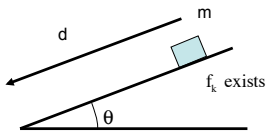
Problem 5.5

A block slides down an incline as shown. During the motion:

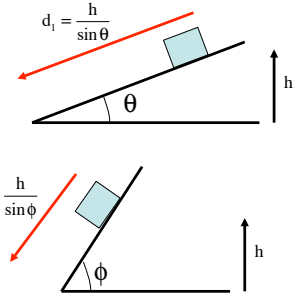


a.) How much work does the gravity do?

b.) How much work does friction do?

c.) How much work does the normal force do?

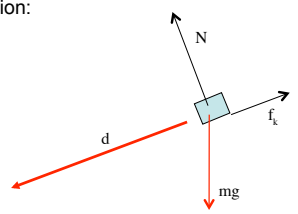
d.) Qualitatively, how would things have changed if a shorter ramp at a steeper angle were used to drop the mass the same vertical distance?



A block slides down an incline as shown. During the motion:

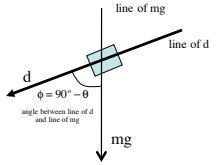
a.) How much work does the gravity do?

There are two dot product and one potential energy way to do this. (Note that the free body diagram shows both the displacement "d" and all of the forces.)



i.) the definition approach: extending the line of "d" and "mg," the vectors look like:

$$W_{mg} = |F||d|\cos\phi$$
$$= (mg)(d)\cos(90^\circ - \theta)$$

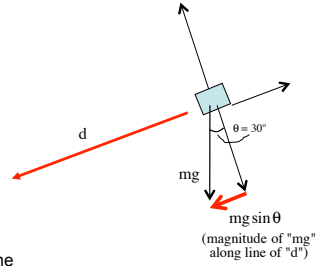


a.) How much work does the gravity do? (con't.)

ii.) the magnitude of the displacement times the magnitude of the component of gravity along the line of displacement:

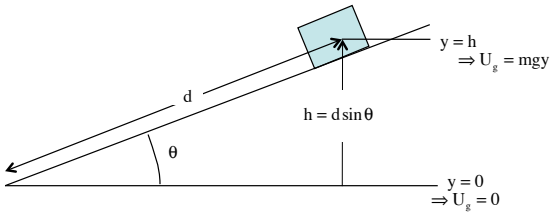
$$W_{mg} = F_{\parallel}|d|$$
$$= (mg\sin\theta)(d)$$

(Notice the the cosine $(90^\circ - \theta)$ is the same as sin θ.)



iii.) THE BEST WAY using potential energy (I'm going to assume the "potential energy equals zero" level is the ground)!:

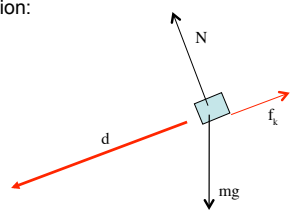
$$W_g = -\Delta U$$
$$= -(0 - mgy)$$
$$= mgd\sin\theta$$



A block slides down an incline as shown. During the motion:

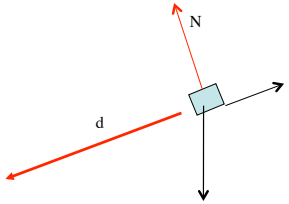
b.) How much work does friction do?

$$\begin{aligned} W_f &= |F||d|\cos\phi \\ &= (f)(d)\cos(180^\circ) \\ &= -fd \\ &= -(\mu_k N)d \\ &= -(\mu_k (mg\cos\theta))d \end{aligned}$$



c.) How much work does the normal force do?

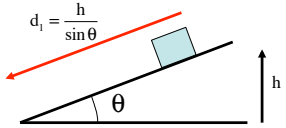
$$\begin{aligned} W_N &= |F||d|\cos\phi \\ &= (f)(d)\cos(90^\circ) \\ &= 0 \end{aligned}$$

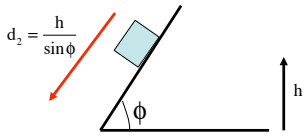


d.) Qualitatively, how would things have changed if a shorter ramp at a steeper angle were used to drop the mass the same vertical distance?

--The amount of work gravity does is based on the net vertical displacement of the body. That hasn't changed, so the amount of work gravity does stays the same.

--The work the normal does is ALWAYS zero.





--Friction is related to the magnitude of the normal force. In this case, the normal force is equal and opposite to the component of gravity perpendicular to the incline. As the angle gets larger, the component of gravity normal to the incline gets smaller (hence the normal force gets smaller—think about it, if the incline is at ninety degrees, the normal force goes to zero). Additionally, the distance over which the frictional force acts gets smaller as the angle gets bigger (look at sketch). In short, the work the frictional force does goes down suggesting that if the block was free falling down the incline, its velocity at the bottom would be greater for the steeper incline.