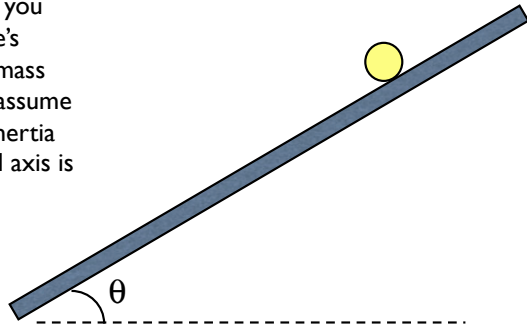


## A rolling stone named "Ball."

Determine the acceleration of the ball as it rolls down the incline. Assume you know the incline's angle, the ball's mass and radius, and assume its moment of inertia about its central axis is  $I = \frac{2}{5}mR^2$



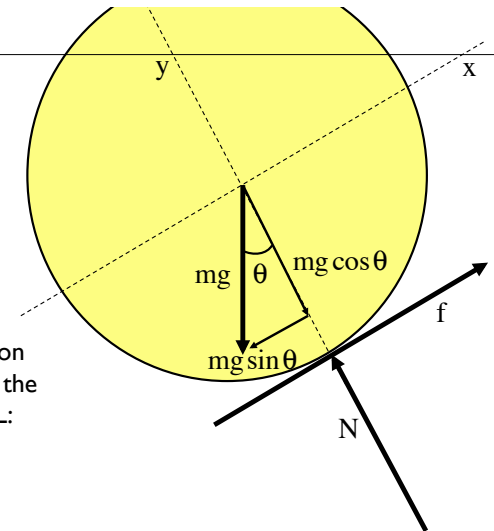
1.)

For the acceleration of the center of mass, the translational version of N.S.L:

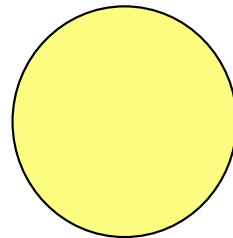
$$\sum F_x : \\ -mg \sin\theta + f = ma$$

For the angular acceleration about the center of mass, the rotational version of N.S.L:

$$\sum \Gamma_{cm} : \\ (f)(R) = I_{cm} \alpha \\ \Rightarrow (f)(R) = \left(\frac{2mR^2}{5}\right) \left(\frac{a}{R}\right) \\ \Rightarrow (f) = \left(\frac{2m}{5}\right) (a)$$



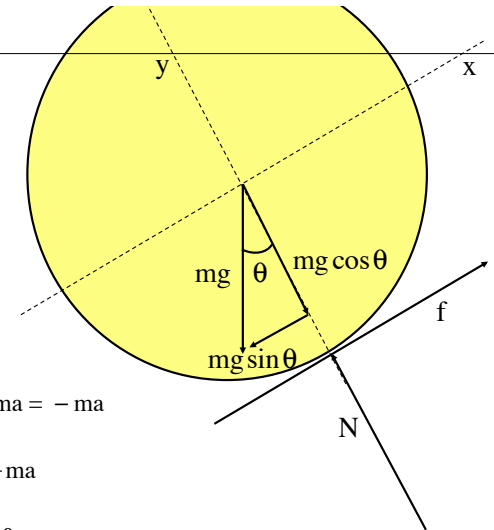
3.)



2.)

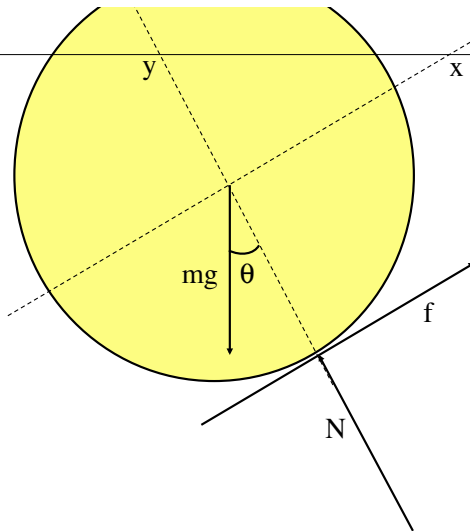
Combining we get:

$$-mg \sin\theta + f = -ma \\ \Rightarrow -mg \sin\theta + \frac{2}{5}ma = -ma \\ \Rightarrow mg \sin\theta = \frac{7}{5}ma \\ \Rightarrow a = \frac{5}{7}g \sin\theta$$



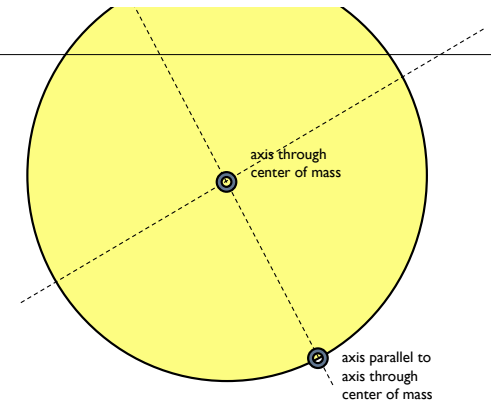
4.)

There is another possibility. The ball's velocity at the contact point is zero (otherwise, it would be sliding over the fixed incline). Let's look at things from that fixed point. Let's sum torques about that point.



5.)

The parallel axis theorem states that the moment of inertia about an axis parallel to one through a body's center of mass is equal to that moment of inertia plus a fudge factor. That fudge factor is equal to the mass of the object times the distance between the axes quantity squared. That is:



$$I_p = I_{cm} + md^2$$

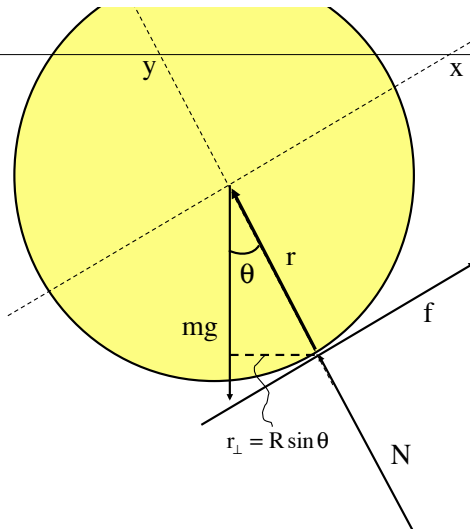
For the ball:

$$\begin{aligned} I_p &= I_{cm} + md^2 \\ &= \frac{2}{5}mR^2 + mR^2 \\ &= \frac{7}{5}mR^2 \end{aligned}$$

7.)

$$\sum \Gamma_p : (mg)(R \sin\theta) = I_p \alpha$$

To finish this off, we need to know the moment of inertia about the contact point "p," and the relationship between the acceleration of the center of mass and the angular acceleration of the ball about the contact point.



Note:  $r_{\perp}$  is the shortest distance between the "line of force" and point about which torque is being taken ...

6.)

With the moment of inertia of the parallel axis theorem and the known relationship between the angular acceleration and the acceleration of the center of mass, we can write:

$$\begin{aligned} \sum \Gamma_p : (mg)(R \sin\theta) &= I_p \alpha \\ \Rightarrow (mg)(R \sin\theta) &= \left(\frac{7}{5}mR^2\right) \left(\frac{a}{R}\right) \\ \Rightarrow a &= \frac{5}{7}mg \sin\theta \end{aligned}$$

8.)