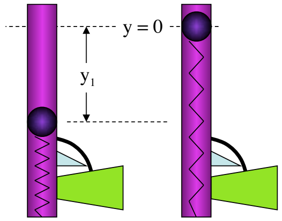
Problem 5.39



A spring in a toy gun is compressed a distance .12 meters. The gun launches a .02 kg bullet from rest to a maximum height of 20 meters *above the starting point of the projectile*. Neglecting air resistance:

a.) Describe the mechanical energy transformation as the bullet rises.

b.) Determine the spring constant "k."

c.) Determine the bullet's velocity as it moves through the equilibrium position of the spring (i.e., at y=0 as shown on the sketch).

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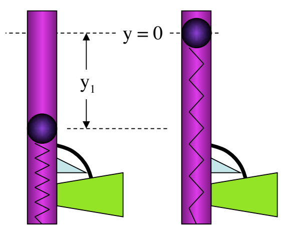

NOTE: THE REASON THE BOOK DIDN'T INCLUDE THE INITIAL GRAVITATIONAL POTENTIAL ENERGY IN THE PROBLEM (as I did), was because with a spring constant of 547 nt/m, the amount of spring potential energy to start using $\frac{1}{2}k(-y_1)^2$ turns out to be 3.938 joules. The amount of gravitational potential energy at the start using $mg(-y_1)$ turns out to be -.024 joules. In other words, there was so little initial gravitational potential energy wrapped up in the body's initial position that it could be ignored. Attempting to be technically correct, though, I included it even though it appears its addition made little difference in the final solution.

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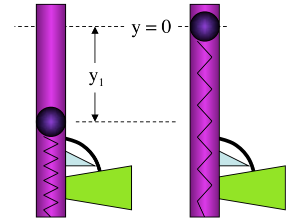

A spring in a toy gun is compressed a distance .12 meters. The gun launches a .02 kg bullet from rest to a maximum height of 20 meters *above the starting point of the projectile*. Neglecting air resistance:

a.) Describe the mechanical energy transformation as the bullet rises.

If we take the y=0 level to be the zero potential energy level for gravity, the object will have negative gravitational potential energy $mg(-y_1)$ and spring potential energy of $\frac{1}{2}k(-y_1)^2$. As the spring uncoils, the spring's potential energy will decrease feeding into the kinetic energy of the bullet AND increasing the bullet's gravitational potential energy. Once the spring has played out completely, the gravitational potential energy will continue to increase as the bullet continues to rise, but the kinetic energy will decrease until the bullet comes to rest at the top of its motion whereupon all the energy will be gravitational potential energy. Note that k=8.7 nt/m as determined in part b.)

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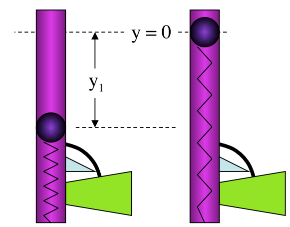

A numerical presentation of this is shown below (neither this nor the graph on the next page were asked for--I've included them both because they are interesting to note). It isn't in the problem, but I'm assuming the spring constant is 8.7 nt/m.

Initially:

$$U_{sp} = \frac{1}{2}k(y_1)^2 = \frac{1}{2}(8.7 \text{ nt/m})(-.12 \text{ m})^2 = 6.26x10^{-2} \text{ J}$$

$$U_{gravity} = mg(y_1) = (.02 \text{ kg})(9.8 \text{ m/s}^2)(-.12 \text{ m}) = -2.35x10^{-2} \text{ J}$$

$KE_{initial} = 0$ $\qquad E_{total\ mechanical} = 3.92x10^{-2}$ J

At spring equilibrium:

$U_{sp} = 0$

$U_{gravity} = 0$ $\qquad E_{total\ mechanical} = 3.92x10^{-2}$ J

$KE_{leaving\ muzzle} = 3.92$ J

At maximum height:

$U_{sp} = 0$

$U_{gravity} = mg(y_{max}) = 3.92x10^{-2}$ J $\qquad E_{total\ mechanical} = 3.92x10^{-2}$ J

