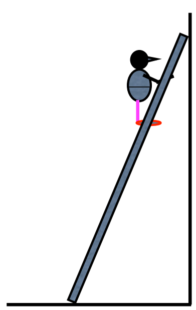
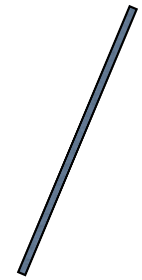
NOTE: In a conceptual sense, if a force on a body produces a torque about some point, it means that force is motivating the body to rotate about that point. Put a little differently, if you do a torque calculation for a force acting on a body and get zero, it means the force will NOT motivate the body to rotate about the point in question.

A ladder of length L and mass m_L sits at an angle θ. A man of mass m_m stands a distance $2L/3$ meters up its length. If the wall is frictionless, what are the forces acting at the wall and at the floor.

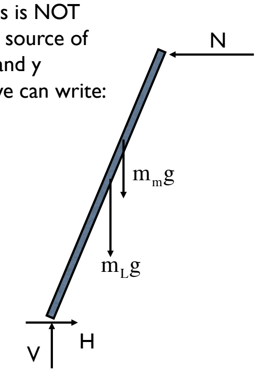




According to the f.b.d., we have three unknowns. Although this is NOT what you will probably do first on your test, the most obvious source of equations is to use the translational version of N.S.L. in the x and y directions. Noting that the acceleration is zero everywhere, we can write:

$$\underline{\sum F_x :}$$
$$H - N = m\cancel{a_x}^{=0}$$
$$\Rightarrow \quad H = N$$

$$\underline{\sum F_y :}$$
$$V - m_L g - m_m g = m\cancel{a_y}^{=0}$$
$$\Rightarrow \quad V = m_L g + m_m g$$



Three unknowns and two equations--we need another equation. That will come from summing the torques about a convenient point and put that equal to zero (as the angular acceleration about any point on the ladder will be zero).

For educational purposes, we are going to do torque calculations about two points, one about the *center of mass* and the other about the *contact point* with the floor.

We also have several ways to do the torque calculations--the definition approach, the "r-perpendicular" approach (i.e., the moment-arm approach), and the "F-perpendicular" approach. Using all three, we will begin with the calculation relative to the center mass of the ladder.

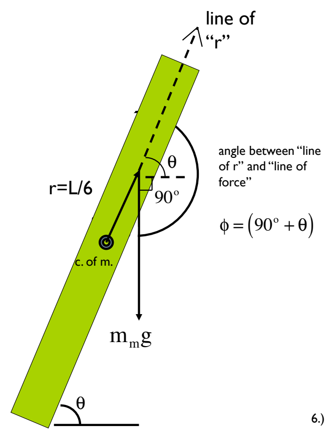
We will do this in pieces so you can follow the drift, then I'll put it all together and write it out in the way you would on a test (assuming you'd used this particular path on the test, which is not likely as it is more complicated than need be . . . but, again, educational, so read it!).

a.) The LADDER'S WEIGHT acts through the ladder's center of mass, so the torque due to that force about the center of mass will be zero. (This makes sense as that force will provide no rotational impetus for rotation about the center of mass.)

b.) The torque generated by the MAN'S WEIGHT, relative to the center of mass, is difficult only in the sense that determine the angle between the line of "F" and "r" is not obvious. The diagram should help. Using the "definition" approach:

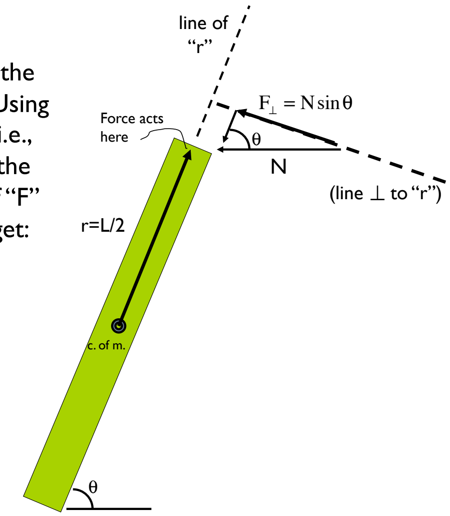
$$\Gamma_{m_m g} = \pm \ |F| \ \ |r| \ \ \sin\phi$$

$$= -\left(m_m g\right)\left(\frac{L}{6}\right)\sin\left(90^\circ + \theta\right)$$



Note: the torque is negative because the force involved is trying to motivate the ladder to rotate in a CLOCKWISE direction, relative to the center of mass.

c.) The torque generated by the NORMAL at the wall, relative to the center of mass, is the next stop. Using the "F-perpendicular" approach (i.e., the product of the magnitude of the "r" vector and the component of "F" that is perpendicular to "r"), we get:



$$\Gamma_N = \pm \ |F_\perp| \ \ |r|$$

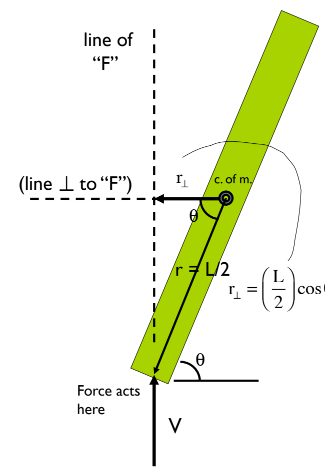
$$= +\left(N\sin\theta\right)\left(\frac{L}{2}\right)$$

