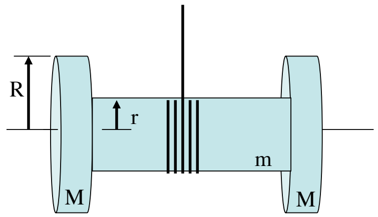
Problem 8.34

A yoyo is made up of two cylinders and a shaft, as shown to the right. The positive direction is to the left along the central axis of the yoyo.

a.) What's the moment of inertia about the central axis?



b.) What torque is generated about the central axis due to gravity?

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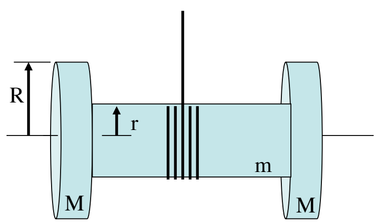


Summing the moment of inertia for all three disks yields:

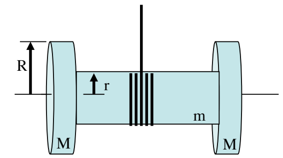
$$I_{total} = \frac{1}{2}mr^2 + \frac{1}{2}MR^2 + \frac{1}{2}MR^2$$

b.) What torque is generated about the central axis due to gravity?

Gravity acts through the center of mass. This is on the central axis, so the net torque about that axis will be zero (that is, gravity isn't going to make this body tend to angular accelerate):

c.) Taking downward to be negative:

i.) What is the direction of the torque due to tension?



ii.) What is the direction of the angular acceleration?

MINOR NOTE: If you use your right hand to mimic the actual motion of the yoyo experiences, you will notice that your thumb will point to the left, in the POSITIVE direction as originally defined.

iii.) What is the direction of the yoyo's acceleration?

c.) Taking downward to be negative:

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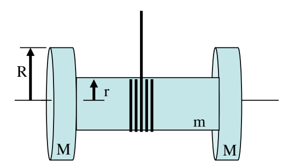


As looking from the left side rightward, tension is motivating the yoyo to rotate counterclockwise. That means the torque it produces is POSITIVE.

ii.) What is the direction of the angular acceleration?

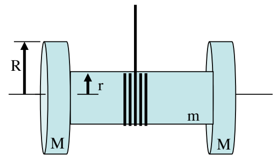
Relative to the central axis, the only force producing a torque is tension. If it's sign is POSITIVE, the angular acceleration's direction must also be POSITIVE.

MINOR NOTE: If you use your right hand to mimic the actual motion of the yoyo experiences, you will notice that your thumb will point to the left, in the POSITIVE direction as originally defined.

iii.) What is the direction of the yoyo's acceleration?

The body is accelerating downward. This has been defined as the NEGATIVE direction.

d.) Write an expression for the angular acceleration of the yoyo with the translational acceleration of the yoyo's center of mass.



e.) Write out N.S.L. for the translational motion of the yoyo.

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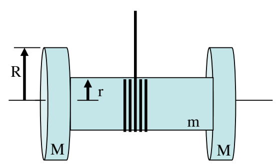


$$a = r\alpha$$

MINOR NOTE: Why wasn't it $a = R\alpha$?

This relationship relates the angular acceleration of a rotating body ABOUT A FIXED POINT, and the translational acceleration of a point from that fixed point. Though it may not be obvious, the point of contact between the string and the yoyo is a fixed point (the string isn't slipping--there would have to be slipping for that point to NOT be have zero velocity and zero acceleration). The radius that matters, in other words, is "r," not "R."

e.) Write out N.S.L. for the translational motion of the yoyo.

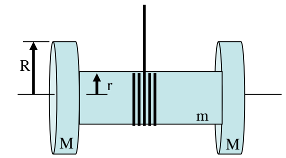
$\sum F_y$:

$$T - mg - Mg - Mg = -(m + 2M)a$$

$$\Rightarrow \quad T = mg + 2Mg - (m + 2M)a$$

$$\Rightarrow \quad T = (m + 2M)(g - a)$$

f.) Write out N.S.L. for the rotational motion of the yoyo.



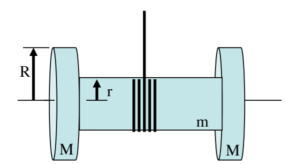
g.) Eliminate the angular acceleration term and substituting in for "T" in the above expression and determine the acceleration of the yoyo's center of mass.

f.) Write out N.S.L. for the rotational motion of the yoyo.

$\sum \Gamma_{\text{central axis}}$:

$$rT = I_{\text{central axis}}\alpha$$

$$rT = \left(\frac{1}{2}mr^2 + \frac{1}{2}MR^2 + \frac{1}{2}MR^2\right)\alpha$$



g.) Eliminate the angular acceleration term and substituting in for "T" in the above expression and determine the acceleration of the yoyo's center of mass.

$$r \quad T = \left(\frac{1}{2}mr^2 + \frac{1}{2}MR^2 + \frac{1}{2}MR^2\right)\alpha$$

$$r \quad (m + 2M)(g - a) = \frac{1}{2}\left(mr^2 + MR^2\right) \left(\frac{a}{r}\right)$$

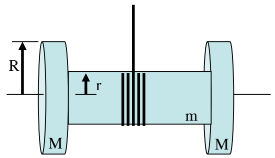
$$(m + 2M)g - (m + 2M)a = \frac{1}{2}\left(mr^2 + MR^2\right) \left(\frac{a}{r^2}\right)$$

$$(m + 2M)g - (m + 2M)a = \left(\frac{1}{2}m + \frac{M}{r^2}R^2\right)a$$

$$2Mg + mg = \left(\frac{1}{2}m + \frac{M}{r^2}R^2 + (m + 2M)\right)a$$

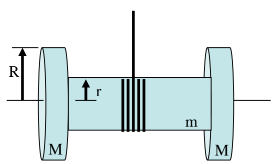
$$\Rightarrow \quad a = \frac{2Mg + mg}{\left(\frac{1}{2}m + \frac{M}{r^2}R^2 + (m + 2M)\right)}$$

h.) The numerical value of the acceleration?



i.) What is the tension?

h.) The numerical value of the acceleration?



$$a = \frac{2Mg + mg}{\left(\frac{1}{2}m + \frac{M}{r^2}R^2 + (m + 2M)\right)}$$

$$= \frac{2(2\text{ kg})(9.8\text{m/s}^2) + (1\text{ kg})(9.8\text{m/s}^2)}{\frac{1}{2}(1\text{ kg}) + \frac{(2\text{ kg})}{(.04\text{ m})^2}(.1\text{ m})^2 + ((1\text{ kg}) + 2(2\text{ kg}))}$$

$$= 2.72\text{ m/s}^2$$

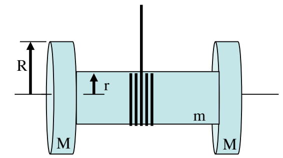
i.) What is the tension?

$$T = (m + 2M)(g - a)$$

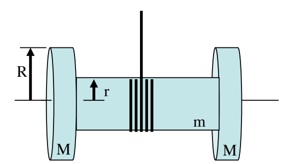
$$= ((1\text{ kg}) + 2(2\text{ kg}))((9.8\text{ m/s}^2) - (2.72\text{ m/s}^2))$$

$$= 35.4\text{ nts}$$

j.) How long does it take for the yoyo to drop 1 meter?



j.) How long does it take for the yoyo to drop 1 meter?



$$(y_2 - y_1) = v_o\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$(-1\text{ m}) = .5(-2.72\text{ m/s}^2)t^2$$

$$\Rightarrow \quad t = .857\text{ sec}$$