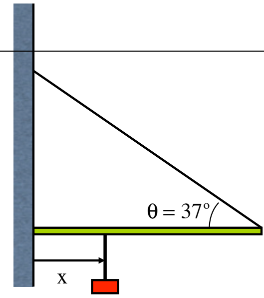
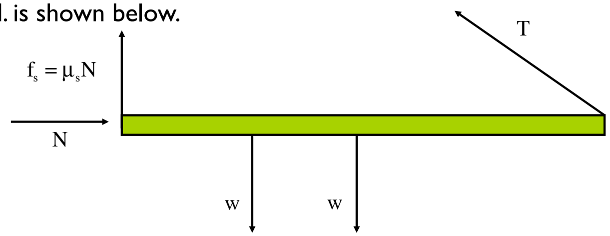
Problem 8.30

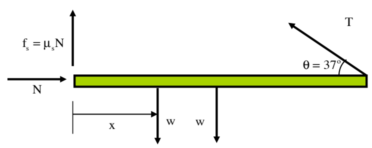
The 4 meters long rod of weigh “w” has a wire at one end and is held by friction against a wall. If the coefficient of friction between the wall and the rod is $\mu_s = .5$, at what minimum distance “x” can a weight “w” be placed and have the rod continue to stay in position.



A f.b.d. is shown below.



The position of “w” is variable and needs to be determined (that’s the question, in fact). We also need to determine the frictional force which means we need the normal force. To get the normal force, we need the tension. In other words, we have three unknowns to deal with. That means we will have to sum the forces in both direction AND sum the torque about some convenient point.



$\sum F_x :$

$$N - T\cos 37° = m a^{=0}$$

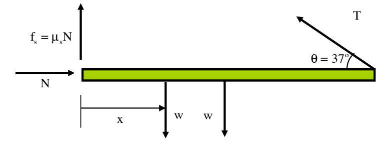
$$\Rightarrow \quad N = T\cos 37°$$

$\sum F_y :$

$$\mu_s N + T\sin 37° - w - w = m a^{=0}$$

$$\Rightarrow \mu_s\left(T\cos 37°\right) + T\sin 37° - 2w = 0$$

$\sum \Gamma_{wall} :$



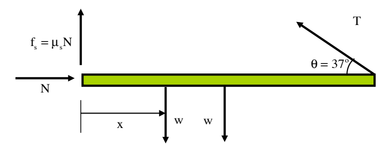
$$\Gamma_N^{=0} + \Gamma_f^{=0} + \Gamma_{wx} + \Gamma_w + \Gamma_T = I\alpha^{=0}$$

$$\Rightarrow \quad -(w)(x) - (w)\left(\frac{L}{2}\right) + \left(T\sin 37°\right)(L) = 0$$

$$\Rightarrow \quad T = \frac{wx + \frac{L}{2}w}{\sin 37°}$$

Note: You should be able to look at the torque calculations above and tell which approach I was using. Do that now before looking at the answers on the next page.

$\sum \Gamma_{wall} :$



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ANSWER TO QUESTION: The first torque calculation on the left: This was done using the $r_\perp$ approach. You can tell because the SHORTEST DISTANCE between “the point about which the torque is taken” and “the line of the weight’s force” is, simply, “x.” Multiplying the force by $r_\perp$ gives you what you are seeing. The same was done for the torque due to the weight of the beam acting at “L/2.”

The last torque calculation was done using the $F_\perp$ approach. You can tell that because the “ $T\sin 37°$ ” term is the component of “T” perpendicular to “r.”

Sooo:

$$\mu_s\left(T\cos 37^\circ\right)+T\sin 37^\circ-2w=0$$

$$\Rightarrow \quad T\left(\mu_s\left(\cos 37^\circ\right)+\sin 37^\circ\right)=2w$$

$$\Rightarrow \quad \left(\frac{wx+\frac{L}{2}w}{\sin 37^\circ}\right)\left(\mu_s\left(\cos 37^\circ\right)+\sin 37^\circ\right)=2w$$

$$\Rightarrow \quad \left(\frac{wx+\frac{L}{2}w}{\sin 37^\circ}\mu_s\left(\cos 37^\circ\right)\right)+\left(\frac{wx+\frac{L}{2}w}{\sin 37^\circ}\sin 37^\circ\right)=2w$$

$$\Rightarrow \quad \left(wx+\frac{L}{2}w\right)\mu_s\left(\cot 37^\circ\right)+\left(wx+\frac{L}{2}w\right)=2w$$

$$\Rightarrow \quad (wx)\mu_s\left(\cot 37^\circ\right)+\left(\frac{L}{2}w\right)\mu_s\left(\cot 37^\circ\right)+wx+\frac{L}{2}w=2w$$

$$\Rightarrow \quad x=\frac{2-\frac{L}{2}-\left(\frac{L}{2}\right)\mu_s\left(\cot 37^\circ\right)}{\mu_s\left(\cot 37^\circ\right)+1}$$

$$\Rightarrow \quad x=\frac{2-\frac{4}{2}-\left(\frac{4}{2}\right)(.5)\left(\cot 37^\circ\right)}{(.5)\left(\cot 37^\circ\right)+1}$$

$$\Rightarrow \quad x=.8 \text{ meter}$$