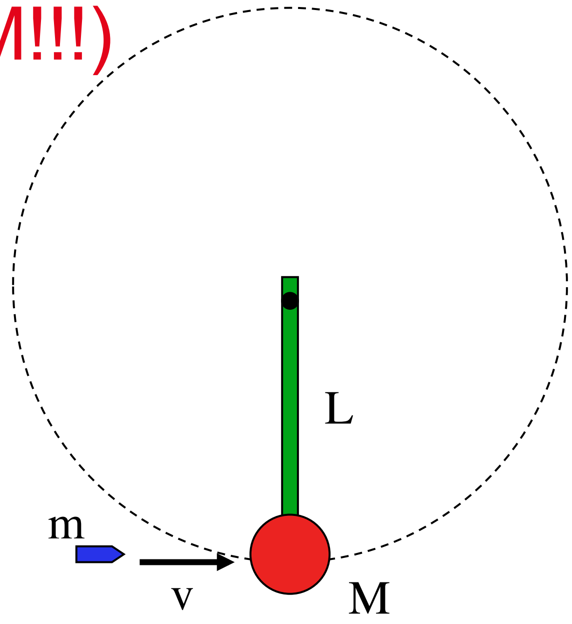
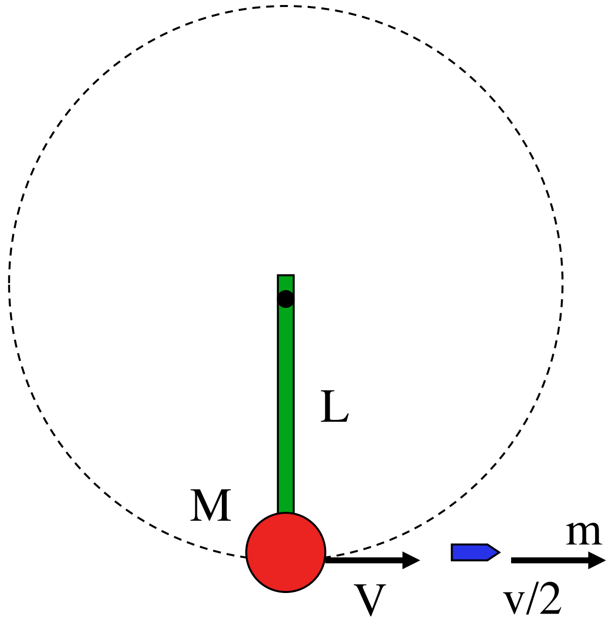
Problem 6.56 (GREAT PROBLEM!!!)

A bullet of mass "m" moving with velocity "v" in the horizontal passes through a pendulum bob of mass "M" leaving at "v/2." Suspended by a stiff but massless rod of length "L," what does "v" have to be so that the pendulum just makes it through the top of the arc?