Note: the torque is positive because the force involved is trying to motivate the ladder to rotate in a COUNTER-CLOCKWISE direction, relative to the *center of mass*.

c.) Next is the torque generated by the normal force "V" at the floor, relative to the center of mass. Using the "r-perpendicular" approach (i.e., the product of the magnitude of the "F" vector and the component of "r" that is perpendicular to "F"--this is called "the moment arm," by the way), we get:

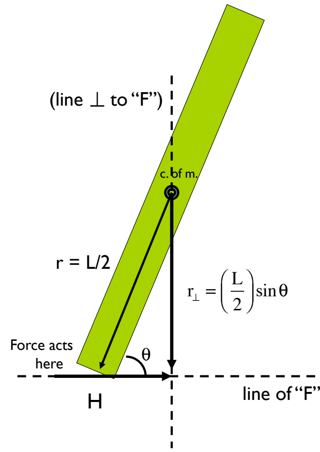
$$\Gamma_N = \pm |F| \ \ |r_\perp|$$

$$= -\left(V\right)\left(\frac{L}{2}\cos\theta\right)$$



Note: the torque is negative because the force involved is trying to motivate the ladder to rotate in a CLOCKWISE direction, relative to the *center of mass*.

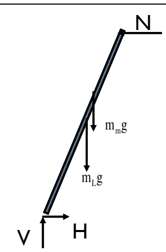
c.) Next is the torque generated by the frictional force "H" at the floor, relative to the center of mass. Using the "r-perpendicular" approach again (i.e., the product of the magnitude of the "F" vector and the component of "r" that is perpendicular to "F"--this is called "the moment arm," by the way), we get:



$$\Gamma_N = \pm|F| \quad |r_\perp|$$
$$= +(H)\left(\frac{L}{2}\sin\theta\right)$$

Note: the torque is positive because the force involved is trying to motivate the ladder to rotate in a COUNTERCLOCKWISE direction, relative to the *center of mass*.

c.) The sum of the torques, therefore, looks like:



$\underline{\sum \Gamma_{cm}}$:

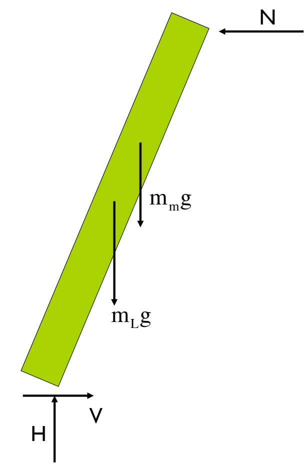
$$\left|\Gamma_{m_L g}\right| \quad + \quad \left|\Gamma_{m_m g}\right| \quad + \quad \left|\Gamma_N\right| \quad + \quad \left|\Gamma_H\right| \quad + \quad \left|\Gamma_V\right| \quad = I\cancel{\alpha}^{=0}$$
$$0 \quad + \left[-(m_m g)\left(\frac{L}{6}\right)\sin(90^\circ+\theta)\right] + \left[(N\sin\theta)\left(\frac{L}{2}\right)\right] + \left[H\left(\frac{L}{2}\sin\theta\right)\right] + \left[-V\left(\frac{L}{2}\cos\theta\right)\right] = 0$$

$$\left|\Gamma_{m_L g}\right| \quad + \quad \left|\Gamma_{m_m g}\right| \quad + \quad \left|\Gamma_N\right| \quad + \quad \left|\Gamma_H\right| \quad + \quad \left|\Gamma_V\right| \quad = I\cancel{\alpha}^{=0}$$
$$0 \quad + \left[-(m_m g)\left(\frac{L}{6}\right)\sin(90^\circ+\theta)\right] + \left[(N\sin\theta)\left(\frac{L}{2}\right)\right] + \left[H\left(\frac{L}{2}\sin\theta\right)\right] + \left[-V\left(\frac{L}{2}\cos\theta\right)\right] = 0$$

From before, "N = H" and "$V = m_L g + m_m g$." As $\sin(90^\circ+\theta) = \cos\theta$ we can write:

