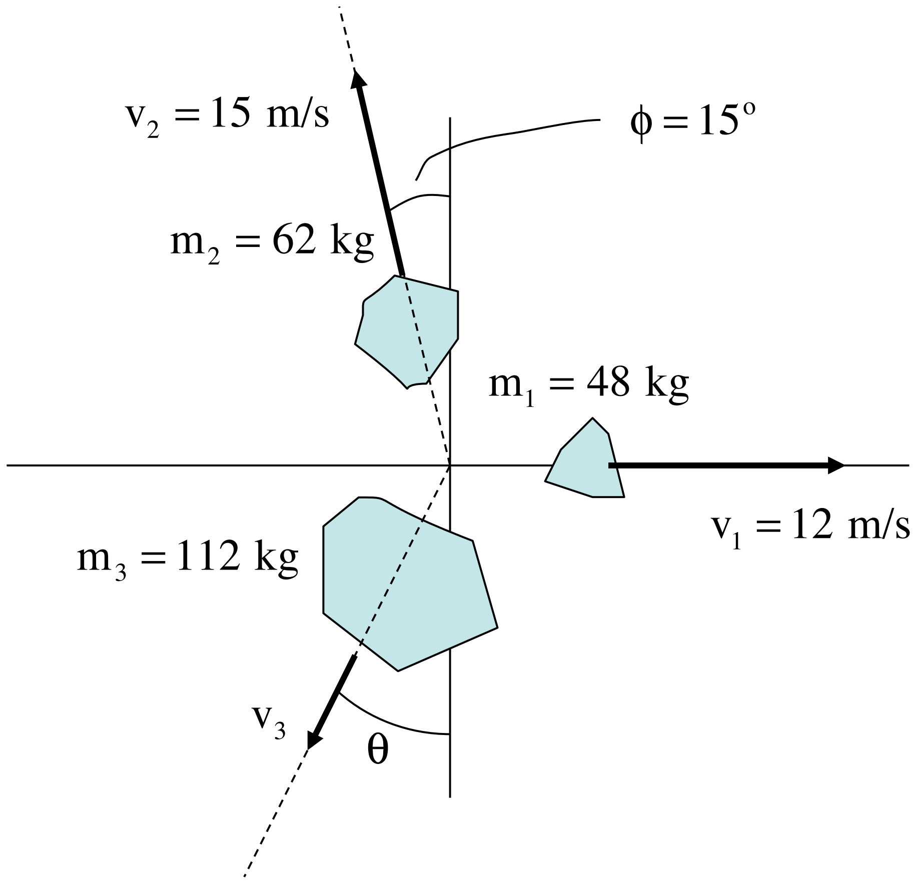
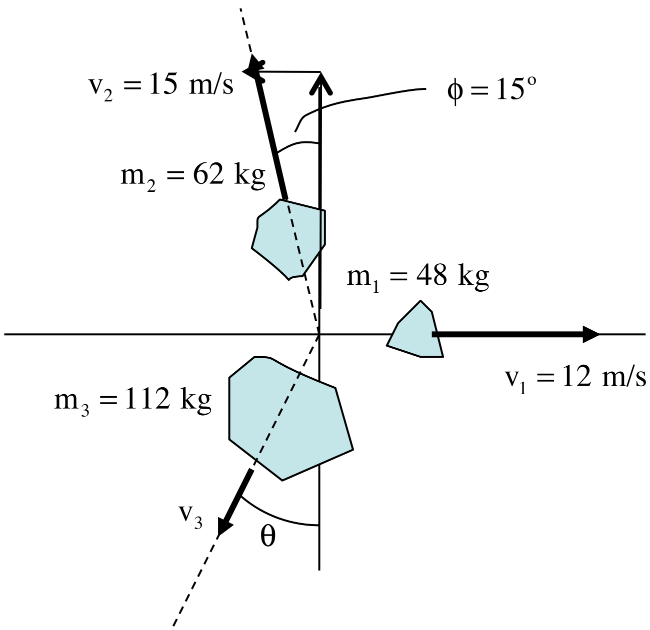
Problem 6.44

Initially, the mass is one. After the explosion, three masses depart as depicted. What is the unknown velocity and angle?



