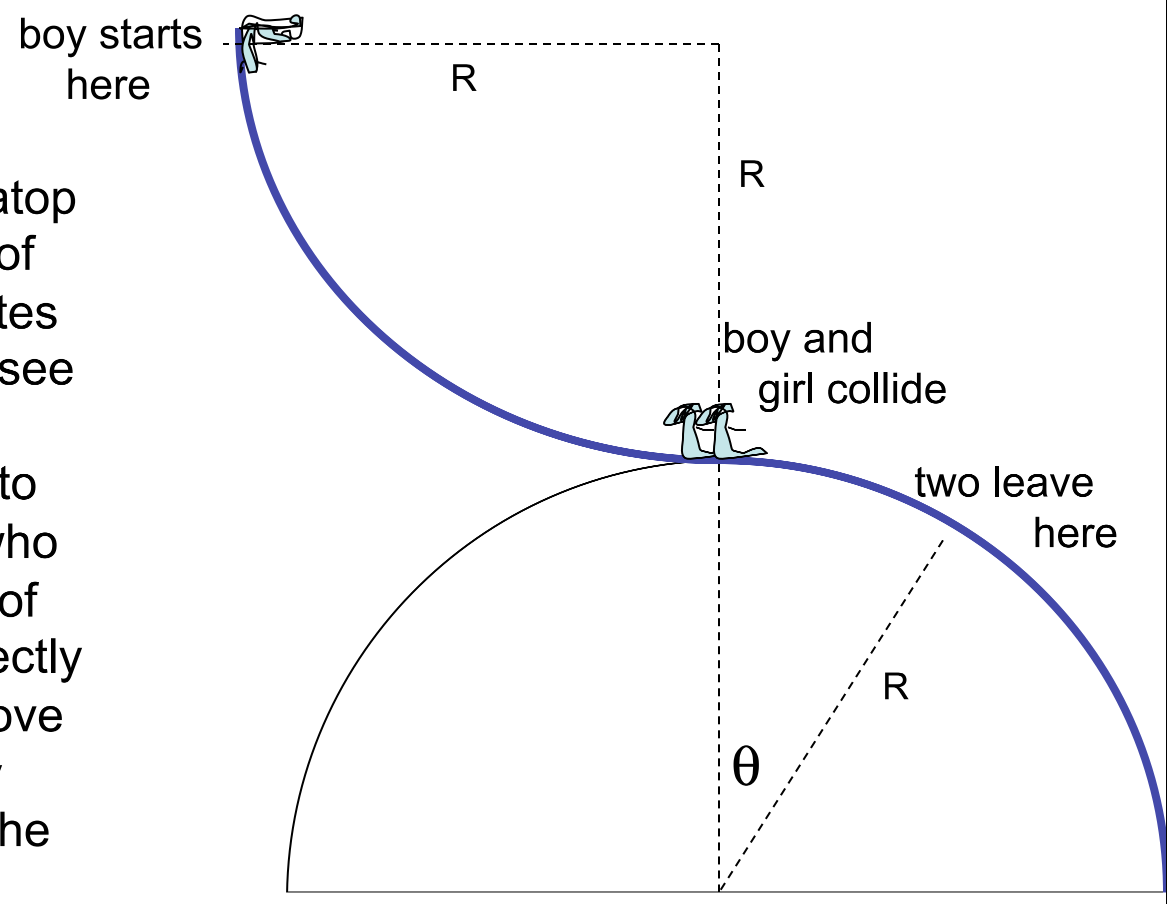
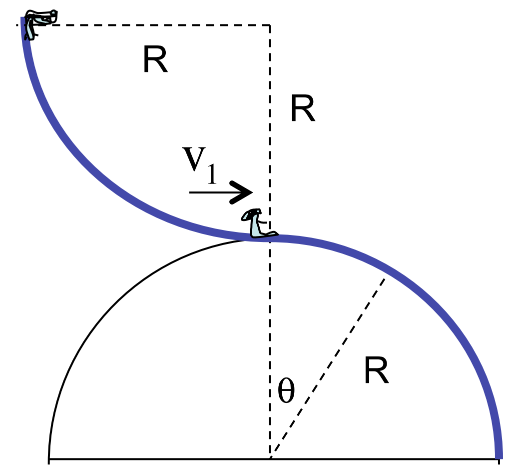
Ice Dome With Collision



Jack with mass of “2m” sits atop a curved, frictionless incline of radius R which itself terminates on an ice dome of radius R (see sketch). He begins to slide down the incline, crashing into Jill whose mass is “m” and who happens to be sitting on top of the ice dome. After the perfectly inelastic collision, the two move down the ice dome until they leave the dome. Determine the angle θ at which they leave.

