Problem 6.4

1 kg ball is thrown straight up into the air with an initial speed of 15 m/s. Find the momentum:

a.) At its maximum height.

b.) Halfway to its maximum.

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a.) At its maximum height.

At its maximum height, its velocity is zero and its momentum is zero.

b.) Halfway to its maximum.

We are going to need the maximum height to determine what's happening at *half* the maximum height. Using conservation of energy, we can write:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}mv_o^2 + 0 + 0 = 0 + mgh$$

$$\Rightarrow \quad h = \frac{v_o^2}{2g}$$

$$\Rightarrow \quad h = \frac{(15 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)}$$

$$\Rightarrow \quad h = 11.48 \text{ m}$$

To determine the momentum at half the maximum height, let's try the conservation of momentum equation: It suggests:

$$\sum p_{1,y} + \sum F_{external,y} \Delta t = \sum p_{2,y}$$

We know the initial momentum and the external force (gravity), so if we could determine the time of flight (using kinematics?), we'd have it. Trying that, we get:

$$\begin{aligned}
\Delta y &= v_{o,y} t + \frac{1}{2} a_y t^2 \\
\Rightarrow \quad \frac{h}{2} &= v_{o,y} t + \frac{1}{2}(-g) t^2 \\
\Rightarrow \quad \frac{(11.48 \text{ m})}{2} &= (15 \text{ m/s}) t - \frac{1}{2}\left(9.8 \text{ m/s}^2\right) t^2 \\
\Rightarrow \quad t &= .45 \text{ seconds and } 2.61 \text{ seconds}
\end{aligned}$$

Noting that F_g =–mg, his means the two possible momenta at the halfway point are:

$$\begin{aligned}
\sum p_{1,y} \quad &+ \quad \sum F_{external,y} \quad \Delta t = \sum p_{2,y} \\
(m) \quad (v_{o,y}) \quad &+ \left[\quad (F_{gravity}) \quad \Delta t \right] = \sum p_{2,y} \\
(1 \text{ kg})(15 \text{ m/s}) &+ \left[(1 \text{ kg})\left(-9.8 \text{ m/s}^2\right)(.45 \text{ s})\right] = p_{2,y} \\
\Rightarrow \quad p_{going\ up,y} &= 10.6 \text{ kg} \bullet \text{m/s}
\end{aligned}$$

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\sum p_{1,y} \quad &+ \quad \sum F_{external,y} \quad \Delta t = \sum p_{2,y} \\
(m) \quad (v_{o,y}) \quad &+ \left[\quad (F_{gravity}) \quad \Delta t \right] = \sum p_{2,y} \\
(1 \text{ kg})(15 \text{ m/s}) &+ \left[(1 \text{ kg})\left(-9.8 \text{ m/s}^2\right)(2.61 \text{ s})\right] = p_{2,y} \\
\Rightarrow \quad p_{going\ down,y} &= -10.6 \text{ kg} \bullet \text{m/s}
\end{aligned}$$

An alternate way to do this problem is to determine the velocity at half the maximum height, then multiply by the mass. If that's what we want to do, conservation of energy yields:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}m{v_o}^2 + \quad 0 \quad + \quad 0 \quad = \frac{1}{2}m{v_{halfway}}^2 + mg\left(\frac{h}{2}\right)$$

$$\Rightarrow \quad v_{halfway} = \sqrt{{v_o}^2 - gh}$$

$$\Rightarrow \quad v_{halfway} = \sqrt{(15\text{ m/s})^2 - (9.8\text{ m/s}^2)(11.48\text{ m})}$$

$$\Rightarrow \quad v_{halfway} = 10.6\text{ m/s}$$

and the momentum at the half-way height point is:

$$\begin{aligned} p &= mv_{halfway} \\ &= (1\text{ kg})(10.6\text{ m/s}) \\ &= 10.6\text{ kg} \bullet \text{m/s} \end{aligned}$$

Note that this approach does not rightly suggest that there is momentum on the way up and momentum on the way down, which the other approach does nail.