The initial momentum in the system is zero, so the "final" momentum in both the x-direction and y-direction must add to zero. With that in mind, we can write:



In the x-direction:

$$\sum p_{initial,x} + \sum F_{ext,x}\Delta t = \sum p_{final,x}$$

$$0 \quad + \quad 0 \quad = m_1 v_1 - m_2 v_2 \sin\left(15^o\right) - m_3 v_3 \sin\theta$$

$$\Rightarrow \quad 0 = (48\text{ kg})(12\text{ m/s}) - (62\text{ kg})(15\text{ m/s})\sin\left(15^o\right) - (112\text{ kg})v_3 \sin\theta$$

$$\Rightarrow \quad v_3 \sin\theta = 3$$

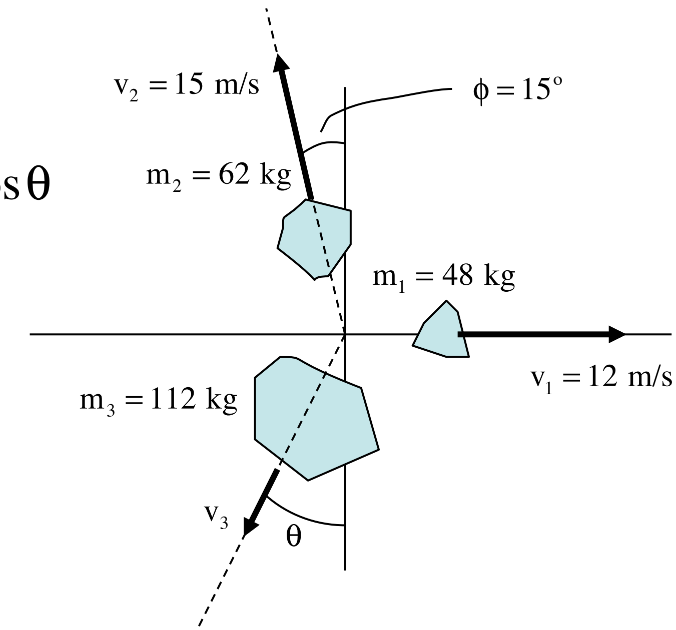
In the y-direction:

$$\sum p_{initial,y} + \sum F_{ext,y}\Delta t = \sum p_{final,y}$$

$$0 \quad + \quad 0 \quad = m_2 v_2 \cos(15^o) - m_3 v_3 \cos\theta$$

$$\Rightarrow \quad 0 = (62\text{ kg})(15\text{ m/s})\cos(15^o) - (112\text{ kg})v_3\cos\theta$$

$$\Rightarrow \quad v_3\cos\theta = 8$$



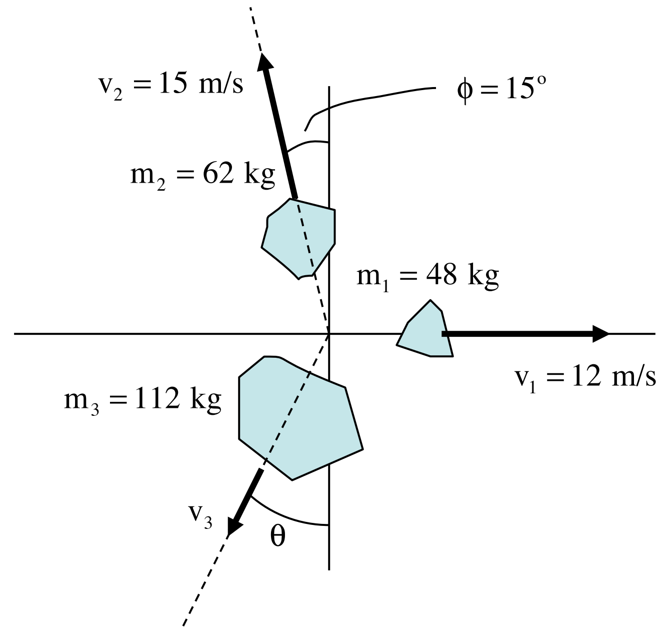
We have two equations:

$$v_3\cos\theta = 8 \qquad \text{and} \qquad v_3\sin\theta = 3$$

Unfortunatley, we have three unknowns (the velocity, the sine of the angle and the cosine of the angle). So what to do? Trickery!!!

$$v_3 \sin\theta = 3 \qquad \text{and} \qquad v_3 \cos\theta = 8$$

Take the two relationships and divide them into one another. That is, divide the left side of the second into the left side of the first, and the right side of the second into the right side of the first. Doing so will yield:



$$\frac{v_3 \sin\theta}{v_3 \cos\theta} = \frac{3}{8}$$

Noting that the velocities will cancel and the sine over cosine is tangent, we can write:

$$\tan\theta = \frac{3}{8} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{3}{8}\right) \quad \Rightarrow \quad \theta = 20.6^o$$

With the angle, we can use either the first or second relationship to write:

$$v_3 \sin\left(20.6^o\right) = 3 \quad \Rightarrow \quad v_3 = 8.53 \text{ m/s} \qquad \text{OR}$$

$$v_3 \cos\left(20.6^o\right) = 8 \quad \Rightarrow \quad v_3 = 8.54 \text{ m/s} \quad \text{(close enough for gov't work)}$$