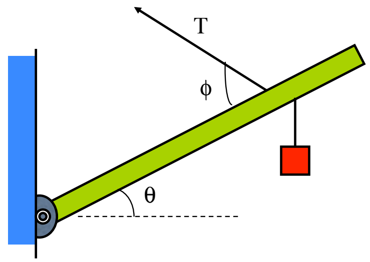
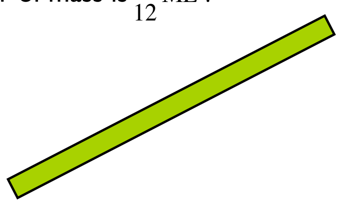
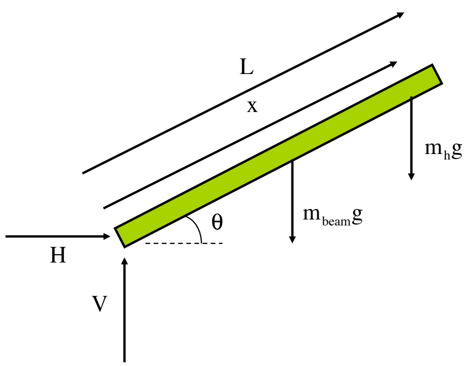
#8.19

You've done a problem in which the tension keeps the rigid body stationary. What is the angular acceleration of the beam if the rope is cut, and what is the acceleration of the hanging mass? You may assume the moment of inertia about the beam's center of mass is $\frac{1}{12}ML^2$.





f.b.d.



$$\sum \Gamma_{pin}:$$
$$\cancel{\Gamma_V}^{=0} + \cancel{\Gamma_H}^{=0} + \Gamma_{m_h g} + \Gamma_{m_b g} = I_{pin}\alpha$$

Notice that we were given the moment of inertia for an axis about the center of mass, but we don't know the moment of inertia for an axis about the pin. We have to use the *Parallel Axis Theorem* to determine that quantity. Doing so yields:

$$\begin{aligned} I_{pin} &= I_{cm} + mr^2 \\ &= \left(\frac{1}{12}mL^2\right) + m\left(\frac{L}{2}\right)^2 \\ &= \frac{1}{3}mL^2 \end{aligned}$$

Note: Remember that the "r" term in the *Parallel Axis Theorem* is the distance between the axis through the center of mass and the axis through the point you are interested in, which in this case is the axis through the pin. That is why r = L/2 in this problem.

With all of this, we can write:

$$\sum \Gamma_{pin}:$$
$$\cancel{\Gamma_V}^{=0} + \cancel{\Gamma_H}^{=0} + \Gamma_{m_h g} + \Gamma_{m_b g} = I_{pin}\alpha$$
$$\Rightarrow \quad -(m_h g)(x\cos\theta) - (m_{beam} g)\left(\frac{L}{2}\cos\theta\right) = \left(\frac{1}{3}m_{beam}L^2\right)\alpha$$
$$\Rightarrow \quad \alpha = \frac{(m_h g)(x\cos\theta) + (m_{beam} g)\left(\frac{L}{2}\cos\theta\right)}{\left(\frac{1}{3}m_{beam}L^2\right)}$$

Note that as $a_{cm} = \left(L/2\right)\alpha$, we can calculate the acceleration of the beam's center of mass, also.