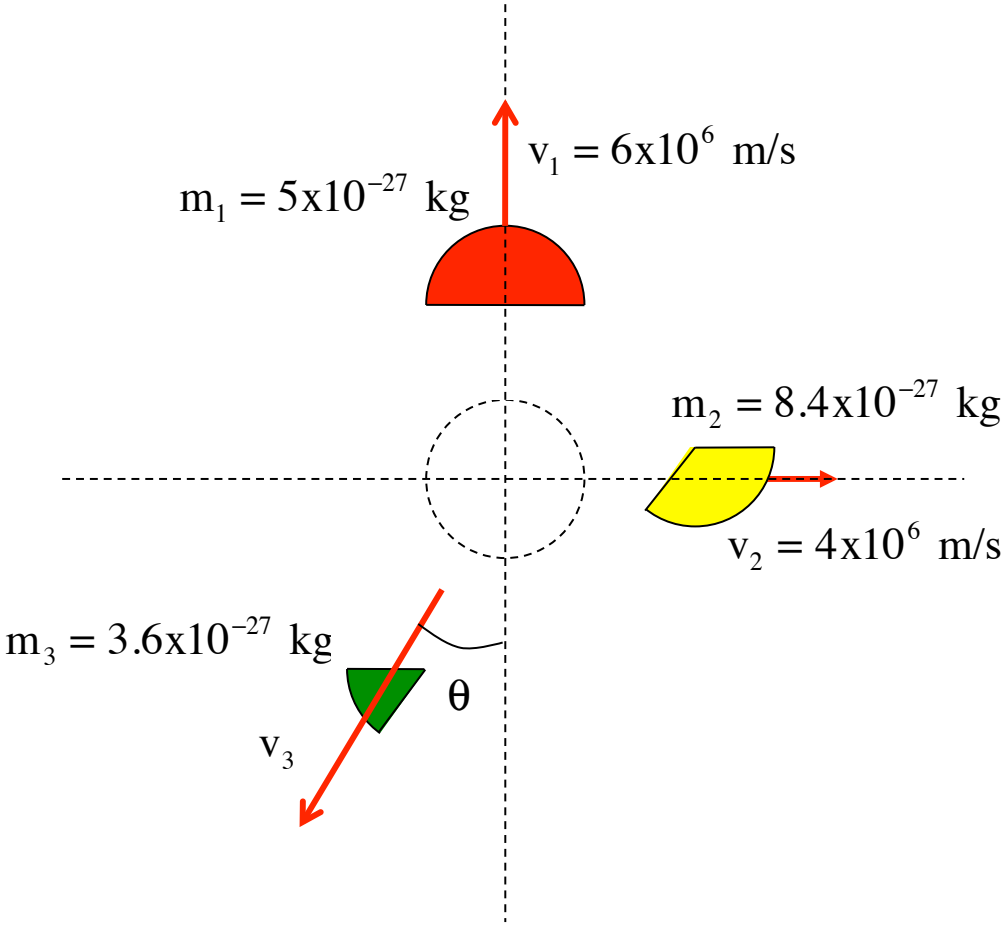
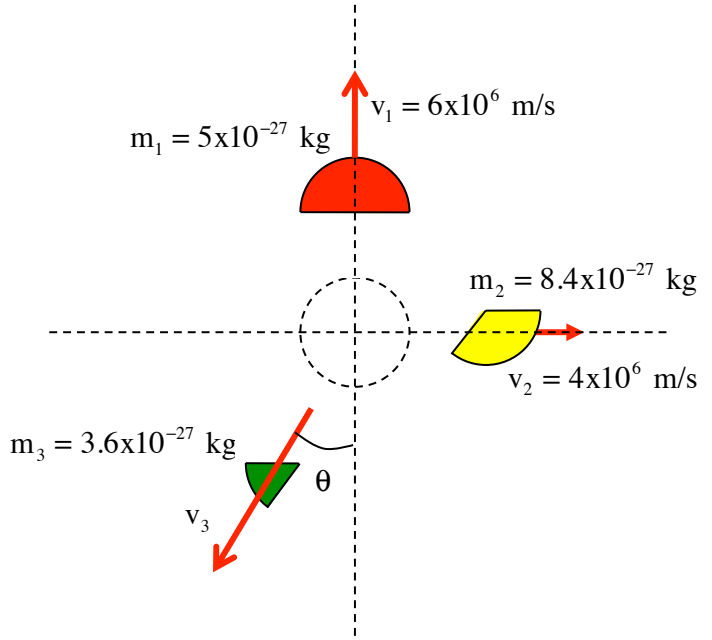
Problem 6.60

A nucleus decays ejecting three particles. Two have known velocity and direction. The third has unknown velocity and direction. Determine the unknown quantities.



