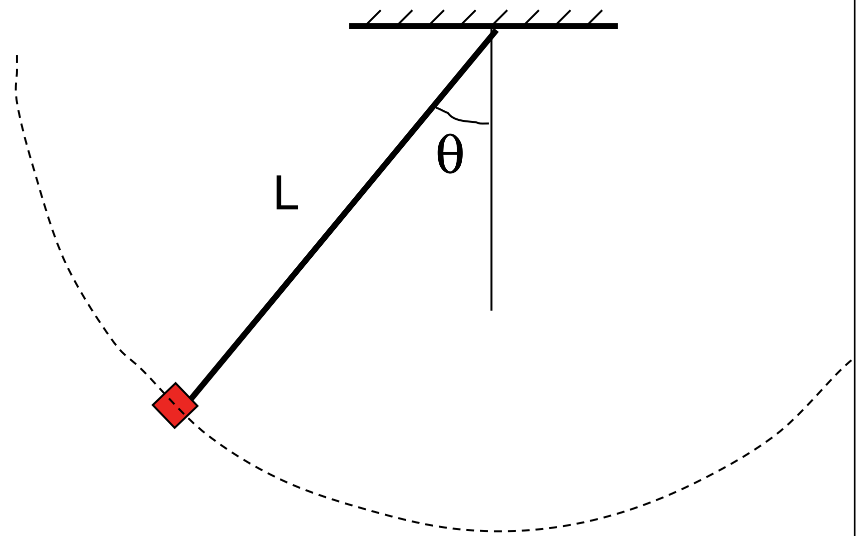
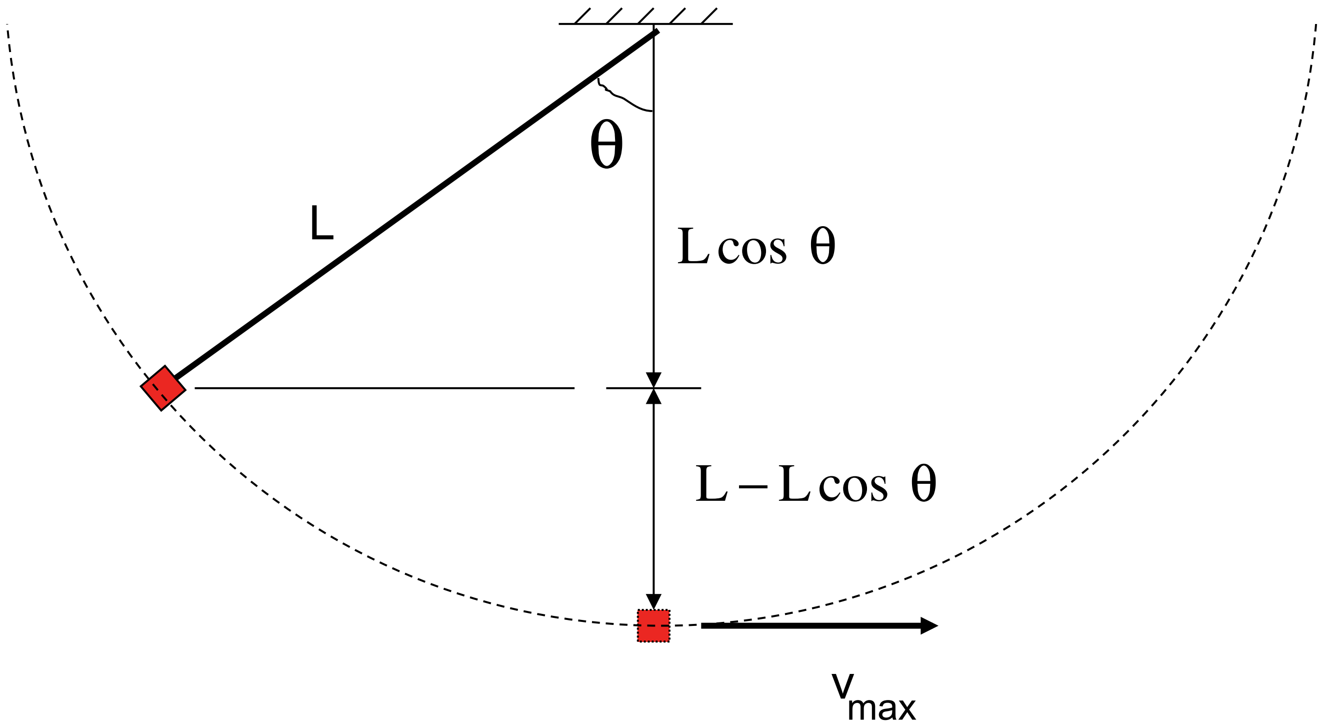
# Pendulum

A block of mass m = 3 kg is attached to a rope of length L=.8 meters that is, itself, attached to the ceiling. The maximum tension the rope can handle without breaking is 33 newtons. Assuming the bob starts from rest, at what angle $\theta$ must the bob start if it is to break the rope as it goes through the bottom of the arc?






The information provided above will be useful assuming the gravitational potential energy is defined as zero (i.e., y=0) at the bottom of the arc.

Noting that the bob is "falling" through a gravitational field, we can start with the conservation of energy used between the bob's start point and it's position at the bottom of the arc where we put y=0.

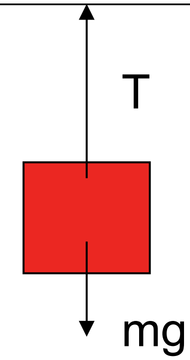
$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + mg(L\text{-}L\cos\theta) + 0 = \frac{1}{2}mv_2{}^2 + 0$$

$$\Rightarrow v^2 = 2gL(1-\cos\theta)$$

We somehow have to deal with the velocity squared term. Maybe noting that the body is following a curved path might motivate us to use N.S.L. coupled with a centripetal acceleration.

Doing so yields a f.b.d. as shown to the right and the following analysis:




$$\sum F_{c1}:$$

$$T - mg = +ma_c$$

$$= m\left(\frac{v^2}{L}\right)$$

Canceling the m's and noting that the radius of the bob's arc is just the length of the string, we can write:

$$v^2 = \frac{L}{m}(T - mg)$$

Combining the two velocity squared expressions, we get:

$$2g\cancel{L}(1-\cos\theta)=\frac{\cancel{L}}{m}\left(T-mg\right)$$

$$\Rightarrow \theta=\cos^{-1}\left[\frac{1}{2g}\left(2g-\frac{T}{m}+g\right)\right]$$

$$\Rightarrow \theta=\cos^{-1}\left[\frac{1}{2(9.8m/s^2)}\left(2(9.8m/s^2)-\frac{(33\text{ N})}{(3\text{ kg})}+(9.8m/s^2)\right)\right]$$

$$\theta\approx 19^{o}$$