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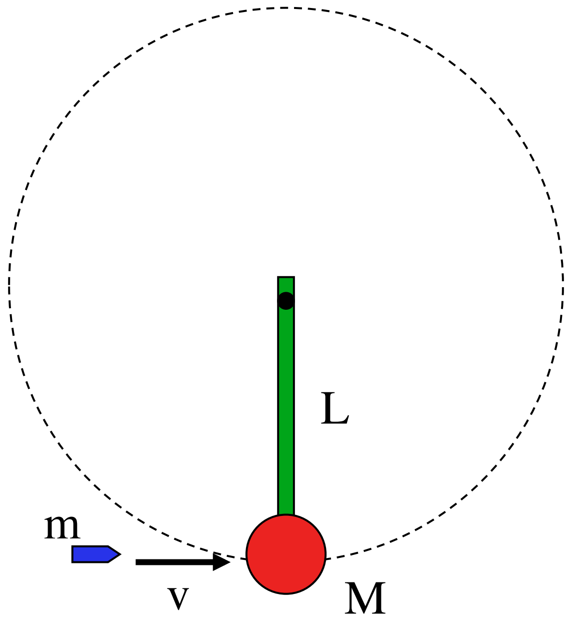
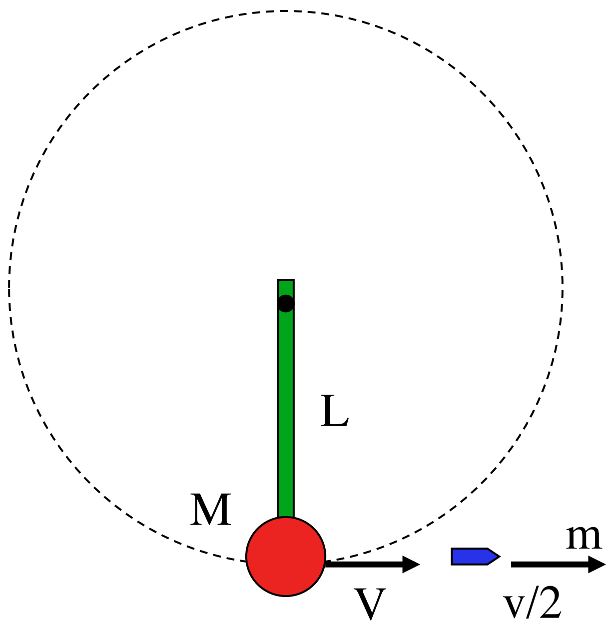


The first thing to notice is that the rod is rigid. In other words, all the ball has to do is get to the top of the arc and it will fall through from there. This is different than a block moving freely through the top of a loop where there needs to be velocity to keep the block on the track. Having said that, we need to use conservation of momentum through the collision and conservation of energy for the bob after the collision.



Conservation of momentum through the collision:

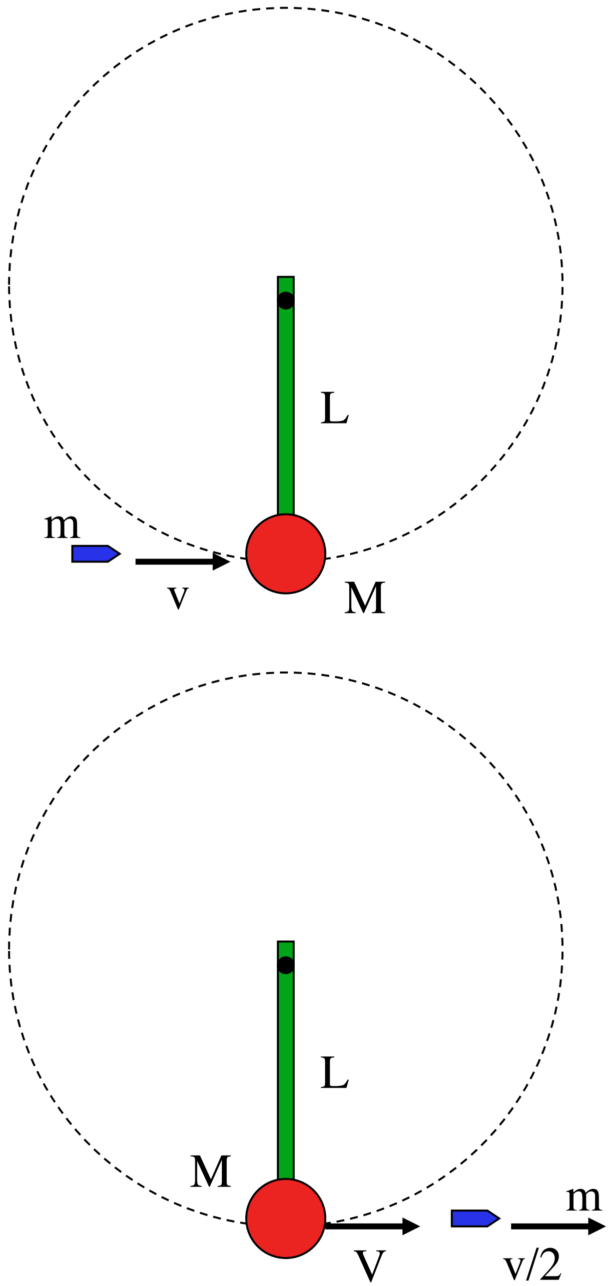
$$\sum p_{1,x} + \sum F_{ext,x}\Delta t = \sum p_{2,x}$$