Because energy is lost in the collision between Jack and Jill, we have to do this problems in little pieces using conservation of energy and momentum where they are applicable. With that in mind, we will start at Jack's start point.



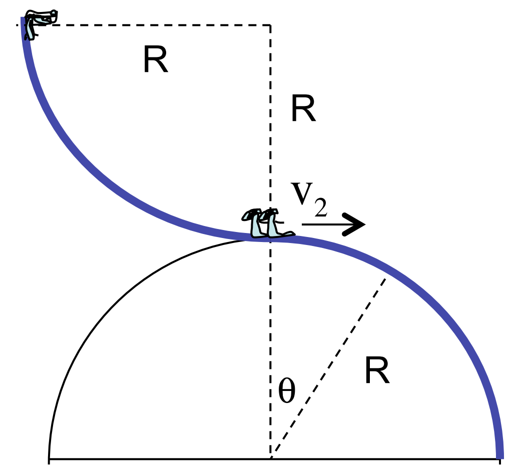
Remembering that Jack's mass is "2m" (and putting the y = 0 level at the ground), we can determine Jack's velocity JUST BEFORE HE HITS THE JILL by using conservation of energy evaluated between his start point and JUST BEFORE HE HITS JILL. That is:

$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$0 + (2m)g(2R) + 0 = \frac{1}{2}(2m)(v_1)^2 + (2m)g(R)$$

$$\Rightarrow \quad v_1 = (2gR)^{1/2}$$

Momentum is conserved through the collision, so we can write:

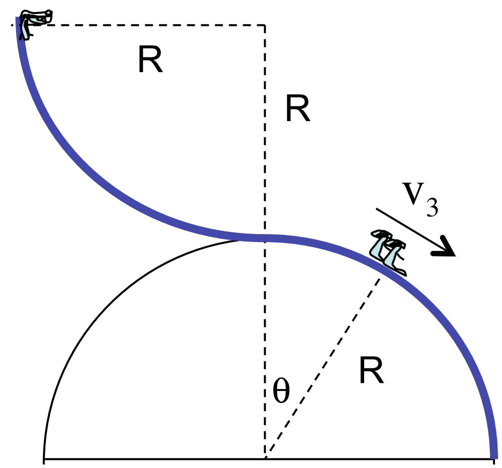


$$\sum p_{1,x} + \sum F_{ext,x}\Delta t = \sum p_{2,x}$$
$$(2m)v_1 + 0 = (2m+m)v_2$$
$$\Rightarrow \quad v_2 = \frac{2}{3}v_1$$
$$\Rightarrow \quad v_2 = \frac{2}{3}(2gR)^{1/2}$$

Knowing the velocity after the collision, we can use conservation of energy to determine the velocity just as the two leave the dome. With the mass of the two-body system now "3m," and noting that the distance above the ground at that second point is $R\cos\theta$, we can write:

$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$
$$\frac{1}{2}(3m)(v_2)^2 + (3m)g(R) + 0 = \frac{1}{2}(3m)(v_1)^2 + (3m)g(R\cos\theta)$$

All we need now is a final expression for the velocity of the two just as they lift off. To get that, we need to use Newton's Second Law. Doing so yields:

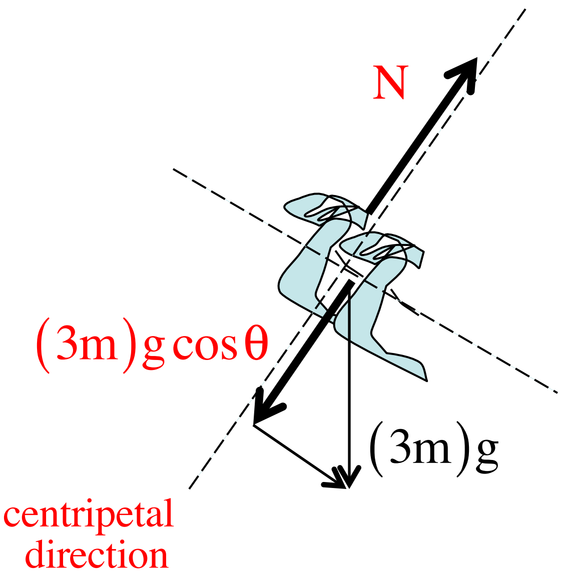


$$\underline{\sum F_c:}$$

$$\overset{0}{\cancel{N}} - (3m)g\cos\theta = -(3m)\left(\frac{v_3^{\,2}}{R}\right)$$

$$\Rightarrow \quad \cancel{(3m)}g\cos\theta = \cancel{(3m)}\left(\frac{v_3^{\,2}}{R}\right)$$

