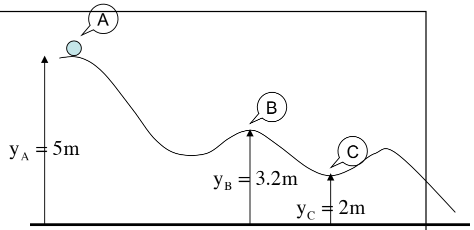
Problem 5.36

A 5 kg bead released from rest slides down a frictionless track.

a.) What is the bead's speed at points B and C?



b.) How much work did gravity do as the bead moved from points A to C?

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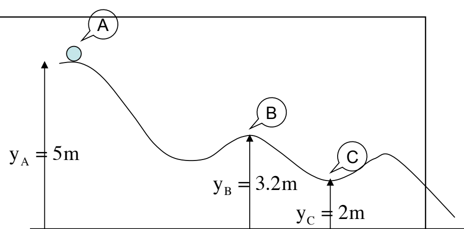

This could be done using Work/Energy, but it is more easily done with conservation of energy. Using that, (and noting that the masses all divide out), we'll start going to Point B:

$$\sum KE_A + \sum U_A + \sum W_{extraneous} = \sum KE_B + \sum U_B$$

$$(0) \quad + (\not{m}gy_A) + \quad 0 \quad = \frac{1}{2}\not{m}v_B^2 + (\not{m}gy_B)$$

$$\Rightarrow \quad v_B = \left[2(gy_A) - 2(gy_B)\right]^{1/2}$$

$$\Rightarrow \quad v_B = \left[2(9.8\text{ m/s}^2)(5\text{ m}) - 2(9.8\text{ m/s}^2)(3.2\text{ m})\right]^{1/2}$$

$$\Rightarrow \quad v_B = 5.94\text{ m/s}$$

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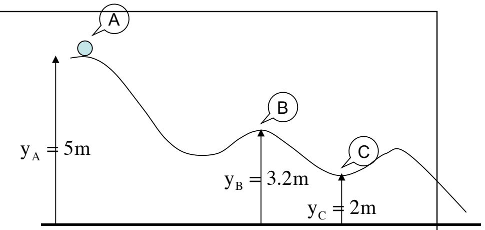
a.) What is the bead's speed at points B and C?

Going to Point C



$$\sum KE_A + \sum U_A + \sum W_{extraneous} = \sum KE_C + \sum U_C$$

$$(0) \quad + (mgy_A) + \quad 0 \quad = \frac{1}{2}mv_C^2 + (mgy_C)$$

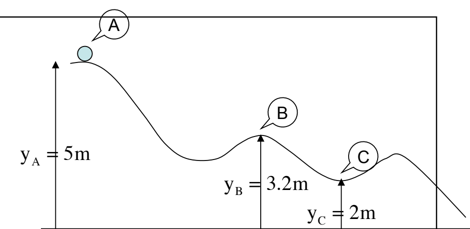
$$\Rightarrow \quad v_C = \left[2(gy_A) - 2(gy_C)\right]^{1/2}$$

$$\Rightarrow \quad v_C = \left[2(9.8\text{ m/s}^2)(5\text{ m}) - 2(9.8\text{ m/s}^2)(2\text{ m})\right]^{1/2}$$

$$\Rightarrow \quad v_C = 7.67\text{ m/s}$$

A 5 kg bead released from rest slides down a frictionless track.

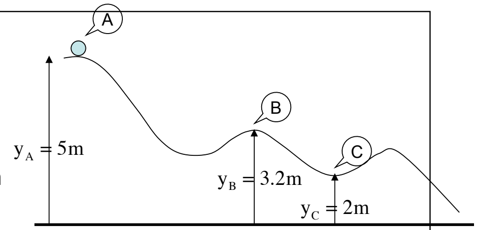
b.) How much work did gravity do as the bead moved from points A to C?



WHENEVER YOU WANT TO DETERMINE HOW MUCH WORK GRAVITY DOES, ALWAYS USE ITS POTENTIAL ENERGY FUNCTION.

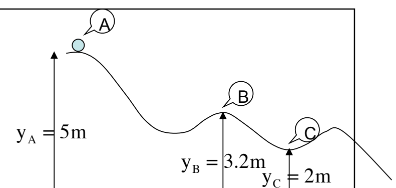
$$\begin{aligned} W_{gravity} &= -\Delta U \\ &= -(U_C - U_A) \\ &= -(mgy_C - mgy_A) \\ &= -\left[(5\text{ kg})(9.8\text{ m/s}^2)(2\text{ m}) - (5\text{ kg})(9.8\text{ m/s}^2)(5\text{ m})\right] \\ &= +147\text{ J} \end{aligned}$$

A little twist: Suppose the total linear distance from A to B was 30 meters and the distance from B to C was 10 meters. How would the problem have changed if you had been told that the average frictional force acting on the bead between A and B was "mg/10," and the average frictional force between B and C was zero, and you wanted the velocity at C?



This is a modified *conservation of energy* problem. Noting that the masses no longer cancel:

With friction from Point A to Point C:



$$\sum KE_A + \sum U_A + \sum W_{extraneous} = \sum KE_C + \sum U_c$$

$$(0) \; + (mgy_A) + \; (\vec{f} \bullet \vec{d}_{AB}) \; = \frac{1}{2} m v_C^2 + (mgy_C)$$

$$\Rightarrow \quad v_C = \frac{\left[2(mgy_A) + 2(-fd_{AB}) - 2(mgy_C)\right]^{1/2}}{m}$$

$$\Rightarrow \quad v_C = \frac{\left[2(mgy_A) + 2\left(-\left(\frac{mg}{10}\right)d_{AB}\right) - 2(mgy_C)\right]^{1/2}}{m}$$

$$\Rightarrow \quad v_C = \frac{\left[2(5\text{ kg})(9.8\text{ m/s}^2)(5\text{ m}) + 2\left(-\frac{(5\text{ kg})(9.8\text{ m/s}^2)(5\text{ m})}{10}\right) - 2(5\text{ kg})(9.8\text{ m/s}^2)(2\text{ m})\right]^{1/2}}{(5\text{ kg})}$$

$$\Rightarrow \quad v_C = 3.13\text{ m/s}$$