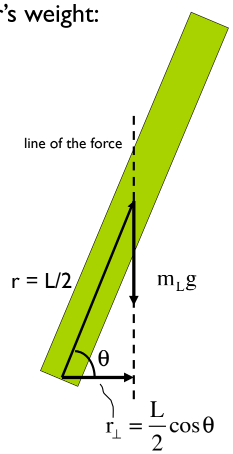
$$-(m_m g)\left(\frac{L}{6}\right)\sin(90^\circ+\theta) + (N\sin\theta)\left(\frac{L}{2}\right) + H\left(\frac{L}{2}\sin\theta\right) - V\left(\frac{L}{2}\cos\theta\right) = 0$$
$$\Rightarrow \quad -(m_m g)\left(\frac{\cancel{L}}{6}\right)\cos\theta + (N\sin\theta)\left(\frac{\cancel{L}}{2}\right) + N\left(\frac{\cancel{L}}{2}\sin\theta\right) - (m_L g + m_m g)\left(\frac{\cancel{L}}{2}\cos\theta\right) = 0$$
$$\Rightarrow \quad -\left(\frac{m_m g}{6}\right)\cos\theta - \left(\frac{m_L g + m_m g}{2}\cos\theta\right) = -\left(\frac{N}{2}\right)\sin\theta - \left(\frac{N}{2}\right)\sin\theta$$
$$\Rightarrow \quad \left(\frac{m_m g}{6} + \frac{m_L g + m_m g}{2}\right)\cos\theta = N\sin\theta$$
$$\Rightarrow \quad \left(\frac{m_m g}{6} + \frac{m_L g + m_m g}{2}\right)\frac{\cos\theta}{\sin\theta} = \left(\frac{m_m g}{6} + \frac{m_L g}{2} + \frac{m_m g}{2}\right)\frac{\cos\theta}{\sin\theta} = N$$
$$\Rightarrow \quad N = \left(\frac{2m_m g}{3} + \frac{m_L g}{2}\right)\cot\theta \qquad (= H)$$

There is an easier way to do all of this. We still have to sum the forces, but if we sum the torques about the *floor* instead of the *center of mass* the torques due to "H" and "V" will be zero and we can ignore them. That leaves torques for only two weights and "N." Using the "moment arm" approach (i.e., the r-perpendicular approach), each calculations will be presented below, then the whole things summarized as you would present the material on a test.



To get the torque due to the ladder's weight:

$$\Gamma_{m_L g} = -\ |F|\ \ |r_\perp|$$
$$= -(m_L g)\left(\frac{L}{2}\cos\theta\right)$$

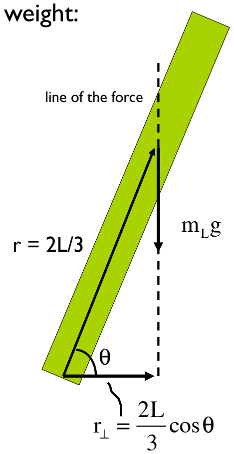


Note: The sign is negative as the force due to gravity on the ladder tends to make the ladder want to rotate clockwise, relative to the point of contact on the floor.

Note: $r_\perp$ is the shortest distance between the "line of force" and point about which torque is being taken . . .

To get the torque due to the man's weight:

$$\Gamma_{m_m g} = -\ |F|\ \ |r_\perp|$$
$$= -(m_m g)\left(\frac{2L}{3}\cos\theta\right)$$

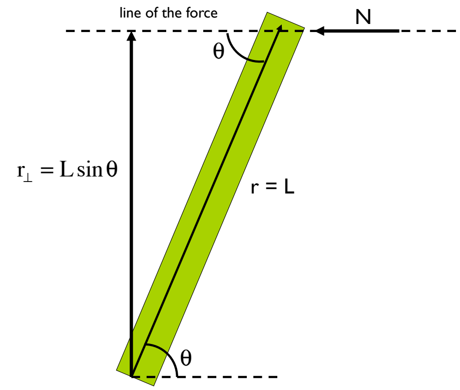


Note: The sign is negative as the force due to gravity on the man tends to make the ladder want to rotate clockwise, relative to the point of contact on the floor.

Note: $r_\perp$ is the shortest distance between the "line of force" and point about which torque is being taken . . .

To get the torque due to "N:"

$$\Gamma_N = |F|\ \ |r_\perp|$$
$$= (N)(L\sin\theta)$$



Note: The sign is positive as "N" makes the ladder want to rotate counterclockwise, relative to the point of contact on the floor.

Note: $r_\perp$ is the shortest distance between the "line of force" and point about which torque is being taken . . .

Summing up the torques yields:

$$\underline{\sum \Gamma_{cm}}:$$

$$-\left|\Gamma_{m_L g}\right| \quad - \quad \left|\Gamma_{m_m g}\right| \quad + \quad \left|\Gamma_N\right| \quad + \left|\Gamma_H\right| + \left|\Gamma_V\right| = I\cancel{\alpha}^{=0}$$

$$-(m_L g)\left(\frac{\cancel{L}}{2}\right)\cos\theta - (m_m g)\left(\frac{2\cancel{L}}{3}\right)\cos\theta + N(\cancel{L}\sin\theta) + \ 0 \ + \ 0 \ = \ 0$$

$$\Rightarrow \quad N = \frac{\left(\frac{m_L g}{2}\right)\cos\theta + \left(\frac{2m_m g}{3}\right)\cos\theta}{\sin\theta}$$

$$\Rightarrow \quad N = \left(\frac{2m_m g}{3} + \frac{m_L g}{2}\right)\cot\theta$$

This expression for "N" is exactly what we got when we summed up the forces in both the "x" and "y" directions, and then summed up the torques about the "center of mass" . . . except that has fewer steps and is easier . . .