The initial momentum in the system is zero, so the "final" momentum in both the x-direction and y-direction must add to zero. With that in mind, we can write:



In the x-direction:

$$\sum p_{initial,x} + \sum F_{ext,x}\Delta t = \sum p_{final,x}$$

$$0 + 0 = m_2 v_2 - m_3 v_3 \sin\theta$$

$$\Rightarrow \quad 0 = \left(8.4x10^{-27}\ \text{kg}\right)\left(4x10^6\ \text{m/s}\right) - \left(3.6x10^{-27}\ \text{kg}\right) v_3 \sin\theta$$

$$\Rightarrow \quad v_3 \sin\theta = 9.33x10^6\ \text{m/s}$$

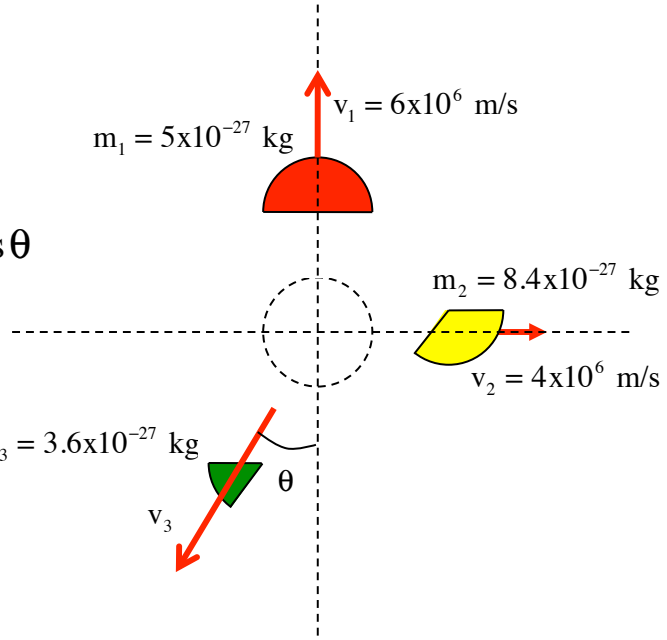
In the y-direction:

$$\sum p_{initial,y} + \sum F_{ext,y}\Delta t = \sum p_{final,y}$$

$$0 + 0 = m_1 v_1 - m_3 v_3 \cos\theta$$

$$\Rightarrow \quad 0 = \left(5x10^{-27}\ kg\right)\left(6x10^{6}\ m/s\right) - \left(3.6x10^{-27}\ kg\right) v_3 \cos\theta$$

$$\Rightarrow \quad v_3 \cos\theta = 8.33x10^{6}\ m/s$$



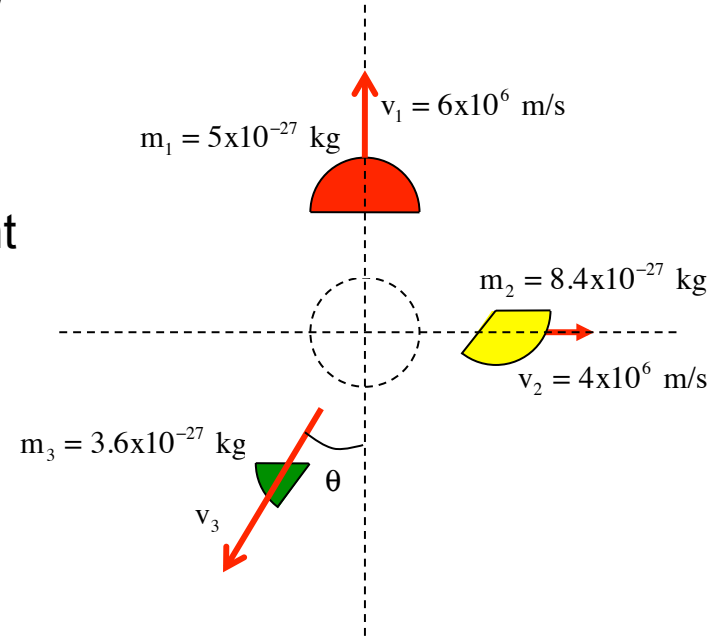
We have two equations:

$$v_3 \cos\theta = 8.33x10^{6} \qquad \text{and} \qquad v_3 \sin\theta = 9.33x10^{6}$$

Unfortunatley, we have three unknowns (the velocity, the sine of the angle and the cosine of the angle). So what to do? Trickery again!!!

$$v_3 \cos\theta = 8.33x10^6 \qquad \text{and} \qquad v_3 \sin\theta = 9.33x10^6$$

Take the two relationships and divide them into one another. That is, divide the left side of the second into the left side of the first, and the right side of the second into the right side of the first. Doing so will yield:



$$\frac{\cancel{v_3}\sin\theta}{\cancel{v_3}\cos\theta} = \frac{9.33x10^6}{8.33x10^6} = \tan\theta$$

With the velocities cancelled and the sine over cosine is tangent, we can write:

$$\tan\theta = \frac{9.33\cancel{x10^6}}{8.33\cancel{x10^6}} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{9.33}{8.33}\right) \quad \Rightarrow \quad \theta = 48.2^o$$

With the angle, we can use either the first or second relationship to write:

$$v_3 \sin\left(48.3^o\right) = 9.33x10^6 \quad \Rightarrow \quad v_3 = 1.25x10^7 \text{ m/s} \qquad \text{OR}$$

$$v_3 \cos\left(48.3^o\right) = 8.33x10^6 \quad \Rightarrow \quad v_3 = 1.25x10^7 \text{ m/s} \quad \text{(yeehaa!)}$$