$KE_{initial} = 0$

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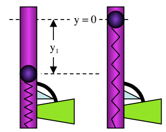
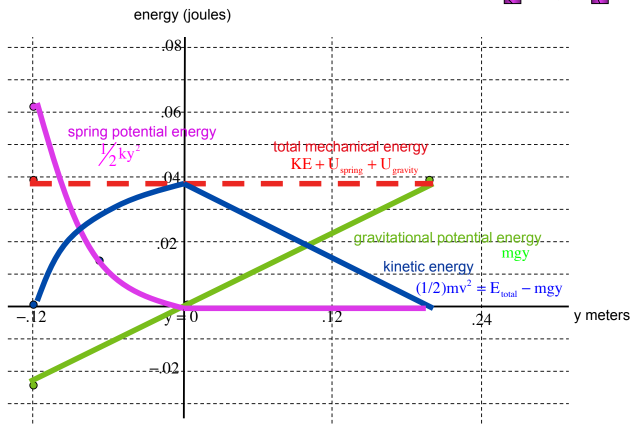


A spring in a toy gun is compressed a distance .12 meters. The gun launches a .02 kg bullet from rest to a maximum height of 20 meters *above the starting point of the projectile*. Neglecting air resistance:

a.) A graphical representation of this follows:



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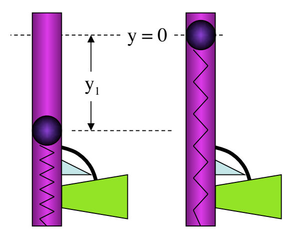

A spring in a toy gun is compressed a distance .12 meters. The gun launches a .02 kg bullet from rest to a maximum height of 20 meters *above the starting point of the projectile*. Neglecting air resistance:

b.) Determine the spring constant "k."

The total energy in the system is the before-firing "initial" energy in the spring ($\frac{1}{2}k(-y_1)^2$) plus the initial gravitational potential energy ($mg(-y_1)$). Energy is conserved throughout, so this number must equal the gravitational potential energy at the top of the bullet's path where the kinetic energy and the spring potential energy are both zero. That is:

$$(E_{total} =)\ \ (1/2)k(-y_1)^2 + mg(-y_1) = mg(y_{max})$$

$$\Rightarrow \ \ (1/2)k(-.12 \text{ m})^2 + (.02 \text{ kg})(9.8 \text{ m/s}^2)(-.12 \text{ m}) = (.02 \text{ kg})(9.8 \text{ m/s}^2)(20 \text{ m})$$

$$\Rightarrow \ \ k = 547 \text{ nt/m}$$

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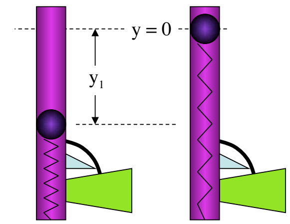

A spring in a toy gun is compressed a distance .12 meters. The gun launches a .02 kg bullet from rest to a maximum height of 20 meters *above the starting point of the projectile*. Neglecting air resistance:

c.) Determine the bullet's velocity as it moves through the equilibrium position of the spring (i.e., at y=0 as shown on the sketch).

$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$(0) + \left[(1/2)k(-y_1)^2 + mg(-y_1)\right] + 0 = \frac{1}{2}mv_{equ}^2 + 0$$

$$\Rightarrow \ \ v_{equ} = \sqrt{\frac{2}{m}\left[(1/2)k(-y_1)^2 + mg(-y_1)\right]}$$

$$\Rightarrow \ \ v_{equ} = \sqrt{\left[\frac{k}{m}(-y_1)^2 + 2mg(-y_1)\right]}$$

$$\Rightarrow \ \ v_{equ} = \sqrt{\left[\frac{(547 \text{ nt/m})}{(.02 \text{ kg})}(-.12 \text{ m})^2 + 2(.02 \text{ kg})(9.8 \text{ m/s}^2)(-.12 \text{ m})\right]}$$

$$\Rightarrow \ \ v_{equ} = 19.8 \text{ m/s}$$