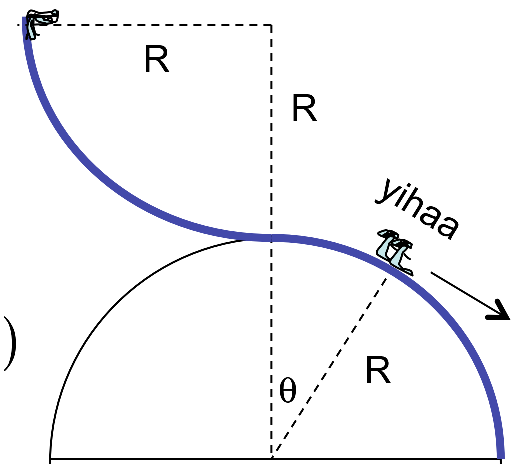
$$\Rightarrow \quad v_3 = (gR\cos\theta)^{1/2}$$



Putting

$$v_3 = (gR\cos\theta)^{1/2} \quad \text{and} \quad v_2 = \frac{2}{3}(2gR)^{1/2}$$

into



$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}(3m)(v_2)^2 + (3m)g(R) + 0 = \frac{1}{2}(3m)(v_1)^2 + (3m)g(R\cos\theta)$$

yields:

$$\frac{1}{2}(3m)\left(\frac{2}{3}[2gR]^{1/2}\right)^2 + (3m)g(R) = \frac{1}{2}(3m)\left([gR\cos\theta]^{1/2}\right)^2 + (3m)g(R\cos\theta)$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{8}{9}gR\right) + g(R) = \frac{1}{2}(gR\cos\theta) + g(R\cos\theta)$$

$$\Rightarrow \quad \left(\frac{4}{9}\right) + 1 = \frac{1}{2}(\cos\theta) + (\cos\theta)$$

$$\Rightarrow \quad \frac{13}{9} = \frac{3}{2}\cos\theta$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{26}{27}\right) = 15.6^{\circ}$$

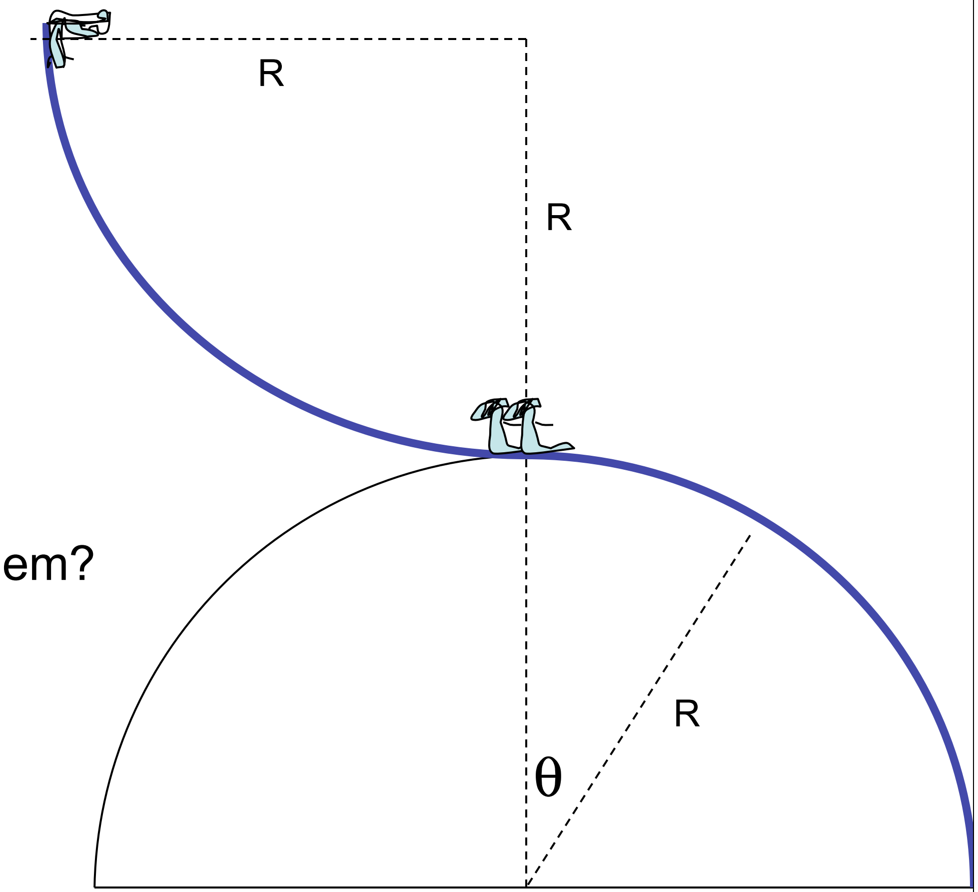
So where was energy conserved?

Where was momentum conserved?

What else did I need to solve the problem?

Could I have added a spring?

Could I have added warm jello?



And the fun never stops . . .