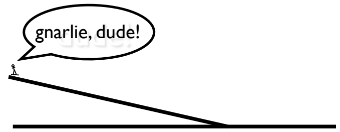
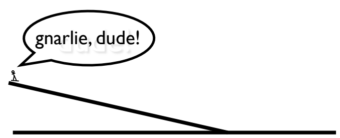
1.) An old but attractive snow boarder sits at the top of a 12 degree hill that is 400 meters long. She begins her run from rest and in a tuck (i.e., she is shoshing). If the coefficient of friction between her skis and the snow is .12, how fast will she be moving at the bottom of the hill? (Don't worry that you don't have "m." The m's will cancel out in the problem.)



1.) An old but attractive snow boarder of 45 kg mass sits at the top of a 12 degree hill that is 400 meters long. She begins her run from rest and in a tuck (i.e., she is shoshing). If the coefficient of friction between her skis and the snow is .12, how fast will she be moving at the bottom of the hill?



$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$(0) + mgh + (-fd) = \frac{1}{2}mv_{bottom}^2 + 0$$

$$(0) + mgh + \left(-(\mu_k mg\cos\theta)d\right) = \frac{1}{2}mv_{bottom}^2 + 0$$

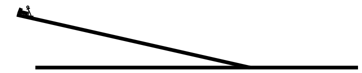
$$\Rightarrow v_{bottom} = \sqrt{\frac{2}{m}\left[mg(h) + \left(-(\mu_k mg\cos\theta)d\right)\right]} \quad \text{(dividing the m's out)}$$

$$\Rightarrow v_{bottom} = \sqrt{2g(d\sin\theta) + 2\left(-(\mu_k g\cos\theta)d\right)}$$

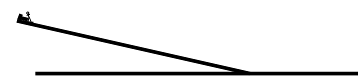
$$\Rightarrow v_{bottom} = \sqrt{2(9.8\text{ m/s}^2)(400\sin 12^\circ) + 2\left(-(.12)(9.8\text{ m/s}^2)(\cos 12^\circ)(400\text{ m})\right)}$$

$$\Rightarrow v_{bottom} = 26.6\text{ m/s} \quad \text{(This is close to 60 mph.)}$$

2.) This hot shoshing momma decides this run is too tame, so she sets up a spring apparatus at the top of the hill, jams herself up against it, and let's loose with a whoop. If the spring constant is 100 nt/m, and if she displaces the spring by a distance of .8 meters before releasing, how fast will she be moving at the bottom of the hill and how much work will friction have done during the motion?



2.) This hot, shoshing, 45 kg momma decides this run is too tame, so she sets up a spring apparatus at the top of the hill, jams herself up against it, and let's loose with a whoop. If the spring constant is 100 nt/m, and if she displaces the spring by a distance of .8 meters before releasing, how fast will she be moving at the bottom of the hill and how much work will friction have done during the motion?



$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$(0) + \left(mgh + (1/2)kx^2\right) + (-fd) = \frac{1}{2}mv_{bottom}^2 + 0$$

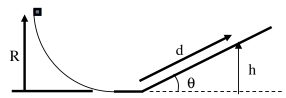
$$(0) + \left(mgh + (1/2)kx^2\right) + \left(-(\mu_k mg\cos\theta)d\right) = \frac{1}{2}mv_{bottom}^2 + 0$$

$$\Rightarrow v_{bottom} = \sqrt{\frac{2}{m}\left[mg(h) + (1/2)kx^2 + \left(-(\mu_k mg\cos\theta)d\right)\right]}$$

$$\Rightarrow v_{bottom} = \sqrt{\frac{2}{(45\text{ kg})}\left[(45\text{ kg})(9.8\text{ m/s}^2)(400\sin 12^\circ) + (1/2)(100\text{ nt})(.8\text{ m})^2 + \left(-(.12)(45\text{ kg})(9.8\text{ m/s}^2)(\cos 12^\circ)(400\text{ m})\right)\right]}$$

$$\Rightarrow v_{bottom} = 27.2\text{ m/s}$$

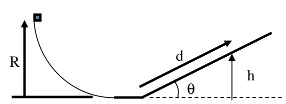
3.) A mass m=4 kgs sits on a curved incline "R=.2 meters" above the ground. At the bottom of the ramp is a flat frictional surface of length .3 meters whose coefficient of friction is u =.2. Attached to the flat is a straight, frictionless, 30 degree incline.



a.) To start with, let's assume the frictional force is negligible. How far up the ramp will the box move before coming to rest? Answer this conceptually.

b.) Now assume the frictional force is not negligible. How far up the ramp will the box move before coming to rest? Answer this mathematically.