$$mv \quad + \quad 0 \quad = m\left(v/2\right) + MV$$

$$\Rightarrow \quad V = \frac{mv - m\left(v/2\right)}{M}$$

$$\Rightarrow \quad V = \frac{m\left(v/2\right)}{M}$$

$$\Rightarrow \quad V = \frac{mv}{2M}$$



Conservation of energy on "M" after the collision:

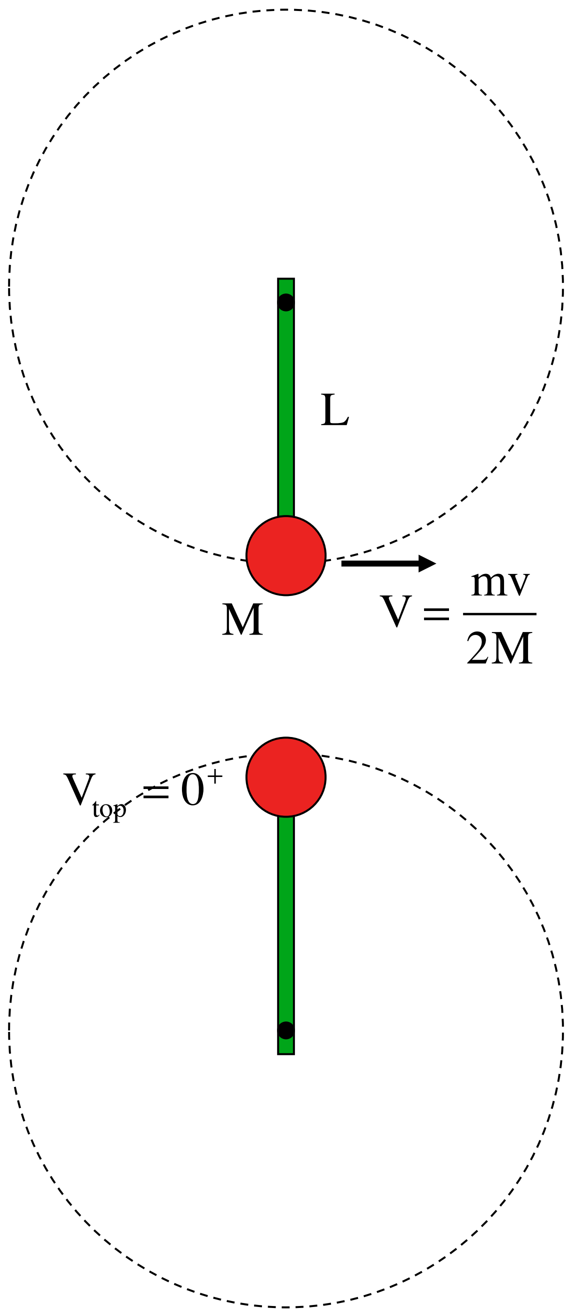
$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}MV^2 + 0 + 0 = 0 + Mg(2L)$$

$$\Rightarrow \frac{1}{2}M\left(\frac{mv}{2M}\right)^2 = 2MgL$$

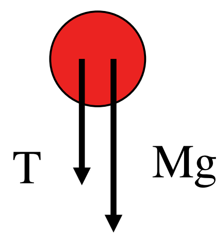
$$\Rightarrow v = \left(\frac{16gLM^2}{m^2}\right)^{1/2}$$

$$\Rightarrow v = \frac{4M}{m}(gL)^{1/2}$$



How would this have differred if the rod had been a string?

