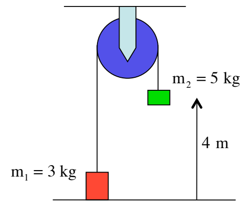
Problem 5.71

Two masses are attached via a string that is positioned over a massless, frictionless pulley. If the masses start from rest:

a.) How fast are they moving when they pass one another?



b.) How fast is the 3 kg mass moving when the 5 kg mass reaches the floor?

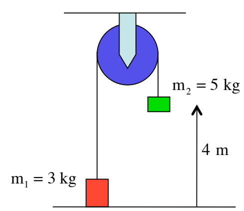
c.) How far will the 3 kg mass rise after the 5 kg mass reaches the floor?

There are two things to notice at the outset.

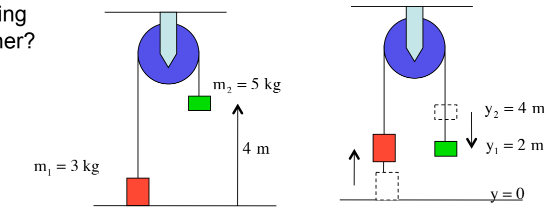
First, there are TWO bodies moving with the same velocity magnitude and displacing the same net distance (though one is moving downward and the other upward).

Second, you can identify the "zero potential energy level" separately for EACH BODY independent of the other (we will do a problem below where that is important). Having said that, I usually make the LOWEST POINT OF TRAVEL the y=0 level for each body. This means that the 3 kg mass will have its y = 0 point at ground level and so will the 5 kg mass (again, they are at the same point in this case, but they don't HAVE to be at the same point.

With that in mind:



a.) How fast are they moving when they pass one another?

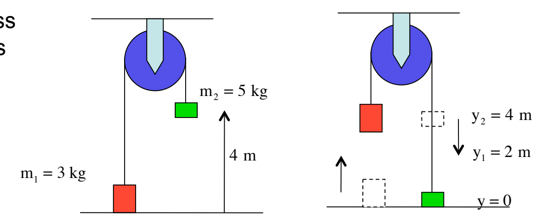


$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$(0) + (m_2 g y_2) + 0 = \left(\frac{1}{2} m_1 v_2^2 + \frac{1}{2} m_2 v_2^2\right) + (m_1 g y_1 + m_2 g y_1)$$

$$\Rightarrow v_2 = \sqrt{\frac{2}{(m_1 + m_2)}\left[m_2 g y_2 - m_1 g y_1 - m_2 g y_1\right]}$$

b.) How fast is the 3 kg mass moving when the 5 kg mass reaches the table?



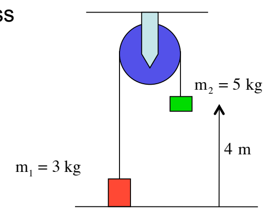
$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

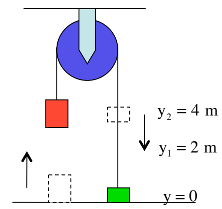
$$(0) + (m_2 g y_2) + 0 = \left(\frac{1}{2} m_1 v_3^2 + \frac{1}{2} m_2 v_3^2\right) + (m_1 g y_2)$$

$$\Rightarrow v_3 = \sqrt{\frac{2}{(m_1 + m_2)}\left[m_2 g y_2 - m_1 g y_2\right]}$$

$$\Rightarrow v_3 = \sqrt{\frac{2 g y_2}{(m_1 + m_2)}\left[m_2 - m_1\right]}$$

c.) How far will the 3 kg mass move above the 4 meter mark (i.e., after the 5 kg mass hits the table)?





Just looking at the 3 kg mass from the 4 meter mark on, we can write:

$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}m_1v_3^2 + (m_1gy_3) + 0 = (0) + (m_1gy_{max})$$

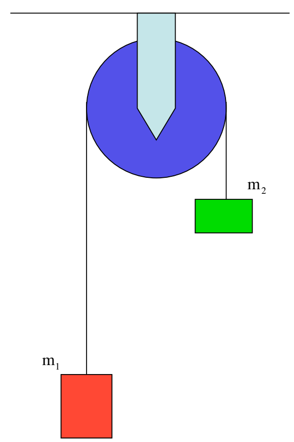
$$\frac{1}{2}m_1\left(\sqrt{\frac{2gy_2}{(m_1+m_2)}[m_2-m_1]}\right)^2 + (m_1gy_3) + 0 = (0) + (m_1gy_{max})$$

$$\frac{1}{2}m_1\left(\frac{2gy_2}{(m_1+m_2)}[m_2-m_1]\right) + (m_1gy_3) + 0 = (0) + (m_1gy_{max})$$

$$\Rightarrow y_{max} = \frac{\frac{1}{2}m_1\left(\frac{2gy_2}{(m_1+m_2)}[m_2-m_1]\right) + (m_1gy_3)}{m_1g}$$

$$\Rightarrow y_{max} = \frac{(m_2-m_1)}{(m_1+m_2)}y_2 + \left(\frac{y_3}{2}\right)$$

NOTE: How should we define the zero points for each mass if the right one will be traveling downward, the the left one traveling upward and neither's position relative to the ground is known?



As I said earlier, I usually take the lowest point each individual mass reaches as the y=0 point. Assuming the right mass moves downward a distance "d" with the left mass moving upward a distance "d", the individual "y=0" points would be as shown!