Ice Dome Problem

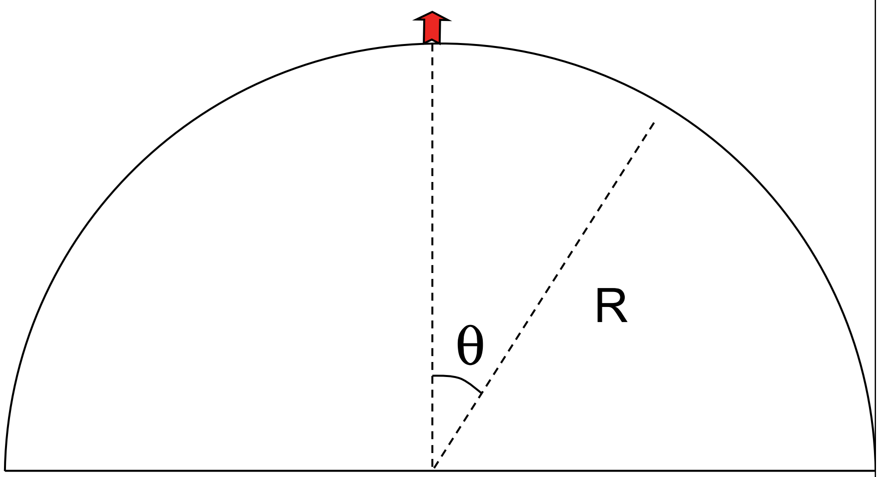
A boy of mass "m" sits directly atop a frictionless ice dome of radius "R." A tiny, tiny breeze passes by displacing the boy just a hair from the top. As a consequence, the boy begins to slide down the dome. At some unknown angle θ the boy leaves the dome. Determine that angle.

HINTS:

1.) Think about the forces that are acting on the boy just as he leaves the dome (i.e., just at the angle you are interested in).

2.) Think about energy considerations.

3.) Think about the KIND of motion you are observing. What is the boy physically doing, and what does that tell you about his acceleration?



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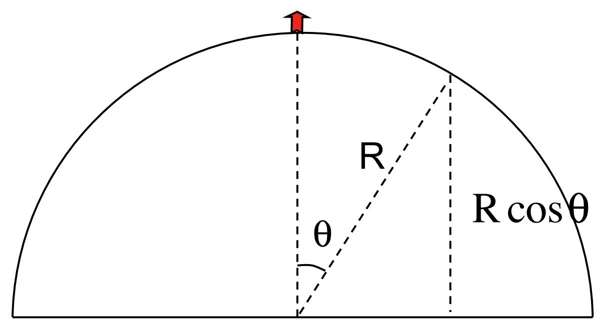
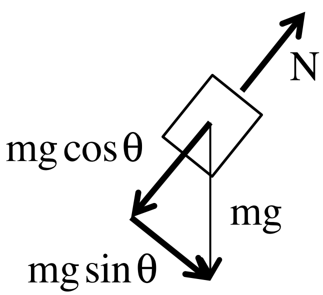
Assuming y=0 at floor level, conservation of energy yields:



$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 \quad + \quad mgR \quad + \quad 0 \quad = \frac{1}{2}mv^2 + mgR\cos\theta$$

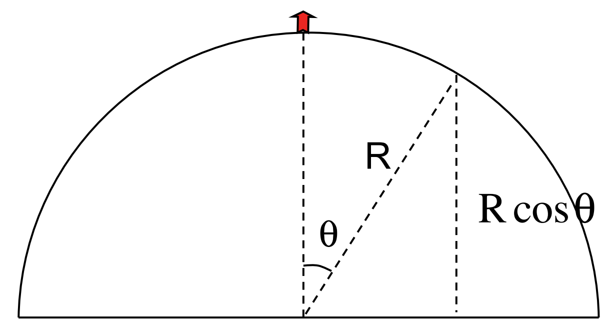
Newton's Second Law in the "centripetal" direction yields:



$$\sum F_c :$$

$$N - mg\cos\theta = -m\frac{v^2}{R}$$

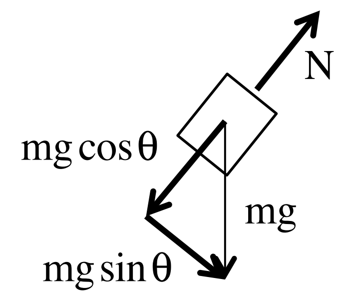
When the boy leaves the dome, the normal force goes to zero. That yields:



$$\text{as } N \Rightarrow 0:$$

$$\cancelto{0}{N} - mg\cos\theta = -m\frac{v^2}{R}$$

$$\Rightarrow \quad v^2 = gR\cos\theta$$



Substituting into the conservation of energy equation yields:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 \quad + \ mgR \ + \quad 0 \quad = \frac{1}{2}mv^2 + mgR\cos\theta$$

$$\Rightarrow \quad mgR = \frac{1}{2}m\left(gR\cos\theta\right) + mgR\cos\theta$$

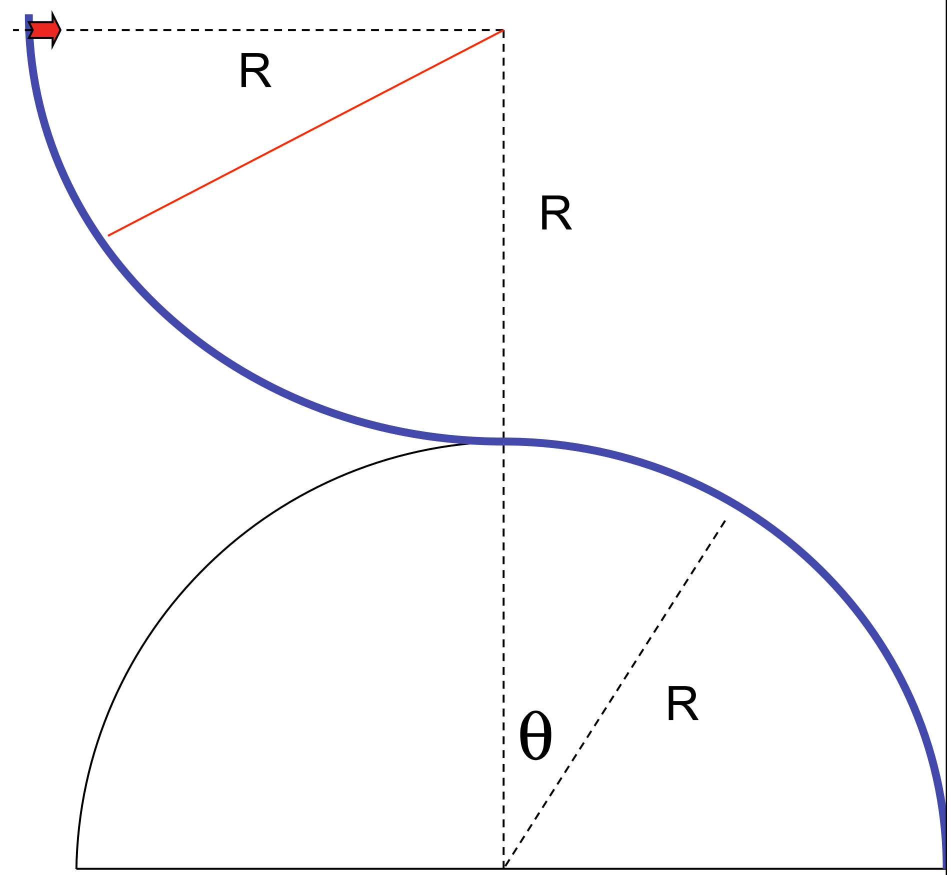
$$\Rightarrow \quad 1 = \frac{1}{2}\cos\theta + \cos\theta$$

$$\Rightarrow \quad 1 = \frac{3}{2}\cos\theta$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(2/3\right) = 48^{\circ}$$

