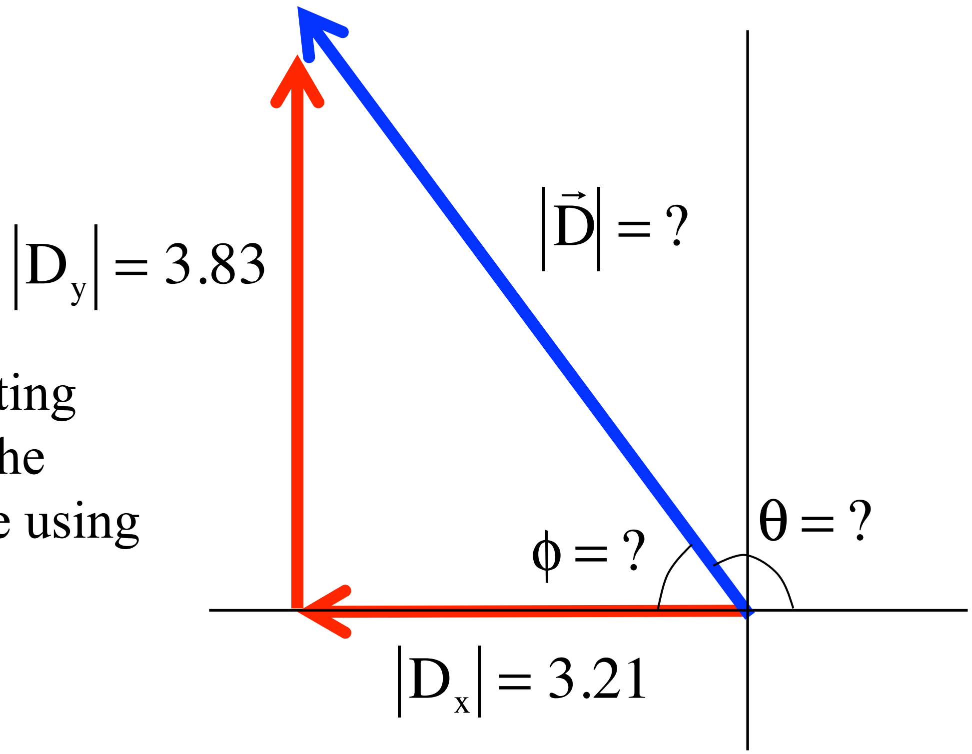
$$\begin{aligned}
\vec{A} &= \left|\vec{A}\right| \measuredangle \theta \\
&= \left[(A_x)^2 + (A_y)^2\right]^{1/2} \measuredangle \tan^{-1}\left(\frac{A_y}{A_x}\right) \\
&= \left[(2)^2 + (3.46)^2\right]^{1/2} \measuredangle \tan^{-1}\left(\frac{3.46}{2}\right) \\
&= 4 \measuredangle 60^{o}
\end{aligned}$$



Note that $\left|\vec{A}\right|$ is ALWAYS positive.

Example 2:

$$\vec{C} = -3.21\hat{i} + 3.83\hat{j} \quad \text{to polar}$$



Again, in looking at the sketch and noting that you can get the magnitude using the Pythagorean relationship and the angle using the tangent function, we can write:

$$\vec{C} = |\vec{C}| \measuredangle \theta$$

$$= \left[(C_x)^2 + (C_y)^2 \right]^{1/2} \measuredangle \tan^{-1}\left(\frac{C_y}{C_x} \right)$$

$$= \left[(-3.21)^2 + (3.83)^2 \right]^{1/2} \measuredangle \tan^{-1}\left(\frac{3.83}{-3.21} \right)$$

$= 5\measuredangle - 40^\circ$ except this clearly isn't a fourth quadrant vector, so we need to add 180° to get it into the third quadrant . . .

$$\Rightarrow \quad \vec{C} = 5\measuredangle(-40^\circ + 180^\circ) = 5\measuredangle 130^\circ$$