3.) A mass m=4 kgs sits on a curved incline "R=.2 meters" above the ground. At the bottom of the ramp is a flat frictional surface of length .3 meters whose coefficient of friction is u =.2. Attached to the flat is a straight, frictionless, 30 degree incline.



a.) To start with, let's assume the frictional force is negligible. How far up the ramp will the box move before coming to rest? Answer this conceptually.

If it starts from rest, the mass will have mgR's worth of potential energy. This number will also be the *total energy* in the system--an amount that will remain constant throughout. When the mass comes to rest at the end of the uphill ramp, all the energy in the system will be gravitational potential energy and must equal mgR. That means the body will rise as high as it was to start with, and d will equal $R/(\sin\theta)$.

b.) Now assume the frictional force is not negligible. How far up the ramp will the box move before coming to rest? Answer this mathematically.

$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$(0) + (mgR) + (\text{-}fx) = 0 + mgh \quad \text{(from the trig, h=d sin 30° = .5d)}$$

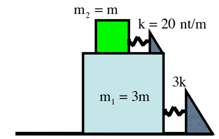
$$(0) + (mgR) + \left(-(\mu_k mg)(x)\right) = 0 + mg\left(d \sin 30^\circ\right) \quad \text{(dividing out the m's and g's)}$$

$$\Rightarrow \quad d = \frac{R - \mu_k x}{.5}$$

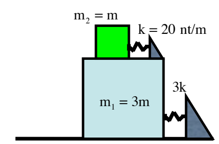
$$\Rightarrow \quad d = \frac{(.2\text{ m}) - (.2)(.3\text{ m})}{.5}$$

$$\Rightarrow \quad d = .28\text{ m}$$

4.) A block of mass "m" sits atop a second block of mass "4m." Each block has a spring attached to it (see sketch). The spring constant of the "m" mass is "k" while the other spring constant is "3k." Both springs are depressed a distance L. If both masses are released from rest at the same time, how fast will "m" be moving as it shoots off the top mass? (Think about how m would act if 3m was held stationary. Then figure out how fast 3m must be moving. Adding that to m's velocity relative to 3m and you have m's velocity relative to the ground.)



4.) A block of mass "m" sits atop a second block of mass "4m." Each block has a spring attached to it (see sketch). The spring constant of the "m" mass is "k" while the other spring constant is "3k." Both springs are depressed a distance L. If both masses are released from rest at the same time, how fast will "m" be moving as it shoots off the top mass? (Think about how m would act if 3m was held stationary. Then figure out how fast 3m must be moving. Adding that to m's velocity relative to 3m and you have m's velocity relative to the ground.)



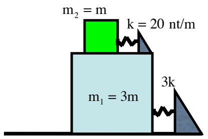
If the 3m mass was stationary, conservation of energy on the "m" mass would yield:

$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$(0) + \left(\frac{1}{2}kL^2\right) + 0 = \left(\frac{1}{2}m_1 v_1^2\right) + 0$$

$$\Rightarrow \quad v_1 = \frac{kL^2}{m_1}$$

If allowed to spring, the spring attached to the 3m mass will puch not only the 3m mass, it will also push "m." Using conservation of energy on that (assuming "m" doesn't spring) yields:



$$\sum KE_1 + \sum U_1 + \sum W_{extraneous} = \sum KE_2 + \sum U_2$$

$$(0) + \left(\frac{1}{2}(3k)L^2\right) + 0 = \left(\frac{1}{2}(3m+m)v_2^{\,2}\right) + 0$$

$$\Rightarrow \quad v_2 = \frac{3kL^2}{4m_2}$$

Superimposing "m's" velocity on top of the velocity supplied by 3m yields:

$$\begin{aligned} v_{\text{net on m}} &= v_1 + v_2 \\ &= \frac{kL^2}{m_1} + \frac{3kL^2}{4m_2} \\ &= kL^2\left(\frac{1}{m_1} + \frac{3^2}{4m_2}\right) \end{aligned}$$

Obscure? Yes! Something you might run into on your test? Definitely NOT, but it was good practice using the conservation of energy relationship . . . yes?