Ice Dome With Twist

Same question with different start point and assuming there is friction that does work equal to .7mgR's worth over the motion:



*Interesting side note: If we tried this without the friction, we would end up with a final relationship that read:

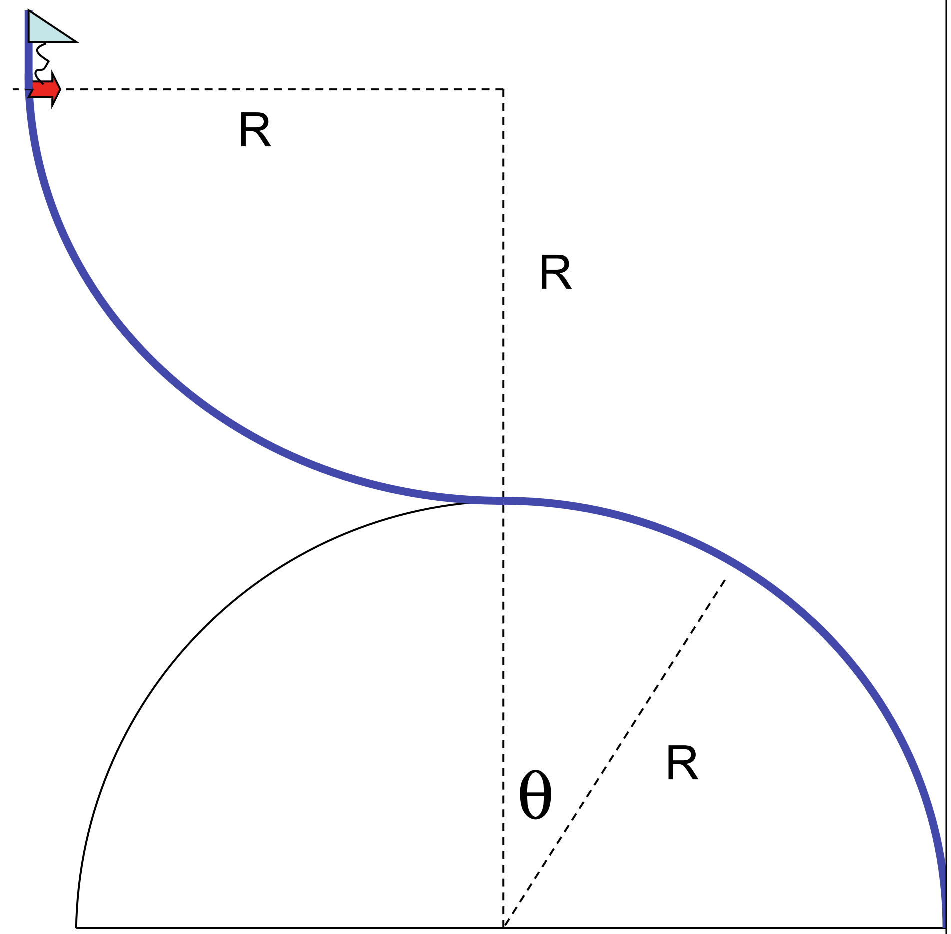
$$\cos\theta = \frac{4}{3}$$

This clearly can't be the solution (after all, the biggest the cosine of an angle can be is 1), so what's the deal?

If we had started the block up the incline at an angle of 60° from the vertical (the red line) , we'd have just enough velocity at the top of the dome to make the guy leave at that point. Starting at the top, in other words, would give the guy *too* much energy. The way the math let us know that we had done something dirty was to give us an nonsensical answer. Interesting, huh?

Ice Dome With Different Twist

Same question with different start point AND the mass shoved a known distance “x” up against a spring of known spring constant “k” at the start:



Ice Dome With Still Different Jello

The only difference here is that when the guy gets to the top of the dome, he ends up in a vat of jello that does mgR/8's worth of frictional work on him between the time he hits it and the time he leaves the dome?