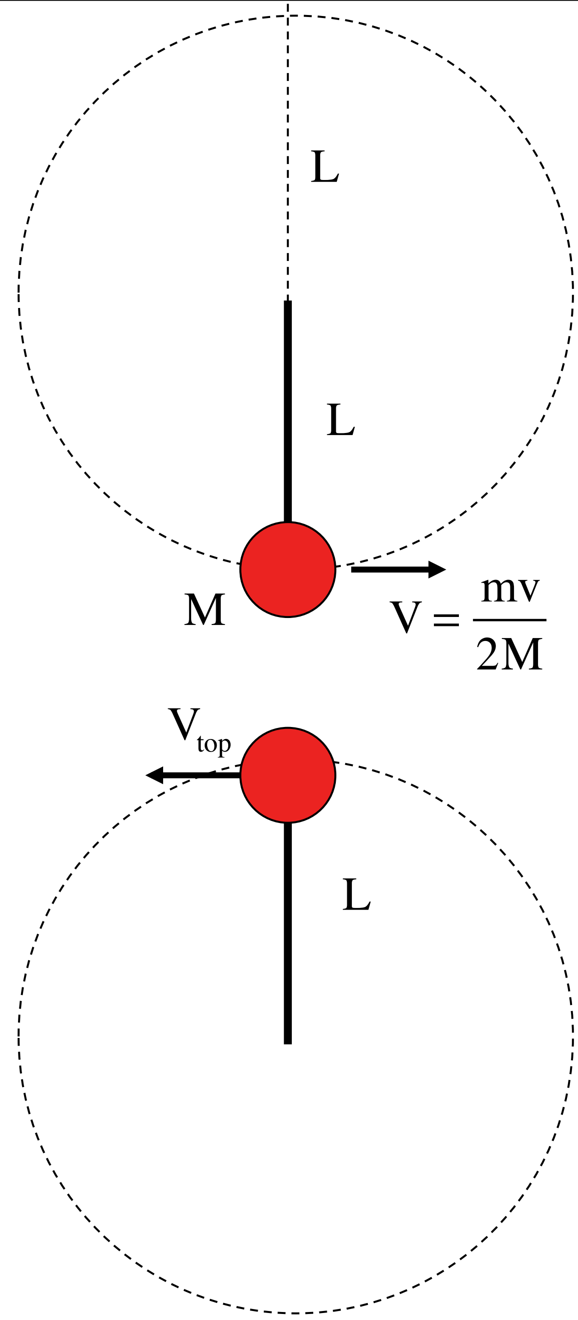
The momentum part would have been the same, but the velocity at the top of the arc would have to satisfy a centripetal force requirement. In that case, a f.b.d. for the forces on the bob at the top would be as shown below with the tension going to zero at the correct velocity. That is:



$$\sum F_c :$$

$$-\cancelto{0}{T} - Mg = -M\frac{v_{top}^{\;2}}{L}$$

$$\Rightarrow \quad v_{top} = (Lg)^{1/2}$$



Conservation of energy would become:

$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}MV^2 + 0 + 0 = \frac{1}{2}Mv_{top}{}^2 + Mg(2L)$$

$$\Rightarrow \frac{1}{2}M\left(\frac{mv}{2M}\right)^2 = \frac{1}{2}M\left[(Lg)^{1/2}\right]^2 + 2MgL$$

$$\Rightarrow v = \left(\frac{4LgM^2}{m^2} + \frac{16gLM^2}{m^2}\right)^{1/2}$$

$$\Rightarrow v = \left(\frac{20LgM^2}{m^2}\right)^{1/2}$$

$$\Rightarrow v = \frac{2M}{m}(5gL)^{1/2}$$